# Chapter 5

# On Abduction

## 5.1 Introductory remarks

Abduction is reasoning to build an explanation for an observation assuming appropriate hypotheses from incomplete knowledge, which is sometimes incorrect due to inconsistency, when the observation is not explainable by complete knowledge, which is always true[32]. Recently abduction on logic programs and its semantics are widely studied[33, 14], probably because of relationship between nonmonotonicity of abduction and meaning of negation in the logic programs. However they have not found any unique applications yet, in contrast to abduction on propositional logic that is being applied to diagnosis and design[9, 10, 11, 43]. In this chapter, the author is creating an application of the approach by implementing analogical reasoning, one form of common sense reasoning, by means of abduction.

Analogical reasoning has long been a subject of study[74]. Recently, numerous studies have focused on the theoretical aspects of analogical reasoning from a logical viewpoint[20, 19, 8]. Among these, the formulation using determination rules[8] has the advantage of offering clear semantics, as it is defined within deduction. The determination rule is a deductive rule as the following.

$$\begin{aligned}\Sigma[x,y] \succ X[x,z] \quad & \text{iff} \\ & \forall y,z(\exists x(\Sigma[x,y] \wedge X[x,z]) \supset \forall x(\Sigma[x,y] \supset X[x,z]))\end{aligned}$$

At the same time, however, ordering of analogy[19] cannot be discussed within this framework. Under [8], all analogically inferred facts are equally plausible, so that the concept of plausibility ordering does not apply.

The special feature of analogical reasoning is that it has a duplicated manner of interpretation. The meaning of analogical reasoning in the given theory is determined depending on the meaning of the theory, which is interpreted without analogical reasoning.

The meaning of the determination rule is "if there is $x$, which has a value $y$ for an attribute $\Sigma$ and has a value $z$ for an attribute $X$, then all the individuals which have the value $y$ for $\Sigma$ have the value $z$ for $X$". In the determination-rule formulation, this semantic information is crystallized into the determination rules. The intuition on analogical reasoning shown above gives a more important suggestion on formulating analogical reasoning. Namely, it suggests the method to define the meaning of analogical reasoning as the meaning of the theory which is extended from the original one. The extension is performed by adding some knowledge which is determined depending on the meaning of the original theory.

In this chapter, a method for regarding analogical reasoning as abduction is proposed. First, an intuitive illustration of the basic ideas is provided. The simplest analogical reasoning is described by the following example[8].

$$\frac{p(s) \wedge q(s) \qquad p(t)}{q(t)}$$

The inference here might be interpreted as "If you know $p(s) \wedge q(s)$ for an individual $s$, then you can conclude $p(x) \supset p(x) \wedge q(x)$." The determination rules are a natural formulation of this intuition. The justification for analogical reasoning is provided by the presence of the determination rules[8]. But this formulation of analogical reasoning is unsatisfactory, because the determination rules are too strong deductive rules. Furthermore, it seems to be difficult to find practically useful determination rules.

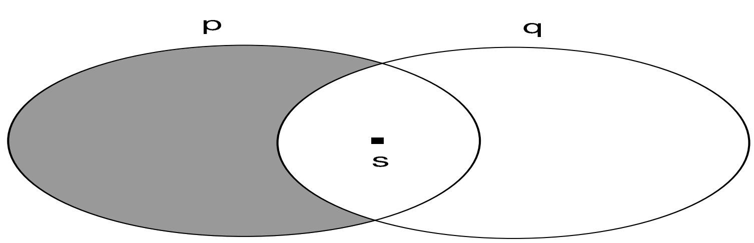


Figure 5.1: Intuitive meaning of analogical reasoning

Let us consider the above example in the following way. "Something, which is $p$, will seem to be $q$, except in the case I know that it is $p$ and that it isn't $q$." Namely, defining predicate $\forall x(ab(x) \Leftrightarrow p(x) \wedge \neg q(x))$, we can infer $q(t)$ by supposing $\neg ab(t)$. This is obvious because $\forall x(p(x) \wedge \neg ab(x) \supset q(x))$. This coincides with minimization of the shaded area in figure 5.1.

The intuitive meaning of predicate *ab* is "That which is $p$ is almost always $q$; exceptions may exist, however, which we call *ab*." The meaning of *ab* is determined according to the meaning of the original theory and it affects the meaning of analogical reasoning in the theory. This coincides with the intuition for analogical reasoning described above.

Based on this viewpoint, we can say that analogical reasoning involves:

- adding definitions of *ab*-like predicates, together with new knowledge that use negations of *ab*-like predicates (i.e., $\forall x(p(x) \wedge \neg ab(x) \supset q(x))$), into the original theory;

- reasoning hypothetically using hypotheses composed of negated *ab*-like predicates.

It is necessary to be able to handle negation within the framework. Here, the framework of abduction proposed in [33] is employed. In this approach, the meanings of abductive frameworks are given by their generalized stable models (GSMs). If we can regard analogical reasoning as a form of abduction, then we can give meaning of analogical reasoning by the GSMs.

The analogical reasoning represented by this formulation has the following meaning: If two individuals share an arbitrary attribute, then all other attributes possessed by one individual are also shared by another (there may be some exceptions). This can be said as a loose framework, because this allows much analogical reasoning with relative small amount of ground. Consequently, it will face the justification problem. But to study which result of analogical reasoning in this formulation is justifiable is effective to clarify the nature of the justification problem, because all styles of analogical reasoning are included by this formulation. Employing the framework of abduction, all assumptions used during analogical reasoning are explicitly kept as hypotheses, which help those studies.

The idea to formulate analogical reasoning by minimizing difference between two attributes are shown in [3], which is based on circumscription. This study differs from that work in the point that it is utilizing hypotheses employed during minimization, in order to clarify the property of analogical reasoning.

In this chapter, it is first described that the method to give semantics for analogical reasoning as GSMs. Several properties of such GSMs are investigated. The procedure for an analogical proof is introduced, and relation between the procedure and justification methods is discussed. This suggests to acquire knowledge by adjusting hypotheses generated by the procedure.

