Kaon B parameter from quenched domain-wall QCD *

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We report on a calculation of B_K with domain wall fermion action in quenched QCD. Simulations are made with a renormalization group improved gauge action at $\beta = 2.6$ and 2.9 corresponding to $a^{-1} \approx 2$GeV and 3GeV. Effects due to finite fifth dimensional size N_5 and finite spatial size N_σ are examined in detail. Matching to the continuum operator is made perturbatively at one loop order. We obtain $B_K(\mu = 2\text{GeV}) = 0.5746(61)$, where the error is statistical only, as an estimate of the continuum value in the $\overline{\text{MS}}$ scheme with naive dimensional regularization. This value is smaller but consistent with $B_K(\mu = 2\text{GeV}) = 0.628(42)$ obtained by the JLQCD Collaboration using the Kogut-Susskind quark action. Results for strange quark mass are also reported.

1. Introduction

The Kaon B parameter B_K is an important quantity to pin down the CKM matrix, thereby gaining an understanding on CP violation in the Standard Model.

A crucial ingredient in a precision calculation of B_K is chiral symmetry. For this reason, the best result so far has been obtained [1] with the Kogut-Susskind (KS) quark action, which keeps $U(1)$ subgroup of chiral symmetry at finite lattice spacings. Carrying out a systematic and extensive set of simulations to control $\mathcal{O}(a^2)$ scaling violation and $\mathcal{O}(\alpha^2_{\overline{\text{MS}}})$ errors that arise with the use of one-loop perturbative renormalization factors, the value $B_K(\mu = 2\text{GeV}) = 0.628(42)$ was obtained in the continuum limit in the $\overline{\text{MS}}$ scheme with naive dimensional regularization (NDR).

The Wilson fermion action has a problem of explicit chiral symmetry breaking, which causes a nontrivial mixing between four-quark operators with different chiralities. This problem has been treated with several non-perturbative renormalization methods [2–5]. The resultant values of B_K are consistent with that of the KS action, but the numerical error is large.

The domain wall fermion formalism offers a possibility of a calculation preserving full chiral symmetry [6]. Here we report a summary of a quenched calculation of B_K with the Shamir's variant [7] of the formalism and a renormalization group (RG) improved gauge action [8]. The latter choice is motivated by our result [9] that chiral symmetry is much better realized with this action than with the plaquette gauge action. We may also expect that scaling behavior of B_K is improved with the use of the RG-improved action. We also report on the strange quark mass obtained from meson mass measurements in our simulation.

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Table 1
Simulation parameters together with the number of configurations analyzed shown in bold letters.

	$\beta = 2.6$			$\beta = 2.9$	
a^{-1}(GeV)	1.81(4)			2.87(7)	
N_t	40			60	
N_σ	16	24	32	24	32
$N_5 = 16$	**122**	**76**	**25**	**76**	**50**
$N_5 = 32$	–	**50**	–	–	–

2. Numerical simulation

Simulations are made at two values of coupling, $\beta = 2.6$ and 2.9 corresponding to a lattice spacing $a^{-1} = 1.81(4)$ GeV and 2.87(7) GeV determined by ρ meson mass, in order to check the scaling behavior.

The choice of lattice size $N_\sigma^3 \times N_t \times N_5$ is made as follows: (i) Our main runs use $24^3 \times 40 \times 16$ at $\beta = 2.6$, and $32^3 \times 60 \times 16$ at $\beta = 2.9$. (ii) Spatial size dependence is examined at $\beta = 2.6$ varying the spatial size from $N_\sigma = 24$ to $N_\sigma = 16$ and 32: They correspond to the physical size $aN_\sigma \sim 1.7, 2.6, 3.4$ fm. (iii) Dependence on the fifth dimensional length N_5 is also examined at $\beta = 2.6, N_\sigma = 24$ using $N_5 = 16$ and $N_5 = 32$.

We choose $m_f = 0.01, 0.02, 0.03, 0.04$ for the bare quark mass, covering the range $m_{PS}/m_V = 0.4 - 0.8$. The u-d and s quarks are assumed degenerate in the analysis for B_K. The domain wall height is taken to be $M = 1.8$. The number of configurations analyzed is given in Table 1.

We measure the Kaon B parameter,

$$B_K = \frac{\langle K\,|\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d|\,K\rangle}{\frac{8}{3}\langle K\,|\bar{s}\gamma_\mu\gamma_5 d|\,0\rangle\,\langle 0\,|\bar{s}\gamma_\mu\gamma_5 d|\,K\rangle}, \qquad (1)$$

and the matrix element divided by the pseudo scalar density,

$$B_P = \frac{\langle K\,|\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d|\,K\rangle}{\langle K\,|\bar{s}\gamma_5 d|\,0\rangle\,\langle 0\,|\bar{s}\gamma_5 d|\,K\rangle} \qquad (2)$$

which should vanish at $m_\pi \to 0$. The extraction of these quantities from Kaon Green functions follows the standard procedure (see, *e.g.,* Ref. [1]) using the Dirichlet boundary condition in time and wall sources for quark propagators. We simultaneously evaluate hadron masses. The physical point for quark masses is calculated by linearly fitting the light hadron masses m_{PS}^2 and m_V as a function of m_f, and using the experimental values of m_π/m_ρ, m_K/m_ρ as input.

