

付録A D_{lm} の表式

ここでは, D_{lm} の具体的な表式を $l = 3$ まで示す. D_{lm} の具体的な表式は, 多重極展開 (2.109) および (2.110) で用いる球面調和関数 Y_{lm} の具体的な表式に依存する. 本研究では, 以下に示す実数型の Y_{lm} を用いる.

$$\begin{aligned}
 Y_{00} &= \sqrt{\frac{1}{4\pi}}, \\
 Y_{11} &= -\sqrt{\frac{3}{4\pi}} \frac{x}{r}, \quad Y_{10} = \sqrt{\frac{3}{4\pi}} \frac{z}{r}, \quad Y_{1-1} = -\sqrt{\frac{3}{4\pi}} \frac{y}{r}, \\
 Y_{22} &= \sqrt{\frac{15}{16\pi}} \frac{x^2 - y^2}{r^2}, \quad Y_{21} = -\sqrt{\frac{15}{4\pi}} \frac{zx}{r^2}, \quad Y_{20} = \sqrt{\frac{5}{16\pi}} \frac{3z^2 - r^2}{r^2}, \\
 Y_{2-1} &= -\sqrt{\frac{15}{4\pi}} \frac{zy}{r^2}, \quad Y_{2-2} = \sqrt{\frac{15}{4\pi}} \frac{xy}{r^2}, \\
 Y_{33} &= -\sqrt{\frac{35}{32\pi}} \frac{x(x^2 - 3y^2)}{r^3}, \quad Y_{32} = \sqrt{\frac{105}{16\pi}} \frac{z(x^2 - y^2)}{r^3}, \\
 Y_{31} &= -\sqrt{\frac{21}{32\pi}} \frac{x(5z^2 - r^2)}{r^3}, \quad Y_{30} = \sqrt{\frac{7}{16\pi}} \frac{z(5z^2 - 3r^2)}{r^3}, \\
 Y_{3-1} &= -\sqrt{\frac{21}{32\pi}} \frac{y(5z^2 - r^2)}{r^3}, \quad Y_{3-2} = \sqrt{\frac{105}{4\pi}} \frac{xyz}{r^3}, \\
 Y_{3-3} &= \sqrt{\frac{35}{32\pi}} \frac{y(3x^2 - y^2)}{r^3}.
 \end{aligned}$$

このとき, D_{lm} は次のようになる.

$$\begin{aligned}
 D_{00} &= \sqrt{\frac{1}{4\pi}}, \\
 D_{11} &= -\sqrt{\frac{3}{4\pi}} \frac{\partial}{\partial x}, \quad D_{10} = \sqrt{\frac{3}{4\pi}} \frac{\partial}{\partial z}, \quad D_{1-1} = -\sqrt{\frac{3}{4\pi}} \frac{\partial}{\partial y}, \\
 D_{22} &= \sqrt{\frac{15}{16\pi}} \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right), \quad D_{21} = -\sqrt{\frac{15}{4\pi}} \frac{\partial^2}{\partial z \partial x}, \quad D_{20} = \sqrt{\frac{5}{16\pi}} \left(2 \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right), \\
 D_{2-1} &= -\sqrt{\frac{15}{4\pi}} \frac{\partial^2}{\partial z \partial y}, \quad D_{2-2} = \sqrt{\frac{15}{4\pi}} \frac{\partial^2}{\partial x \partial y}, \\
 D_{33} &= -\sqrt{\frac{35}{32\pi}} \frac{\partial}{\partial x} \left(\frac{\partial^2}{\partial x^2} - 3 \frac{\partial^2}{\partial y^2} \right), \quad D_{32} = \sqrt{\frac{105}{16\pi}} \frac{\partial}{\partial z} \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right), \\
 D_{31} &= -\sqrt{\frac{21}{32\pi}} \frac{\partial}{\partial x} \left(4 \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right), \quad D_{30} = \sqrt{\frac{7}{16\pi}} \frac{\partial}{\partial z} \left(2 \frac{\partial^2}{\partial z^2} - 3 \frac{\partial^2}{\partial x^2} - 3 \frac{\partial^2}{\partial y^2} \right),
 \end{aligned}$$

$$D_{3-1} = -\sqrt{\frac{21}{32\pi}} \frac{\partial}{\partial y} \left(4 \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right), \quad D_{3-2} = \sqrt{\frac{105}{4\pi}} \frac{\partial^3}{\partial x \partial y \partial z}$$

$$D_{3-3} = -\sqrt{\frac{35}{32\pi}} \frac{\partial}{\partial y} \left(3 \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right).$$