## 5.2 Analogical reasoning as a form of abduction

In this section, we briefly review the framework of abduction given in[33]. Then the method for representing analogical reasoning by means of the abductive framework is illustrated, defining *ab*-like predicates such as described in the previous section. The object theory is locally stratified logic programs that only use unary predicates. This simple object is suitable for the chapter, which is not to construct practical systems but to investigate characteristics of analogical reasoning.

### 5.2.1 Generalized stable model semantics for abduction

**Definition 5.2.1 (Abductive framework[33])** An abductive framework *is a triple* $\langle L, A, IC\rangle$, *where* $L$ *is a normal logic program[42] which has no head with a predicate in* $A$*;* $A$ *is a set of* abducible predicates*; and* $IC$ *are* integrity constraints*, which is a set of closed formulae. All ground atoms composed of the element of* $A$ *are* abducibles.

**Definition 5.2.2 (Pre-GSM for an abductive framework[33])** *Let* $\langle L, A, IC\rangle$ *be an abductive framework and* $\Delta$ *be a set of abducibles. Then a pre-generalized stable model(pre-GSM),* $M(\Delta)$*, is a stable model[15] of* $L \cup \Delta$.

A pre-GSM is a stable model of the program $L$, where $L$ has been extended by the hypotheses $\Delta$. $IC$ is not taken into account.

**Definition 5.2.3 (GSM[33])** *Let* $\langle L, A, IC\rangle$ *be an abductive framework and* $\Delta$ *be a set of abducibles. Then a generalized stable model(GSM),* $M(\Delta)$*, is a pre-GSM such that for each* $\psi$ *in* $IC$*,* $M(\Delta) \models \psi$.

The semantics of $\langle L, A, IC\rangle$ is the set of all of its GSMs[33].

An interesting relation between abduction and negation as failure is shown in [14]. In [33], a new abductive framework$\langle L^*, A \cup A^*, IC \cup IC^*\rangle$[1] is constructed from $\langle L, A, IC\rangle$, where $L^*$ is a program in which all occurrences of $\neg p$ in $L$ are replaced by $p^*$; $A^*$ is the set of such $p^*$s; and $IC^*$ is the set of closed formulae $\forall x \neg(p(x) \wedge p^*(x)), \forall x(p(x) \vee p^*(x))$ for all elements $p^*$ of $A^*$[2]. Intuitively, $p^*$ represents the negation as failure of $p$. [33] shows that there

[1] Referred to as the *transformed framework* of the original.

[2] In the remaining part of this chapter, we call integrity constraints with this form as integrity constraints of $A^*$.

exists a one-to-one correspondence between the set of GSMs of $\langle L, A, IC\rangle$ and the set of GSMs of $\langle L^*, A \cup A^*, IC \cup IC^*\rangle$.

We will show an example of GSM semantics. In the example, because $L$ is a locally stratified program[66], $L$'s unique stable model is identical to its perfect model([15], Corollary 1). Note that the GSM semantics gives meaning to the effect that the predicates which occur in the body with negation represent unusual cases, just as with perfect model semantics[66].

**Example 5.2.4** *Suppose the following logic program $L$ is given:*

$$\begin{array}{l} robin(a) \\ robin(b) \\ injured(a) \\ fly(x) \leftarrow robin(x), \neg injured(x) \end{array}$$

*Transformed framework for $L$ is $F = \langle L^*, A^*, IC^*\rangle$ where $L^*$ is the program*

$$\begin{array}{l} robin(a) \\ robin(b) \\ injured(a) \\ fly(x) \leftarrow robin(x), injured^*(x) \end{array}$$

*and*

$$\begin{array}{rl} A^* = & \{injured^*\} \\ IC^* = & \{\forall x \neg(injured(x) \wedge injured^*(x)), \\ & \forall x(injured(x) \vee injured^*(x))\} \end{array}$$

*Pre-GSMs for $F$ are:*

$$\begin{array}{rl} M(\Delta_1) = & \{robin(a), robin(b), injured(a)\} \\ \Delta_1 = & \phi \\ M(\Delta_2) = & \{robin(a), robin(b), injured(a), injured^*(a), fly(a)\} \\ \Delta_2 = & \{injured^*(a)\} \\ M(\Delta_3) = & \{robin(a), robin(b), injured(a), injured^*(b), fly(b)\} \\ \Delta_3 = & \{injured^*(b)\} \\ M(\Delta_4) = & \{robin(a), robin(b), injured(a), injured^*(a), injured^*(b), fly(a), fly(b)\} \\ \Delta_4 = & \{injured^*(a), injured^*(b)\} \end{array}$$

*Only GSM for $F$ is $M(\Delta_3)$.*

### 5.2.2 How to convert analogical reasoning into an abductive framework

This subsection shows how to deal with analogical reasoning for a given logic program by means of abductive framework. The basic idea is to apply GSM semantics to the transformed framework of extended program by means of the rules which has negation of *ab*-like predicates (as introduced in section 5.1), thereby obtaining models in which *ab*-like predicates represent unusual cases.

For the remainder of this chapter, we will assume that the given logic program $L$ is locally stratified[66]. This means that $L$ possesses a unique stable model([15], Corollary 1). Thus, by definition of GSM, the abductive framework $E = \langle L, \phi, \phi \rangle$ possesses a unique GSM.

Because there exists a one-to-one correspondence between the set of GSMs of abductive framework and the set of GSMs of its transformed framework, we can show the following corollary.

**Corollary 5.2.5** *If logic program $L$ is locally stratified, then the transformed framework $F = \langle L^*, A^*, IC^* \rangle$ of the abductive framework $E = \langle L, \phi, \phi \rangle$ has a unique GSM.*

The uniqueness of the GSM of given program's transformed framework is important. In the subsequent formulation of analogical reasoning, we will determine the abductive framework that takes analogical reasoning into account according to the meaning of the program. Therefore, if there are several meanings of the program, then we will get plural abductive frameworks for analogical reasoning. Although it would be interesting to consider how we might select, from among several possible meanings for the program, the meaning on which we would analogically reason, and the way in which this selection would affect the result of the reasoning, we will not deal with these issues here.

