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Equation of state in finite-temperature QCD with improved Wilson quarks*

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We study finite-temperature phase transition and equation of state for two-flavor QCD at $N_t = 4$ using an RG-improved gauge action and a meanfield-improved clover quark action. The pressure is computed using the integral method. The O(4) scaling of chiral order parameter is also examined.

1. Introduction

The transition temperature and the equation of state (EOS) of QCD at finite temperature belong to the most basic information for understanding the early Universe and heavy ion collisions. Full QCD studies of these quantities have been made mainly with the Kogut-Susskind quarks, particularly for the EOS [1]. In this paper we present the first result of the EOS from Wilson-type quarks.

We study two-flavor QCD on $N_t = 4$ lattices. In order to suppress lattice artifacts, which are known to be severe for the combination of the plaquette gauge and Wilson quark actions, we adopt a renormalization-group (RG) improved gauge action [2] combined with a meanfield-improved clover quark action. See Ref. [3] for details of our action.

In Fig. 1 we show the phase diagram with our action at $N_t = 4$. The line of finite-temperature transition is determined by the Polyakov loop and its susceptibility. The parity-broken phase [4] is not yet identified. Dashed lines are used in a test discussed in Sec. 2.

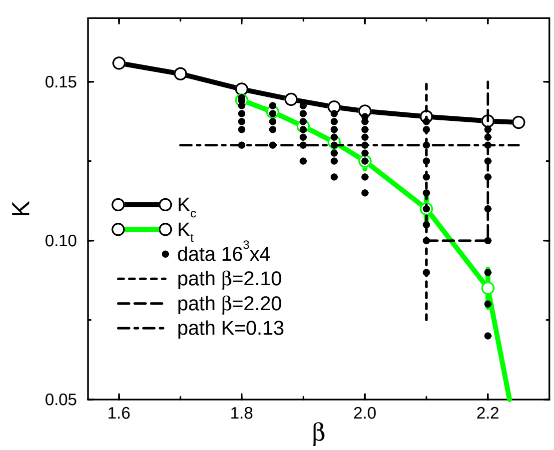

Figure 1. Phase diagram for $N_t = 4$. The solid line represents the critical line $K_c(\beta)$ of vanishing pion mass at $T = 0$. The shaded line $K_t(\beta)$ is the location of finite-temperature transition. Dots represent the simulation points on $16^3 \times 4$ and 16^4 lattices carried out so far in the present work.

*presented by S. Ejiri

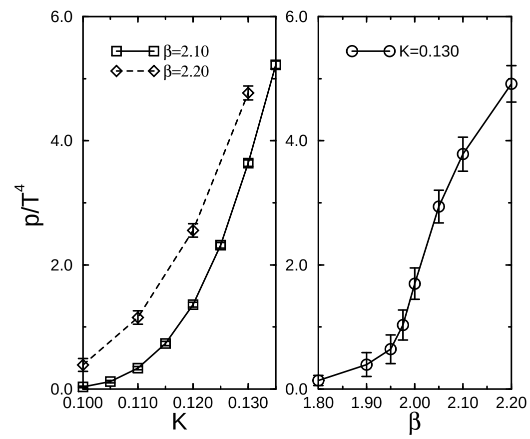

Figure 2. Pressure on an $8^3 \times 4$ lattice calculated from three paths in Fig. 1.

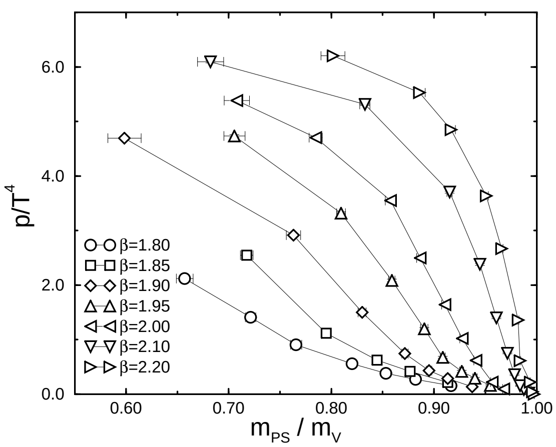

Figure 3. Pressure on a $16^3 \times 4$ lattice as a function of $m_{\rm PS}/m_{\rm V}|_{T=0}$.

2. Equation of state

We compute the pressure p by the integral method [5], which is based on the formula, valid for large homogeneous systems, that

$$\frac{p}{T^4} = -\frac{N_t^3}{N_s^3}\int^{(\beta,K)} \mathrm{d}\xi \left\{ \left\langle \frac{\partial S}{\partial \xi} \right\rangle - \left\langle \frac{\partial S}{\partial \xi} \right\rangle_{T=0} \right\}. \quad (1)$$

The integration path in the parameter space (β, K) should start from a point in the low temperature phase where the integrand approximately vanishes. We evaluate the quark contributions to the derivatives $\frac{\partial S}{\partial \beta}$ and $\frac{\partial S}{\partial K}$ by the method of noisy source using U(1) noise vectors.

The value for the pressure computed in (1) should be independent of the choice of the integration path. To check this point, we make a series of test runs on $8^3 \times 4$ and 8^4 lattices along three paths shown in Fig. 1, generating 500 HMC trajectories at each point. The results for p/T^4 obtained from these paths are plotted in Fig. 2. We find that p/T^4 at $(\beta, K) = (2.1, 0.13)$ and $(2.2, 0.13)$ in the two figures are in good agreement, confirming the path independence of the integral.

