CHAPTER 3

PROPOSED MODEL FOR THE SUBCOOLED FLOW BOILING CHF

3.1 Basic Assumptions

As mentioned in chapter 2, based on the experimental observations reported in the previous works, for the happening of the most often encountered flow pattern condition (fig.1-2), the liquid sublayer dryout mechanism is adopted as the CHF triggering mechanism by the present author. The basic assumptions in this new model are the same as what were used by Lee-Mudarwar, Katto and Celata. The vapor blanket is assumed overlying a very thin liquid sublayer adjacent to the wall and the CHF is assumed happening when the liquid sublayer initial thickness is extinguished by evaporation during the passage time of the vapor blanket $\tau = L_B / U_B$.

![Diagram of liquid sublayer dryout mechanism](image)

**Fig.3-1 Conceptual View of the Liquid Sublayer Dryout Mechanism**

Let's first give a brief depiction to the formation of a stable vapor blanket. As shown in fig.3-1, bubbles detach tube wall frequently in the onward region of the NVG point. Vapor blankets are formed as the consequence of the coalescence of the small bubbles. As assumed by Lee-Mudarwar (1988), the development of each blanket is strongly influenced by neighboring blankets that tend to confine the blanket circumferential growth, it's therefore
reasonable to assume the equivalent diameter of the vapor blanket is approximately equal to the diameter of a bubble at the departure from the wall. The departed bubbles are assumed to coalesce into a long blanket, which maintains a fairly constant equivalent diameter while stretching in the direction of fluid flow due to the generation of more vapors by sublayer evaporation.

The vapor blanket divides flow area into two parts. One is near wall region filled with superheated liquid and is called liquid sublayer. The other is core region that is filled with gas-liquid two-phase flow.

Considering waves existing both at interfaces I (the interface of the liquid sublayer and the vapor blanket) and II (the interface of the vapor blanket and the core region), as shown in fig.3-2, we assume that the two waves are always of same wavelength (see appendix 2 for mathematics demonstration), that is, $\lambda_1 = \lambda_2$. The two wavelengths are assumed to be equal to the Helmholtz instability wavelengths at the interfaces I and II respectively. A stable vapor blanket is assumed containing only 1 complete wavelength. Otherwise, if a vapor blanket contains more than one wavelength, the blanket would be unstable and has the tendency to break down to form the stable vapor blanket at its thinnest points, which is reached when the two waves come to opposite phases (see appendix 2 for physics proof). With these assumptions, the vapor blanket length $L_B$ can be calculated from $L_B = \lambda_1 = \lambda_2$.

Fig.3-2 Conceptual View of the Liquid Sublayer Dryout Mechanism in the Present Model
From the above-mentioned assumptions, by writing the Helmholtz critical wavelengths to both the interfaces and by supposing they are equal to each other, the vapor blanket velocity can be written as a simple function of core region two-phase flow average velocity that can be got by the knowledge we have known. The vapor blanket length is then calculated simply from the expression of the Helmholtz instability wavelength. On the base of the vapor blanket velocity, the sublayer thickness is obtained from the Karman velocity distribution equations if the liquid velocity at the centerline of the vapor blanket, \( U_{BL} \), is known. \( U_{BL} \) is calculated as the difference of the vapor blanket velocity, \( U_B \), and the relative vapor blanket velocity determined by the balance between buoyancy and drag forces exerted on the vapor blanket in the axial direction.

The above thought, based only on a fundamental physical consideration, is the main difference between the current model and those proposed by predecessors.

### 3.2. Calculation of Vapor Blanket Velocity \( U_B \)

First we write the critical Helmholtz instability wavelength at the interface I. Because the liquid sublayer is near the tube wall and always be very thin, the average velocity in the liquid sublayer is assumed to be 0. The critical Helmholtz wavelength at the interface I therefore can be written as:

\[
\lambda_1 = \frac{2\pi \sigma}{\rho_g U_B^2}
\]  

(3-1)

Second, write the critical Helmholtz instability wavelength at the interface II as:

\[
\lambda_2 = \frac{2\pi \sigma (\rho_c + \rho_g)}{\rho_c \rho_g (V - U_B)^2}
\]  

(3-2)