Through the reminder of this chapter we will not distinguish between the logic program $L$ and the abductive framework $\langle L, \phi, \phi \rangle$.

**Definition 5.2.6 (A list of attributes, LA)** *Let $L$ be a given logic program, and let $\mathcal{M}$ be the GSM of $L$'s transformed framework $\langle L^*, A^*, IC^* \rangle$. If there exists a set of predicates $P = \{p_1, \ldots, p_n\}(n \geq 2)$ in $L$ and $\mathcal{M} \models p_1(h) \wedge \ldots \wedge p_n(h)$ for a ground term $h$, then $P$ is called* a list of attributes(LA)[3] *of $L$ by $h$.*

[3] LA was called as *class* in the previous paper[64]. The name of this concept is changed in order to avoid confusion in terminology.

A LA denotes attributes which are possessed by specific individuals.

**Definition 5.2.7 (Analogical addenda)** *Let $L$ be a given logic program. If $P = \{p_1, \ldots, p_n\}$ is a LA of $L$, then the* analogical addenda *for $L$ with respect to $P$ are as follows.*

$$\bigcup_{p_i \in P} \{a_{Pp_i}(x) \leftarrow p_1(x), \ldots, p_{i-1}(x), p_{i+1}(x), \ldots, p_n(x), p_i^*(x)\} \quad \cup$$
$$\bigcup_{p_i \in P} \{p_i(x) \leftarrow p_1(x), \ldots, p_{i-1}(x), p_{i+1}(x), \ldots, p_n(x), a_{Pp_i}^*(x)\}$$

*(where $a_{Pp_i}$ is a predicate which does not occur in $L$)*

*Let $G$ be the union of all analogical addenda for $L$ with respect to all LAs of $L$ by each individual constants, let $A_G$ be the set of predicates in $G$ with $^*$, and let $IC_{A_G}$ be the integrity constraints for $A_G$[33]. Then $\langle L^* \cup G, A^* \cup A_G, IC^* \cup IC_{A_G}\rangle$ is called* a framework with analogical addenda *upon $L$, and is denoted by $FA(L)$. $a_{Pp_i}$ is referred to as* a predicate representing $p_i$-exception with respect to $P$.

Analogical addenda with respect to $P$ are composed of the following.

1. The definitions of predicates $a_{Pp_i}$, which represents unusual cases with respect to $P$.

2. The rules which conclude attributes $p_i$ in $P$, using negation of $a_{Pp_i}$ and other attributes.

**Definition 5.2.8 (Strongly analogical addenda)** *Let $L$ be a given logic program. If $P = \{p_1, \ldots, p_n\}$ is a LA of $L$ by $h$, then the* strongly analogical addenda *for $L$ with respect to $P$ are as follows.*

$$\bigcup_{p_i \in P} \{p_i(x) \leftarrow p_1(x), \ldots, p_{i-1}(x), p_{i+1}(x), \ldots, p_n(x), a_{Pp_i}^*(x)\}$$

*(where $a_{Pp_i}$ is a predicate which does not occur in $L$)*

*Let $G$ be the union of all strongly analogical addenda for $L$ with respect to all LAs of $L$ by each individual constants, let $A_G$ be the set of predicates in $G$ with $^*$, and let $IC_{A_G}$ be the integrity constraints for $A_G$. Then $\langle L \cup G, A_G, IC_{A_G}\rangle$ is called* a framework with strongly analogical addenda *upon $L$, and is denoted by $FSA(L)$. $a_{Pp_i}$ is referred to as* a predicate representing $p_i$-exception with respect to $P$.

Strongly analogical addenda with respect to $P$ are rules which conclude attributes $p_i$ in $P$, using negation of $a_{Pp_i}$. When $P$ has three or more elements, the rules, which is used to reason an existing attribute using two or more hypotheses, seem to lack in the above definition. In fact, such cases are covered by the (strongly) analogical addenda with respect to subsets of $P$, which are also LAs. Namely, the effect of the rule

$$p(x) \leftarrow p_1(x), a^*_{\{p_1,p_2,p\}p}(x), a^*_{\{p_1,p_2\}p_2}(x)$$

is accomplished by rules

$$\begin{array}{l} p_2(x) \leftarrow p_1(x), a^*_{\{p_1,p_2\}p_2}(x) \\ p(x) \leftarrow p_1(x), p_2(x), a^*_{\{p_1,p_2,p\}p}(x) \end{array}$$

**Example 5.2.9** *Suppose the following logic program $L$ is given:*

$$\begin{array}{l} p(a) \\ q(a) \\ p(b) \end{array}$$

*The GSM of $L$'s transformed framework is $\{p(a), q(a), p(b)\}$.*
*Only LA for $L$ is $P = \{p, q\}$.*
*Analogical addenda $G$ for $L$ with respect to $P$ are:*

$$\begin{array}{l} a_p(x) \leftarrow q(x), p^*(x) \\ a_q(x) \leftarrow p(x), q^*(x) \\ p(x) \leftarrow q(x), a_p^*(x) \\ q(x) \leftarrow p(x), a_q^*(x) \end{array}$$

$$\begin{array}{rll} A_G = & \{p^*, q^*, a_p^*, a_q^*\} & \\ IC_{A_G} = \{ & \forall x \neg (p(x) \wedge p^*(x)), & \forall x (p(x) \vee p^*(x)), \\ & \forall x \neg (q(x) \wedge q^*(x)), & \forall x (q(x) \vee q^*(x)), \\ & \forall x \neg (a_p(x) \wedge a_p^*(x)), & \forall x (a_p(x) \vee a_p^*(x)), \\ & \forall x \neg (a_q(x) \wedge a_q^*(x)), & \forall x (a_q(x) \vee a_q^*(x))\} \\ FA(F) = & \langle L \cup G, A_G, IC_{A_G} \rangle & \end{array}$$