Encouraged by this result, we perform production runs on $16^3 \times 4$ and 16^4 lattices. At each dots plotted in Fig. 1, we generate 500–2000 trajectories on the $16^3 \times 4$ lattice and 200–300 trajectories on the 16^4 lattice. Measurement of the derivatives is made at every trajectory. Hadron propagators are calculated at every fifth trajectory to compute pseudo scalar ($m_{\rm PS}$) and vector (m_V) meson masses.

As is seen from Fig. 2, paths along the K-direction give much smaller errors in p/T^4 than those from paths in the β-direction. Therefore, we carry out the integral in the K-direction. We then obtain pressure plotted in Fig. 3 as a function of the mass ratio $m_{\rm PS}/m_{\rm V}$ at zero temperature. Interpolating these data, we find p/T^4 for each value of $m_{\rm PS}/m_{\rm V}$.

Figure 4 shows the pressure as a function of T/T_{pc} at fixed $m_{\rm PS}/m_{\rm V}$. Here T_{pc} is the pseudo-critical temperature at the same value of $m_{\rm PS}/m_{\rm V}$. The temperature scale is set by $m_{\rm V}$ through $T/T_{pc} = m_{\rm V}(\beta_{pc})/m_{\rm V}(\beta)$ with β_{pc} the pseudo-critical coupling.

We find that the pressure for fixed T/T_{pc} depends only weakly on the quark mass even for relatively heavy quark in the range $m_{\rm PS}/m_{\rm V} = 0.7$–$0.8$. For heavier quark masses, the pressure decreases toward the pure gauge value (dashed line) [6] as expected.

We also note that the magnitude of pressure is much larger than that for the pure gauge system for $N_t = 4$, and that it overshoots the Stefan-Boltzman value in the continuum at high temperatures. These features are probably the result of

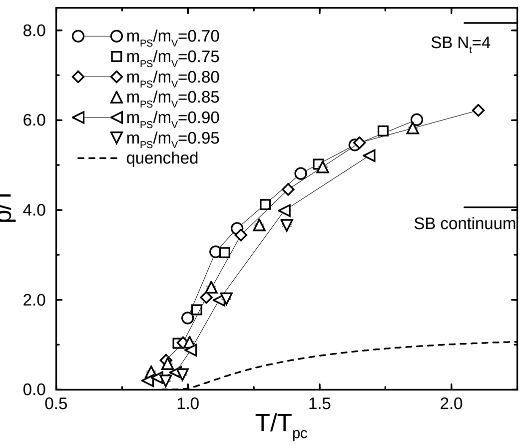

Figure 4. Pressure on a $16^3 \times 4$ lattice as a function of T/T_{pc}. The dashed curve shows pressure for pure gauge theory with the RG-improved action on a $16^3 \times 4$ lattice [6].

large discretization errors from the clover quark action here [1]. Indeed the large Stefan-Boltzman value on an $N_t = 4$ lattice shown at the top-right in Fig. 4 is dominated by the quark contribution.

3. O(4) Scaling

The chiral phase transition of two-flavor QCD is expected to belong to the universality class of O(4) spin system in three dimensions. In particular, identifying the magnetization, external magnetic field, and reduced temperature of the spin model with $M = \langle\bar{\Psi}\Psi\rangle$, $h = 2m_q a$, and $t = \beta - \beta_{ct}$, where β_{ct} is the chiral transition point, we expect a scaling relation

$$M/h^{1/\delta} = f(t/h^{1/\beta\delta}) \qquad (2)$$

to hold with the O(4) scaling function $f(x)$ [8] and the O(4) critical exponents β and δ [9]. A previous study using the RG-improved gauge action and unimproved Wilson quark action [7] found this relation to be well satisfied for the quark mass and the chiral condensate defined by axial Ward identities [10].

Figure 5 shows the result of a similar analysis from the present work. Data for $\langle\bar{\Psi}\Psi\rangle_{\rm sub} = 2m_q a(2K)^2 \sum_x \langle\pi(x)\pi(0)\rangle$ are fitted to the scaling relation, adjusting β_{ct} and the scales for t and h. The scaling ansatz works well, yielding $\beta_{ct} = 1.47(7)$ for the best fit of data for $2m_q a < 0.9$ and $\beta \leq 1.95$ with $\chi^2/{\rm df} = 1.1$.

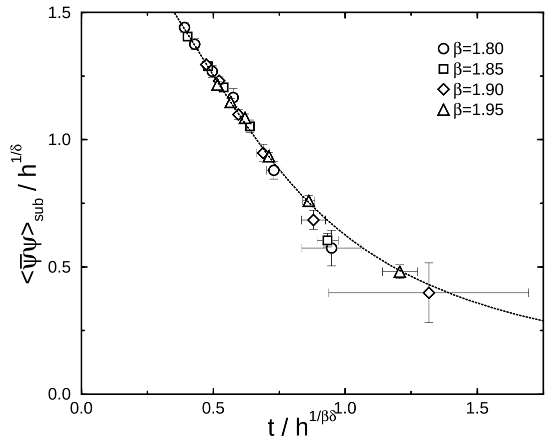

Figure 5. O(4) scaling relation.

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