Considering the assumption that the two waves are of the same wavelength, we get \( \lambda_1 = \lambda_2 \). Then \( U_B \) is got as:

\[
U_B = \frac{V_c}{1 + b}
\]  

(3-3)

with \( b = \sqrt{(\rho_c + \rho_g) / \rho_c} \)  

(3-3a)

If the CHF is assumed to occur at the tube exit, \( V_c \) and \( \rho_c \) are the core region two-phase average velocity and average density at the tube exit respectively. \( V_c \) can be simply calculated from:

\[
V_c = G / \rho_c
\]  

(3-4)

\( \rho_c \) is calculated from:

\[
\rho_c = (1 - \alpha_e) \times \rho_{\text{liquid}} + \alpha_e \times \rho_g
\]  

(3-4a)
where $\alpha_e$ and $\rho_{\text{out}}$ are the exit core region void fraction and liquid density respectively. From the calculation results, the liquid sublayer thickness and vapor blanket diameter are always shown to be very thin, $\alpha_e$ therefore can be written simply as:

$$\alpha_e = \alpha_{\text{out}}$$  \hspace{1cm} (3-4b)

where $\alpha_{\text{out}}$ is exit void fraction. The evaluation of $\alpha_{\text{out}}$ can be obtained either by Ahmad (1970) or Kroger-Zuber (1968) or Dix (1971) models. Present authors tested all the models and found no very big difference existing in the prediction results. While with the Ahmad model, $\alpha_{\text{out}}$ is given as:

$$\alpha_{\text{out}} = \frac{\chi_{\text{out}}}{\chi_{\text{out}} + \left( \frac{\rho_e}{\rho_f} \right) S (1 - \chi_{\text{out}})}$$  \hspace{1cm} (3-5)

where $\chi_{\text{out}}$ is exit true quality and $S$ is slip ratio. $S$ is calculated in the Ahmad model as:

$$S = \left( \frac{\rho_f}{\rho_g} \right)^{0.205} \left( \frac{GD}{\mu_f} \right)^{-0.015}$$  \hspace{1cm} (3-5a)

$\chi_{\text{out}}$ can be calculated either form Jafri et al. (1995) model (eq.3-6) or the model (eq.3-7) recommended by Ahmad (1970), Saha-Zuber (1974) or Levy (1967). Although the latter is considered only an approximation of the former, the latter one is adopted for the general accuracy and simplicity.

$$\frac{dx}{d\chi_{\text{equi}}} = 1 + \frac{x - \chi_{\text{equi}}}{(1 - x)\chi_d}$$  \hspace{1cm} (3-6)

with initial condition: at NVG point ($\chi_{\text{equi}}$=\chi_d), $\chi$=0  \hspace{1cm} (3-6a)

$$\chi_{\text{out}} = \frac{\chi_{\text{equi}} - \chi_d \exp \left( \frac{\chi_{\text{equi}}}{\chi_d} - 1 \right)}{1 - \chi_d \exp \left( \frac{\chi_{\text{equi}}}{\chi_d} - 1 \right)}$$  \hspace{1cm} (3-7)

where $\chi_{\text{equi}}$ and $\chi_d$ are the thermal equilibrium qualities at the tube exit and the NVG point respectively.

$$\chi_{\text{equi}} = \left( H_{\text{in}} + \frac{4q}{G} \frac{L}{D} - H_f \right) / H_{\text{gb}}$$  \hspace{1cm} (3-7a)

It seems $\chi_{\text{equi}}$ is the function of pressure $P$, mass velocity $G$, inlet liquid thermal condition and the ratio of the heated length to the inside diameter $L/D$. If $L/D$ is maintained as a certain value, $\chi_{\text{equi}}$ shows no relation with the inner diameter $D$.