*Strongly analogical addenda $SG$ for $L$ with respect to $P$ are:*

$$\begin{array}{l} p(x) \leftarrow q(x), a_p^*(x) \\ q(x) \leftarrow p(x), a_q^*(x) \end{array}$$

$$\begin{array}{rll} A_{SG} = & \{a_p^*, a_q^*\} & \\ IC_{A_{SG}} = \{ & \forall x \neg (a_p(x) \wedge a_p^*(x)), & \forall x (a_p(x) \vee a_p^*(x)), \\ & \forall x \neg (a_q(x) \wedge a_q^*(x)), & \forall x (a_q(x) \vee a_q^*(x))\} \\ FSA(L) = & \langle L \cup SG, A_{SG}, IC_{A_{SG}} \rangle & \end{array}$$

The meaning of analogical reasoning for logic program $L$ is given by (a subset of) the set of GSMs of $FA(L)$.

## 5.3 GSM semantics for analogical reasoning

### 5.3.1 Basic definitions

Generally, a framework with analogical addenda has plural GSMs. For the ground atom $p(h)$ whose truth value is not decidable only from $L \cup D \cup G$, these GSMs correspond to a case that $a^*_{Pp}(h)$ is hypothesized and a case that $p^*(h)$ is hypothesized.

**Example 5.3.1** *In the example 5.2.9, $FA(L)$ has the following two GSMs.*
$\{p(a), p(b), q(a), q(b), a^*_p(a), a^*_p(b), a^*_q(a), a^*_q(b)\}$
*(corresponds to $\Delta = \{a^*_p(a), a^*_p(b), a^*_q(a), a^*_q(b)\}$)*
$\{p(a), p(b), q(a), q^*(b), a^*_p(a), a^*_p(b), a^*_q(a), a_q(b)\}$
*(corresponds to $\Delta = \{q^*(b), a^*_p(a), a^*_p(b), a^*_q(a)\}$)*

As described intuitively in section 5.1, analogical reasoning minimizes unusual cases with respect to LAs. Semantics for analogical reasoning must identify such cases. For the above example, $a_q(b)$ holds in the second GSM but does not in the first GSM. We can say the first GSM gives the semantics for analogical reasoning.

**Definition 5.3.2 (Analogical GSM)** *Let $L$ be a given logic program and $F = \langle L^*, A^*, IC^*\rangle$ its transformed framework. Also let $FA(L) = \langle L^* \cup G, A^* \cup A_G, IC^* \cup IC_{A_G}\rangle$. Let $\mathcal{M}$ be a GSM of $F$. If a GSM of $FA(L)$ $\mathcal{M}'$ is a superset of $\mathcal{M} \cup \{u\}$, such that $u \in B_{L^*}$ and $u \notin \mathcal{M}$, then $\mathcal{M}'$ is* an analogical GSM *of $FA(L)$. ($B_{L^*}$ stands for the Herbrand base for $L^*$.)*

There are two intuitions in the above definition.

First, $\mathcal{M}' \supseteq \mathcal{M}$ guarantees *conservativeness* of analogical reasoning. For $FA(L)$, its GSM $\mathcal{M}'$ is called *conservative*, iff $\mathcal{M}' \supseteq \mathcal{M}$, where $\mathcal{M}$ shows GSM of $L$'s transformed framework.

Second, existence of $u$ means that there is at least one atom which is analogically inferred, and cannot be deductively inferred.

**Example 5.3.3** *In the example 5.3.1, $FA(L)$ has the following unique analogical GSM:*
$\{p(a), p(b), q(a), q(b), a^*_p(a), a^*_p(b), a^*_q(a), a^*_q(b)\}$
*(corresponds to $\Delta = \{a^*_p(a), a^*_p(b), a^*_q(a), a^*_q(b)\}$)*
*Note $q^*(b) \notin B_{L^*}$.*

**Definition 5.3.4 (Analogical extension)** *An analogical GSM for $FA(L)$ is called* an analogical extension *of $L$.*

The analogical extension does not always exist.

**Example 5.3.5** *Consider the following program L:*

$$p(x) \leftarrow \neg q(x)$$
$$p(a)$$
$$p(b)$$
$$q(a)$$

*The unique GSM for* $F = \langle L^*, A^*, IC^* \rangle$ *is* $\{p(a), p(b), q(a), q^*(b)\}$. *The only LA of L is* $\{p,q\}$. *L's all analogical addenda is G:*

$$p(x) \leftarrow q(x), a^*_{\{p,q\}p}(x)$$
$$q(x) \leftarrow p(x), a^*_{\{p,q\}q}(x)$$
$$a_{\{p,q\}p}(x) \leftarrow q(x), p^*(x)$$
$$a_{\{p,q\}q}(x) \leftarrow p(x), q^*(x)$$

$FA(L)$*'s GSMs are:*

$$\{p(a), p(b), q(a), q^*(b), a^*_{\{p,q\}p}(a), a^*_{\{p,q\}q}(a), a^*_{\{p,q\}p}(b), a_{\{p,q\}q}(b)\}$$
$$\{p(a), p(b), q(a), q(b), a^*_{\{p,q\}p}(a), a^*_{\{p,q\}q}(a), a^*_{\{p,q\}p}(b), a^*_{\{p,q\}q}(b)\}$$

*Both are not analogical.*

We are interested in when an analogical extension of given program exists. Next theorem shows a sufficient condition of the existence of the analogical extension. First, the concept of "saturation" is defined.