3. Operator matching

We carry out matching of the lattice and continuum operators at a scale $q^* = 1/a$ using one-loop perturbation theory [10] and the $\overline{\rm MS}$ scheme with NDR in the continuum. In the domain wall formalism the renormalization factor of an n-quark operator O_n has a generic form

$$Z = (1-w_0^2)^{n/2} Z_w^{n/2} Z_{O_n} \qquad (3)$$

where $w_0 = 1 - M$ and $Z_{w,O_n} = 1 + O(g^2)$. We apply tadpole improvement by explicitly moving the one-loop correction to the domain wall height M from Z_w to w_0, and by factoring out a tadpole factor $u^{n/2} = P^{n/8}$ with P the plaquette from Z_{O_n}. A mean-field estimate appropriate for the RG-improved action is used for calculating the coupling constant $g^2_{\overline{\rm MS}}(\mu)$ [11]. The continuum value at a physical scale *e.g.,* $\mu = 2.0$ GeV, is obtained *via* a renormalization group running from $q^* = 1/a$ to μ.

For B_K the factor $(1-w_0^2)^2 Z_w^2$ cancels out, and the one-loop value given by the ratio

$$\frac{Z_{O_4}}{Z_A^2} = \frac{1 - (4\log q^* a + 13.6)g^2/(16\pi^2)}{(1 - 6.26 g^2/(16\pi^2))^2} \qquad (4)$$

turned out to be very near unity *i.e.,* $Z_{B_K}^{\overline{\rm MS}}(q^* = 1/a) = 0.984(\beta = 2.6)$ and $0.988(\beta = 2.9)$. The Z factor at the scale $\mu = 2$GeV obtained with a 2-loop running becomes $Z_{B_K}^{\overline{\rm MS}}(\mu = 2\text{GeV}) = 0.979(\beta = 2.6)$ and $1.006(\beta = 2.9)$.

4. Results for B_K

4.1. Chiral property

We start by investigating the chiral property of the system through B_P. The renormalized value of B_P, extrapolated to $m_f = 0$ linearly in m_f, is plotted as a function of the inverse of spatial size in Fig. 1.

Since results for the fifth dimensional size $N_5 = 16$ (filled circle) and 32 (open square) agree with each other, the observed non-zero value, albeit

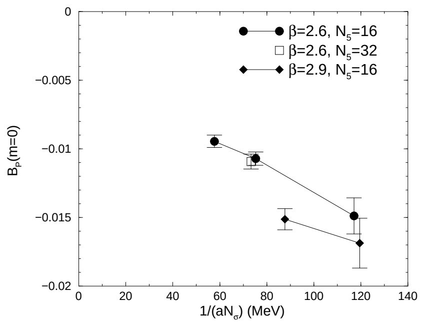

Figure 1. The matrix element B_P at $m_f = 0$ as a function of spatial size.

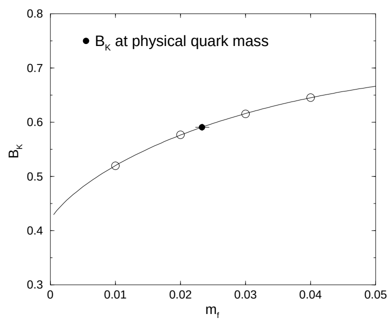

Figure 2. Bare B_K interpolated as a function of $m_f a$ at $\beta = 2.6$ for a $24^3 \times 40 \times 16$ lattice.

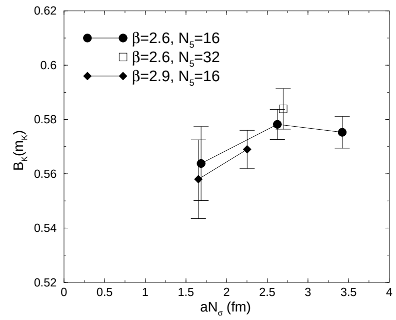

Figure 3. Renormalized B_K as a function of spatial size.