$\chi_d$ is calculated from:
\[ \chi_d = (H_{li} - H_f) / H_{ls} = -C_{pl,T_d} \Delta T_d / H_{fs} \tag{3-7b} \]

where \( H_{li} \) and \( \Delta T_d \) are the liquid enthalpy and liquid subcooling at the NVG point respectively. As to the NVG determination, Ahmad (1970), Levy (1967) and Saha-Zuber (1974) raised different models. It seems the NVG is basically a function of pressure, inner diameter and mass flux and shows no relation with the heated length. Either the Levy or the Ahmad model is found can be used in the present model. While with the Ahmad model, the NVG is calculated from:

\[ \Delta T_d = q / h_{t,A} \tag{3-7c} \]

where \( h_{t,A} \) is calculated by:

\[ h_{t,A} = 2.44 \frac{k_f}{D} \left( \frac{GD}{\mu_f} \right)^{1/2} \left( \frac{C_{pl,T_d} \mu_f}{k_f} \right)^{1/3} \left( \frac{H_{li}}{H_f} \right)^{1/3} \left( \frac{H_{ls}}{H_f} \right)^{1/3} \tag{3-7d} \]

If the calculated \( \Delta T_d \) is higher than the inlet subcooling (\( \Delta T_d > \Delta T_{in} \)), which means tube inlet is the physically valid NVG point, the value of the \( \Delta T_d \) is substituted by \( \Delta T_{in} \).

As to the exit liquid temperature \( T_{exit} \), Ahmad (1970), Sauter (1969) and Kroger-Zuber (1968) recommended almost the same exponential expression as:

\[ \Delta T_{exit} = \Delta T_d e^{-A} \tag{3-7e} \]

where \( A \) has a special physical meaning as the ratio of the heat absorbed by the liquid from the NVG point to the tube exit to the whole heat needed to raise the liquid at the NVG to the saturation condition. In the Ahmad model, \( A \) is written as:

\[ A = \frac{q * Z_{sb}}{(GDC_{pl,T_d} \Delta T_d / 4)} \tag{3-7f} \]

where \( Z_{sb} \) is significant boiling length and is calculated as the difference of heated length \( L \) and \( Z_0 \), the length from the tube inlet to the NVG point, as:

\[ Z_{sb} = L - Z_0 \]

\[ Z_0 = GDC_{pl,T_d} (\Delta T_{in} - \Delta T_d) / (4q) \tag{3-7g} \]

With rearrangement, \( A \) also can be written as:

\[ A = 4q \left[ \frac{L}{D} \frac{G(H_{li} - H_{ls})}{4q} \right] \left[ \frac{G(H_f - H_{li})}{4q} \right] \tag{3-7h} \]

### 3.3 Calculations for \( L_B \) and \( D_B \)

With \( U_B \) calculated, \( L_B \) is calculated from:

\[ L_B = \lambda_1 = \lambda_2 = \frac{2 \pi \sigma}{\rho_{sat} U_B^2} \tag{3-8} \]

The diameter of vapor blanket is calculated from the Levy model (1967) as:
\[ D_B = 0.015 \left( \frac{\alpha D}{\tau_w} \right)^{0.5} \]  \hspace{1cm} (3-9)

where \( \tau_w = \frac{fG^2}{8\rho_f} \)  \hspace{1cm} (3-9a)

The friction factor \( f \), calculated by Colebrook equation (1938), is written as:

\[ \frac{1}{\sqrt{f}} = 1.14 - 2.0\log\left( \frac{e}{D} + \frac{9.35}{Re\sqrt{f}} \right) \]  \hspace{1cm} (3-10)

where \( e \) is the surface roughness, which is assumed to be close to \( 0.75D_B \) in Celata model (1994a). Considering \( e = 0.75D_B \), making use of eq.3-9, eq.3-10 then turns to:

\[ \frac{1}{\sqrt{f}} = 1.14 - 2.0\log\left( 0.75 * 0.015 \sqrt{\frac{8\rho_f}{fG^2D}} + \frac{9.35}{Re\sqrt{f}} \right) \]  \hspace{1cm} (3-10a)

3.4. Calculation of Liquid Sublayer Thickness \( \delta \)

3.4.1 Calculation of \( U_{BL} \)

![Diagram of forces exerted on the vapor blanket in axial direction]

Fig. 3-3 Forces Exerted on the Vapor Blanket in Axial Direction

As reported by Lee-Mudarwar (1988), for a vapor blanket in vertical turbulent flow, a force balance exists between buoyancy force \( F_B \) and drag force \( F_D \) (fig.3-3). That is:

\[ F_n = F_D \]  \hspace{1cm} (3-11)
The buoyancy force is proportional to the relative density between the two phases and the volume of the vapor blanket and is written as:

$$F_b = \frac{\pi}{4} D_b^2 L_B g (\rho_f - \rho_s)$$  \hspace{1cm} (3-12)