**Definition 5.3.6 (Saturation)** *Suppose* $P = \{p_1, \ldots, p_n\}$ *is a LA of program L by h. If and only if* $\mathcal{M} \models p_{k_1}(t) \wedge \ldots \wedge p_{k_m}(t)$ *implies* $\mathcal{M} \models p_1(t) \wedge \ldots \wedge p_n(t)$ *holds, for all t, every subset* $P' = \{p_{k_1}, \ldots, p_{k_m}\} (m \leq n)$ *of P and a GSM* $\mathcal{M}$ *of L's transformed framework, then P is called* saturated.

If given program $L$ has a LA with a predicate which occurs in L as a negated literal, then GSM of $FA(L)$ may be not conservative. In the next theorem, we utilize the fact that any GSMs of $FA(L)$ are conservative until $L$ has such a LA.

**Theorem 5.3.7** *Suppose logic program L's transformed framework* $F = \langle L^*, A^*, IC^* \rangle$ *has unique GSM* $\mathcal{M}$, *L has at least one LA, there is no predicate in LAs of L which occur in* $L^*$ *with* $^*$, *and for at least one LA P, P is not saturated. Then L has an analogical extension.*

**Proof** The outline of the proof is followings. We first show that the pre-GSM of $FA(L')$ is also the pre-GSM of the $FA(L)$, where $L' = L \cup Y$ and $Y$ denotes a set of atoms which are going to be *analogically* inferred[4]. Next we show that this pre-GSM is indeed a GSM of $FA(L)$.

By assumption, there is a not-saturated LA of $L$. Let $P = \{p_1, \ldots, p_n\}$ be such an LA. By definition of saturation, there is a term $h$ and a subset $P' = \{p_{k_1}, \ldots, p_{k_m}\}$ of $P$ such that $\mathcal{M} \not\models p_i(h)$, where $i \notin \{k_1, \ldots, k_m\}$. Let $W$ be an union of all such $p_i(h)$ and let $Y$ be a non-empty subset of $W$. Because $Y$ is a set of ground atoms without $^*$ and $L$ is locally stratified, $L \cup Y$ is locally stratified. Let $F' = \langle L^* \cup Y, A^*, IC^* \rangle$ be a transformed framework of $L \cup Y$. Consequently, $F'$ has a unique GSM. We will call this $\mathcal{M}_1$.

Let $L' = L \cup Y$ and suppose $FA(L') = \langle L^* \cup Y \cup G, A^* \cup A_G, IC^* \cup IC_{A_G} \rangle$. Let $G(Q)$ be analogical addenda with respect to the LA $Q = \{q_1, \ldots, q_j\}$. By definition, $G(Q) = G_1(Q) \cup G_2(Q)$, where

$$G_1(Q) = \bigcup_{q_i \in Q} \{a_{Qq_i}(x) \leftarrow q_1(x), \ldots, q_{i-1}(x), q_{i+1}(x), \ldots, q_j(x), q_i^*(x)\}$$
$$G_2(Q) = \bigcup_{q_i \in Q} \{q_i(x) \leftarrow q_1(x), \ldots, q_{i-1}(x), q_{i+1}(x), \ldots, q_j(x), a_{Qq_i}^*(x)\}$$

We now make the following subsets of the Herbrand base:

$$\begin{aligned}
m_1 &= \{p^*(h) | \mathcal{M}_1 \not\models p(h) \wedge p^* \in A^*\} \\
m_2(Q) &= \{a_{Qq_i}(h) | \exists (a_{Qq_i}(x) \leftarrow q_1(x), \ldots, q_{i-1}(x), \\
&\qquad\qquad q_{i+1}(x), \ldots, q_j(x), q_i^*(x)) \in G_1(Q) \\
&\quad \text{where} \quad \mathcal{M}_1 \models q_1(h), \ldots, q_{i-1}(h), q_{i+1}(h), \ldots, q_j(h) \\
&\qquad\qquad \wedge \mathcal{M}_1 \not\models q_i(h)\} \\
m_3(Q) &= \{a_{Qq_i}^*(h) | \exists (a_{Qq_i}(x) \leftarrow q_1(x), \ldots, q_{i-1}(x), \\
&\qquad\qquad q_{i+1}(x), \ldots, q_j(x), q_i^*(x)) \in G_1(Q) \\
&\quad \text{where} \quad \mathcal{M}_1 \not\models q_1(h), \ldots, q_{i-1}(h), q_{i+1}(h), \ldots, q_j(h) \\
&\qquad\qquad \vee \mathcal{M}_1 \models q_i(h)\}
\end{aligned}$$

Now, if $\mathcal{M}_1 \cup m_1 \cup m_2(Q) \cup m_3(Q) \models a_{Qp}^*(h)$, then either $\mathcal{M}_1 \cup m_1 \cup m_2(Q) \cup m_3(Q) \not\models q_1(h) \wedge \ldots \wedge q_{i-1}(h) \wedge q_{i+1}(h) \wedge \ldots \wedge q_j(h)$, or else $\mathcal{M}_1 \cup m_1 \cup m_2(Q) \cup m_3(Q) \models p(h)$.

In the former case, the premises of rules $q_i(x) \leftarrow q_1(x), \ldots, q_{i-1}(x), q_{i+1}(x), \ldots, q_j(x), a_{Qq_i}^*(x)$ in $G_2(Q)$ cannot be satisfied for $x = h$. In the latter case, $\mathcal{M}_1 \models q_1(h) \wedge \ldots \wedge q_j(h)$ for the $h$ which satisfies these premises. So we can say that any atoms with the predicates in $F'$ cannot be in the least model of $L^* \cup Y \cup G(Q)$, except for such atoms in $\mathcal{M}_1$.

[4]Such atoms are indicated as $u$ in definition 5.3.2.

According to above argument, it is clear that $\mathcal{M}_1 \cup m_1 \cup m_2(Q) \cup m_3(Q)$ is the least model of $L^* \cup Y \cup G(Q) \cup m_1 \cup m_3(Q)$. And we can say $\mathcal{M}_1 \cup m_1 \cup \bigcup_Q m_2(Q) \cup \bigcup_Q m_3(Q)$ is a pre-GSM of $FA(L')$. This is because $a_{Qq_i}$ and $a^*_{Qq_i}$ do not occur in $G_1(S), G_2(S)$ for which $S \neq Q$.