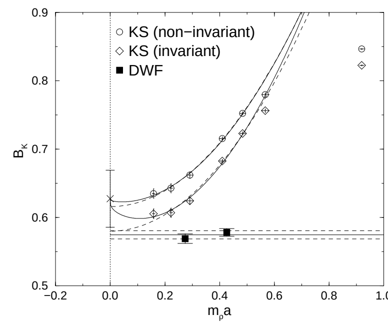

Figure 4. Scaling behavior of B_K. Previous results with the KS action[1] are also shown.

very small, should be due to a finite spatial-size effect. The decrease of the magnitude of B_P toward larger spatial sizes supports this interpretation. The existence of finite-size effect is consistent with the result for the pion mass in Ref. [9], where a non-zero pion mass of a magnitude expected from finite-size effects observed for the KS fermion was found for $N_5 \geq 16$ at $\beta = 2.6$. It is a future problem to see whether and how values of B_P vanish toward the infinite volume limit.

4.2. B_K

The bare value of B_K is interpolated as a function of $m_f a$ with the formula given by chiral perturbation theory,

$$B_K = B\,(1 - 3c(m_f a)\log(m_f a) + b(m_f a)) \qquad (5)$$

as shown in Fig. 2. The physical value of B_K is obtained at half the strange quark mass $m_s a/2$ (solid circle in Fig. 2) which is estimated from the experimental value of m_K/m_ρ.

We plot the renormalized value of $B_K(\mathrm{NDR}; \mu = 2\mathrm{GeV})$ as a function of the spatial size in Fig. 3. Filled circles and diamonds are data at $\beta = 2.6$ and 2.9 keeping the same fifth dimensional size $N_5 = 16$. The spatial size dependence is mild especially for $N_\sigma a \geq 2.6$ fm at $\beta = 2.6$. This result is consistent with that of a previous finite spatial size study with the staggered quark action [1]. We conclude that the size of about 2.4 fm used in our main runs is sufficient to avoid spatial size effects for B_K.

In Fig. 3 an open square shows the result for $N_5 = 32$ on a $24^3 \times 40$ four-dimensional lattice at $\beta = 2.6$. Since this data is consistent with that at $N_5 = 16$ within the statistical error, the fifth dimensional size of $N_5 = 16$ is sufficient for B_K.

Our final results from the main runs are shown in Fig. 4. We observe a good scaling behavior, and making a constant extrapolation $B_K(a) = B_K$, we find $B_K(\mu = 2\mathrm{GeV}) = 0.5746(61)$ where the error is statistical only. The open symbols and the associated lines represent results from a previous calculation with the staggered quark action

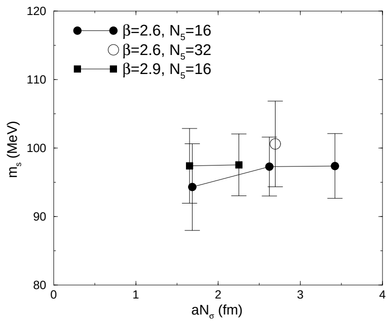

Figure 5. Renormalized strange quark mass as a function of spatial size.

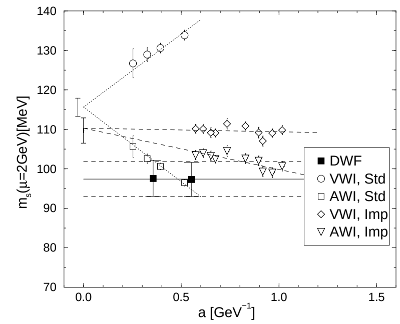

Figure 6. Scaling behavior of the strange quark mass compared with those from the standard (Std) [13] and an improved (Imp)[11] 4-d actions. VWI and AWI represent vector and axial-vector Ward-identity masses, respectively.

[1], where gauge invariant and non-invariant four-quark operators are used. Compared to the result in the continuum limit $B_K = 0.628(42)$ from this work, our present result is smaller but consistent within the error.

5. Strange quark mass

We show results for the strange quark mass in the $\overline{\rm MS}$ scheme at $\mu = 2$ GeV in Fig. 5, where m_K is used as input. We observe that the fifth dimensional size $N_5 = 16$ and spatial size $N_\sigma a = 2.4$ fm is sufficient for a reliable determination of strange quark mass, giving $m_s \approx 97$ MeV. A recent result $m_s = 110(2)(22)$ MeV [12] using the domain wall fermions but with the plaquette gauge action at $\beta = 6.0$ is also consistent with ours, if the systematic error from the conversion to the $\overline{\rm MS}$ scheme is taken into account.

Our results exhibit a good scaling behavior as shown in Fig. 6. Making a constant fit yields $m_s^{\overline{\rm MS}}(2{\rm GeV}) = 97.4(4.4)$ MeV. Compared with results from the conventional 4-dimensional formalisms (the plaquette and Wilson quark action [13] and an RG-improved gauge action and the clover quark action [11]), our result is 3–4σ (13–18 MeV) smaller. Reduction of systematic errors from one-loop renormalization factors and finite-size effects would be needed to see if there is an actual disagreement.

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