The drag force is determined by the relative velocity between the two phases and the drag coefficient as:

$$F_D = \frac{1}{2} \rho_f C_D (U_b - U_{BL})^2 \frac{\pi D_b^2}{4}$$  \hspace{1cm} (3-13)

Considering eq.3-11, making use of eqs.3-12 and 3-13, \( U_{BL} \) is got as:

$$U_{BL} = U_b - \left( \frac{2L_B g (\rho_f - \rho_s)}{\rho_f C_D} \right)^{0.5}$$  \hspace{1cm} (3-14)

Drag coefficient \( C_D \) can be obtained either by Harmathy (1960) or Chan & Prince (1965) expressions. The former determined by buoyancy and surface tension forces is recommended in the present model at low pressure (P<1MPa). The latter one proposed for small bubble that is dominated by viscous forces is recommended at medium and high pressure (P≥1MPa).

Harmathy: \( C_D = \frac{2}{3} \frac{D_b}{\left( \frac{\sigma}{g (\rho_f - \rho_s)} \right)^{0.5}} \)  \hspace{1cm} (3-14a)

Chan & Prince: \( C_D = \frac{48 \mu_f}{\rho_f D_b (U_b - U_{BL})} \)  \hspace{1cm} (3-14b)

where \( (U_b - U_{BL}) \) is the relative velocity of the bubble with respect to the liquid at the position corresponding to the centerline of the vapor blanket.

At low pressure, \( C_D \) is obtained from eq.3-14a. Otherwise eq.3-14b is used, and then the \( U_{BL} \) is expressed as:

$$U_{BL} = U_b - 2L_B g (\rho_f - \rho_s) D_b / (48 \mu_f)$$  \hspace{1cm} (3-14c)

3.4.2 Calculation of Liquid Sublayer Thickness \( \delta \)

By knowing \( U_{BL} \), the distance \( y \), which is the distance from wall to the bubble centerline (fig.3-4), can be got from Karman velocity distribution equation as:

\[
\begin{align*}
U_{BL}^+ &= y^+ & 0 \leq y^+ < 5 \\
U_{BL}^+ &= 5.0 \ln y^+ - 3.05 & 5 \leq y^+ < 30 \\
U_{BL}^+ &= 2.5 \ln y^+ + 5.5 & y^+ \geq 30
\end{align*}
\]  \hspace{1cm} (3-15)

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where \( U_{nl}^* = \frac{U_{nl}}{U_r} \), \( U_r = \left( \frac{\tau_w}{\rho_f} \right)^{0.5} \), \( y^+ = y \frac{U_r}{\mu_f} \), \( \tau_w = \frac{fG^2}{8\rho_f} \)

Then \( \delta \) is got from:

\[
\delta = y - \frac{D_B}{2} \quad (3-16)
\]

![Fig.3-4 Velocity Profile in Liquid Phase](image)

3.5 Computation of the CHF

The critical heat flux, which is the minimum heat flux necessary to extinguish the liquid sublayer by evaporation during the passage time of the vapor blanket, is calculated from:

\[
CHF = \frac{\rho_f \Delta H_{lk}}{L_B} U_B \quad (3-17)
\]

For a given geometric and inlet thermal hydraulic conditions, the critical heat flux can be predicted by an iterative procedure through the foregoing equations. Fig.3-5 shows out the CHF computation flow chart and the detailed CHF calculation procedure is shown in Appendix 3.
Fig. 3-5 CHF Computation Flow Chart

Input $G, P, D, L, T_n$

Calculate $f$ and $D_{th}$
Assume a $q_m$

Calculate $\Delta T_{th}, Z_0$

$Z_0 < L$

$\chi_{out} < 1$

Calculate $\chi_{out}, \alpha_{out}, T_{out}$

Calculate $V_c, U_{th}, L_{th}, U_{th}$

$U_{th} > 0$

Calculate $\delta$

$\delta > 0$

Calculate $q$

$|q - q_m| \leq 0.01$

$q < q_m$

$CHF = q$