Next we show that this pre-GSM is in fact a GSM. Obviously, $\mathcal{M}_1 \cup m_1 \cup \bigcup_Q m_2(Q) \cup \bigcup_Q m_3(Q) \models IC_{A_G}$. And $\mathcal{M}_1 \models IC^*$, because $\mathcal{M}_1$ is a GSM for $F'$. So we can say that $\mathcal{M}_1 \cup m_1 \cup \bigcup_Q m_2(Q) \cup \bigcup_Q m_3(Q)$ is a GSM for $FA(L')$. We denote this GSM by $\mathcal{N}'$ and assume that $\mathcal{N}'$ corresponds to the set of hypotheses $\Delta$. In fact, $\Delta = m_1 \cup \bigcup_Q m_3(Q)$.

Note that the premises imply that, for an arbitrary atom $p_i(h)$ in $Y$, there exists a LA $P = \{p_1, \ldots, p_n\}$ that contains $p_i$, and that $\mathcal{M} \models p_1(h) \wedge \ldots \wedge p_{i-1}(h) \wedge p_{i+1}(h) \wedge \ldots \wedge p_n(h)$. From the way in which we construct $F'$, we have $\mathcal{M}_1 \models p_1(h) \wedge \ldots \wedge p_{i-1}(h) \wedge p_{i+1}(h) \wedge \ldots \wedge p_n(h)$. For such $P$ and $p_i$, there exist

$$a_{Pp_i}(x) \leftarrow p_1(x), \ldots, p_{i-1}(x), p_{i+1}(x), \ldots, p_n(x), p_i^*(x) \in G_1(P)$$
$$p_i(x) \leftarrow p_1(x), \ldots, p_{i-1}(x), p_{i+1}(x), \ldots, p_n(x), a^*_{Pp_i}(x) \in G_2(P)$$

Clearly $\mathcal{N}' \models p_1(h) \wedge \ldots \wedge p_{i-1}(h) \wedge p_{i+1}(h) \wedge \ldots \wedge p_n(h)$ and $\mathcal{N}' \models a^*_{Pp_i}(h)$. From this, together with the subset of $G_2(P)$ shown above, we can derive $p_i(h)$. This results that if $\mathcal{N}'$ is the least model for $L^* \cup Y \cup G \cup \Delta$ then it is also the least model for $L^* \cup (Y - \{p_i(h)\}) \cup G \cup \Delta$. Since $p_i(h)$ is an arbitrary atom in $Y$, we have shown that $\mathcal{N}'$ is a least model for $L^* \cup G \cup \Delta$. By definition, this is a GSM for $FA(L)$. Because $\mathcal{N}'$ is obviously an analogical GSM of $FA(L)$, the theorem is proven. ■

**Example 5.3.8** *Example 5.2.9 satisfies the premises of theorem 5.3.7.*

Unfortunately, it is impossible to show the converse of theorem 5.3.7. The counter example of the converse of theorem 5.3.7 can be seen in example 5.2.4.

**Example 5.3.9** *Example 5.2.4 doesn't satisfy the premises of theorem 5.3.7, but there is an analogical extension shown as the following.*

$$\{ \; robin(a), injured(a), fly(a), robin(b), injured^*(b), fly(b),$$
$$a^*_{\{robin,fly\}fly}(a), a^*_{\{robin,fly\}fly}(b), a^*_{\{robin,fly\}robin}(a),$$
$$a^*_{\{robin,fly\}robin}(b), a^*_{\{robin,injured\}robin}(a), a^*_{\{robin,injured\}robin}(b),$$
$$a^*_{\{robin,injured\}injured}(a), a_{\{robin,injured\}injured}(b) \; \}$$

Because an analogical model requires conservativeness, any atoms with the predicate which occurs in $L$ as a negated literal cannot be analogically inferred. It seems too restrictive, but this is indispensable. In the above example, we could analogically infer $injured(b)$ if we abandoned conservativeness requirements. In that case, we don't longer have a ground to infer $fly(b)$. This relaxation affects the LA on which analogical reasoning is based. According to our standpoint, such mutual dependencies should not be allowed.

**Definition 5.3.10 (Strongly analogical extension)** *A strongly analogical GSM for $FSA(L)$ is called* a strongly analogical extension *of $L$.*

**Theorem 5.3.11** *Let $F$ be a transformed framework of logic program $L$. If, for each predicate belonging to LA of $L$, the predicate formed by attaching $^*$ to it does not occur in $F$, then the strongly analogical extension of $L$ is unique, if exists.*

**Proof** Suppose $F = \langle L^*, A^*, IC^* \rangle$ and let M be its GSM. Since an analogical extension is conservative, $L$'s strongly analogical extensions are identical to those of $L' = L \cup \mathcal{M}$, if it exists. Since there is no rule whose head has a exceptional predicate, all $a^*_{Pp}(h)$ must be hypothesized, for all LA $P$, $p \in P$ and ground term $h$, in order to satisfy the integrity constraints. There is only one way to select hypotheses. Therefore, a pre-GSM of the framework with strongly analogical addenda for $L'$ is unique. By definition, strongly analogical extension of $L'$ is unique, if it exists. ■

**Theorem 5.3.12** *If $F = \langle L^*, A^*, IC^* \rangle$ satisfies the premises of theorem 5.3.7, then a strongly analogical extension of $L$ exists, and is an analogical extension of $L$.*

**Proof** Assume that, in the proof of theorem 5.3.7, $W$ is selected for $Y$. In this case, for any LA $P$ of $L$, the premises of the elements of $G_1(P)$ are never satisfied. Consequently, the $\mathcal{N}'$ constructed in this case is identical to a GSM for $FSA(L)$. ■

**Theorem 5.3.13** *The interpretation, which is obtained by removing atoms with predicates representing exceptional and atoms with predicates with $^*$ from $L$'s analogical extension, is a model for $L$.*

**Proof** $L$'s analogical extension is clearly a model for $L$ (which is extended by extra vocabulary) because it is a model for $FA(L)$. So, the interpretation, which is obtained by removing atoms with predicates representing exceptional and atoms with predicates with $^*$ from $L$'s analogical extension, is a model for $L$, because there is no clause in $L$ which has predicates representing exceptional or predicates with $^*$ in its head. ■

**Definition 5.3.14 (Analogical model)** *When an analogical extension of $L$ exists, the interpretation obtained from $L$'s analogical extension by removing all predicates representing unusual cases, together with all predicates with $^*$, is a (non-minimal) model for $L$. Specifically, it is called an analogical model for $L$.*

**Theorem 5.3.15** *Let $SAM(L)$ be an analogical model of $L$ which corresponds to $L$'s strongly analogical extension, and $AM_i(L)$ be an analogical model of $L$. If $SAM(L)$ exists, then $SAM(L) = \bigcup_i AM_i(L)$.*

**Proof** The existence of $SAM(L)$ implies that there is a model of $FA(L)$ free of any atoms with exceptional predicates without $^*$, except those which hold within the GSM of $L$'s transformed framework. Because $\exists i(SAM(L) = AM_i(L))$, $SAM(L) \subseteq \bigcup_i AM_i(L)$ is obvious. Next, we must show $SAM(L) \supseteq \bigcup_i AM_i(L)$ to prove the theorem. Suppose that there is an analogical model which is not included by the analogical model which corresponds to a strongly analogical extension. Let $\mathcal{M}$ be an analogical extension which corresponds to such an analogical model and let $\mathcal{N}$ be the strongly analogical extension. Then there is at least one atom which belongs to $\mathcal{M}$ and does not belong to $\mathcal{N}$. Obviously, there is a LA which contains a predicate of this atom. So this atom must be contained by $\mathcal{N}$, which is made by the way shown in the proof of theorem 5.3.7 under the condition that $W = Y$. This leads contradiction. ■

**Definition 5.3.16 (Analogical explanation)** *Let $L$ be a given logic program and let $F = \langle L^*, A^*, IC^* \rangle$ be its transformed framework. Goal $Q$ has* an analogical explanation *iff there exists an analogical GSM $\mathcal{M}$ and $\mathcal{M} \models Q$. If this $\mathcal{M}$ corresponds to a set of abducibles $\Delta$, then we say that $Q$ has an analogical explanation* with a set of hypotheses $\Delta$.

As described by theorem 5.3.15, a strongly analogical extension is equivalent to applying as much analogical reasoning as possible. So it is unique, if it exists. But applying as much analogical reasoning as possible may cause violations against integrity constraints. In contrast, in $FA(L) = \langle L^* \cup G, A^*, IC^* \rangle$, let $G'$ be a program obtained from $G$ by replacing all $p^*(x)$

by $\neg p(x)$. $L \cup G'$ is not locally stratified. The priority for minimization[66] is undecidable. Consequently, for a GSM $\mathcal{M}$ of $\langle L^*, A^*, IC^* \rangle$ and arbitrary LA $P$, predicate $p$, and term $h$ such that $p \in P, \mathcal{M} \not\models p(h)$, it is possible to selectively hypothesize either $p^*(h)$ or $a^*_{Pp}(h)$. These correspond to (generally plural) analogical extensions of $L$. There is no domain-independent solution on the selection, and we must employ new knowledge to select an appropriate model[14]. Specifically, we will require knowledge on plausibility of the sets of hypotheses to which each analogical GSM corresponds.

### 5.3.2 Active usage of $p$-exception

It is assumed that we can allow rules whose head contain predicates representing $p$-exceptions with respect to $P$. These rules are used to decide whether it is possible to make a hypothesis or not. Namely, $a^*_{Pp}(t)$ is hypothesized iff $a_{Pp}(t)$ cannot be proven. This convention can be quite useful: for example, we can reflect the constraint "birds almost fly, but penguins don't" into the analogical reasoning by the following program.

$$penguin(p)$$
$$a_{\{bird,fly\}fly}(p)$$

But it must be noted that the unlimited usage of such rules causes a problem. A framework with such extensions is not guaranteed to have a unique GSM — even if we employ the strongly analogical extension — because whether atoms with exceptional predicates should be hypothesized or not becomes the problem which is not trivial.

This fact could be troublesome to construct the analogical proof procedure. To cope with this problem, we restrict ourselves to use such rules only as unit clauses, in order to ensure that the program is still locally stratified.

## 5.4 Analogical proof procedure

Suppose we only deal with programs that satisfy the premises of theorem 5.3.7. Those programs have unique GSM for their frameworks with strongly analogical addenda, by theorem 5.3.11 and 5.3.12.

Therefore, if we want to construct a procedure which proves whether a goal has an analogical explanation or not for the given logic program, it is sufficient to construct the procedure corresponds to the strongly analogical extension of the given program, by theorem 5.3.15. The logic program, which corresponds to the framework with strongly analogical addenda upon the given one, is locally stratified, because each predicate belonging to the LA of

$L$ does not occur in its transformed framework attached by $^*$. So we can use the procedure introduced by [14] which is shown in figure 5.2 and figure 5.3.

An abductive derivation from $(G_1 \quad \Delta_1)$ to $(G_n \quad \Delta_n)$ is a sequence

$$(G_1 \quad \Delta_1), (G_2 \quad \Delta_2), \ldots, (G_n \quad \Delta_n)$$

such that, for each $i$, $1 \leq i < n$, $G_i$ has the form $\leftarrow l, l'$, where (without loss of generality) computation rule $\mathbf{R}$ selects $l$, and $l'$ is a (possibly empty) collection of atoms, and

**abd1)** if $l$ is not abducible then
$G_{i+1} = C$; $\Delta_{i+1} = \Delta_i$;
where $C$ is the resolvent of some clause
in the program with the clause $G_i$ on the selected literal $l$

**abd2)** if $l$ is abducible and $l \in \Delta_i$ then
$G_{i+1} = \leftarrow l'$; $\Delta_{i+1} = \Delta_i$;

**abd3)** if $l$ is abducible, $l \notin \Delta_i$,
$l$ has the form $k^*$, and
there is a consistency derivation
from $(\{\leftarrow k\} \quad \Delta_i \cup \{k^*\})$ to $(\phi \quad \Delta')$ then
$G_{i+1} = \leftarrow l'$; $\Delta_{i+1} = \Delta'$;

A refutation is an abductive derivation to a pair $(\Box \quad \Delta')$.

Figure 5.2: Abductive derivation

This procedure is a natural generalization of SLDNF. The special feature of the procedure is that it records used hypotheses to prove a goal.

This procedure is correct for locally stratified programs. Therefore, provided that the framework with analogical addenda has already been established according to the definitions, this procedure can be used to decide whether goal $Q$ has an analogical explanation.

Now we consider a procedure by which we can analogically prove goals without the framework with analogical addenda. To do so, the mechanism to select a LA including an attribute in the goal is required. When the goal has variables, it is needed to determine their value temporarily in order to find the appropriate LA.

The proof procedure for this framework is obtained by attaching a procedure in figure 5.4 to that in figure 5.2.

A consistency derivation from $(F_1 \quad \Delta_1)$ to $(F_n \quad \Delta_n)$ is a sequence

$$(F_1 \quad \Delta_1), (F_2 \quad \Delta_2), \ldots, (F_n \quad \Delta_n)$$

such that, for each $i$, $1 \leq i < n$, $F_i$ has the form $\{\leftarrow k, k'\} \cup F_i'$, where (without loss of generality) the clause $\leftarrow \quad k, k'$ has been selected (to continue the search), computation rule $\mathbf{R}$ selects $k$, and

**con1)** if $k$ is not abducible then
  $F_{i+1} = C' \cup F_i'$; $\Delta_{i+1} = \Delta_i$;
   where $C'$ is the set of all resolvents of clauses
   in the program with the selected clause on
   the selected literal, and $\Box \notin C'$

**con2)** if $k$ is abducible, $k \in \Delta_i$ and $k'$ is not empty then
  $F_{i+1} = \{\leftarrow k'\} \cup F_i'$; $\Delta_{i+1} = \Delta_i$;

**con3)** if $k$ is abducible, $k \notin \Delta_i$ and
  $k$ has the form $l^*$ then
   if there is an abductive derivation
    from $(\leftarrow l \quad \Delta_i)$ to $(\Box \quad \Delta')$ then
     $F_{i+1} = F_i'$; $\Delta_{i+1} = \Delta'$;
   else if no such derivation and $k'$ is not empty then
    $F_{i+1} = \{\leftarrow k'\} \cup F_i'$; $\Delta_{i+1} = \Delta_i \cup \{l^*\}$;

Figure 5.3: Consistency derivation

## 5.5 The related work

Iwayama et al.[30] proposes a formulation of analogical reasoning for logic programs, which is in similar manner to the thesis. They also add clauses to the original program to implement analogical reasoning and employ the stable model semantics. The main differences are the treatment of negative information and the standpoint for justification.

In this chapter, negative information on analogical reasoning can be dealt by $p$-exception predicates. In [30], because their object program is Horn program with integrity constraints, negative information can be dealt in more direct manner.

[30] assumes that analogical reasoning should be done only when the causal relation from similarity to projected property is consistent with original program. Therefore, if there is one exceptional individual for analogy, then analogical reasoning must be given up. To implement this viewpoint, they remove the variable from the head of the rules, which coincide with analogical addenda.

```
abd4) if l is not abducible and l has the form p(t) then
         if t is not a variable then
            if there is an abductive derivation
               from ({← p(s)}   φ) to (φ   φ) and Analog(P, p, t) is true
               for an appropriate set of predicates P ∌ p, P ≠ φ then
                  G_{i+1} = ← l'; Δ_{i+1} = Δ' ∪ a*_{P∪{p}p}(x);
         if t is a variable then
            label:  get a new term t' from the Herbrand term enumerator;
            if s = t' then goto label;
            if there is an abductive derivation
               from ({← p(s)}   φ) to (φ   φ) and Analog(P, p, t') is true
               for an appropriate set of predicates P ∌ p, P ≠ φ then
                  G_{i+1} = ← l'; Δ_{i+1} = Δ' ∪ a*_{P∪{p}p}(x);
            else
               goto label;

Analog(P, p, t)
begin
      if for all c in P
         there is an abductive derivation from ({← c(s)}   φ) to (φ   φ),
         there is an abductive derivation
         from ({← c(t)}   Δ_i) to (φ   Δ') and
         there is a consistency derivation
         from ({← a_{P∪{p}p}(t)}   φ) to (φ   φ) then
            return true;
      else
            return false;
end
```

Figure 5.4: The addendum for the analogical proof procedure

## 5.6 Summary

Analogical models for locally stratified logic programs were given as analogical GSMs for their transformed framework. This yields a semantics for analogical reasoning over these programs. A sufficient condition that an analogical extension exists was shown, and the strongly analogical extension was shown to be unique under this condition. In general, analogical extensions are not unique; there is one state in which unusual cases with respect to the LA are minimized, and the other state in which predicates which already exist in the original program are minimized. The choice among these must be based on another piece of knowledge. This would seem to be a quite natural result from the intuitive viewpoint of analogical reasoning.

Because a proof procedure for the transformed framework exists, we can construct a procedure for analogically proving the goal, provided that we first create the framework with analogical addenda for the given program. We then construct a procedure for proving the given goal by introducing appropriate LAs or unusual cases *as needed.* In this case, we can use a fact which contains a predicate representing an exceptional case.