Chapter 5

CONDENSATION COEFFICIENT OF He II

5.1 Introduction

The condensation coefficient that is the physical properties of substances is defined by Eq. (1.2). The background and the problems of the previous studies of the condensation coefficient of He II are already described in Sec. 1.2. In the present study, the condensation coefficient of He II is estimated from the direct comparison of such experimental results, as the temperature, the pressure and physical quantities which are derived from them, with the most reliable kinetic theory analysis results as described in Sec. 4.6. This report may be the first one describing the temperature dependence of the condensation coefficient of He II.

5.2 Temperature Dependence of Condensation Coefficient

The condensation coefficient result is plotted as a function of temperature in Fig. 5.1. This may be the first experimental result that presents the temperature dependence of He II condensation coefficient, as far as the authors know. It is seen that the condensation coefficient considerably decreases as the temperature approaches to the λ point. The impinging molecules onto a He II free surface from vapor phase are apt to instan-
Fig. 5.1: The condensation coefficient $\alpha_c$ as a function of temperature.

taneously lose the energy to be absorbed in He II due to so called super thermal conduction, and consequently condense there in high probability. As the temperature approaches to the $\lambda$ point, this excellent heat transfer mechanism deteriorates and thus the condensation coefficient drops down presumably to the value for He I. It is also seen that the condensation coefficient exhibits a tendency to increase as the temperature lowers. This tendency seems consistent with the well known fact in quantum evaporation experiments that the striking coefficient[58] is unity at temperatures as low as 0.03 $K$. The effect of super thermal conduction on condensation coefficient of He II is far discussed in the Chap. 7.

It was already discussed in Sec. 4.7 that the effect of non-linear slip boundary condition begins to appear as the rise of free surface temperature. It is found in Fig 5.1 that the effect of condensation coefficient change must be taken into account for the estimation of theoretical pressure rise in the evaporation wave in addition to the non-linear effect of slip boundary condition at the temperature near $\lambda$ point because the gradient $\frac{\partial \alpha_c}{\partial T_w}$ becomes to large as the temperature approaches to the $\lambda$ point.

The present result of $\alpha_c$ seems, in general, to contradict the existing
experimental results that the condensation coefficient is unity or larger than 0.9 as reported by Atkins et al. [20], Hunter and Osborne [21] and Kessler and Osborne [22]. The cause for this contradiction can be attributed to the use of wrong energy balance relation at a He II-vapor boundary and of the inadequate theory based on the kinetic theory for free molecular gases in their researches as mentioned in introduction. In the study by Atkins et al. [20], the theoretical maximum evaporation mass flux, corresponding to \( \alpha_c = 1.0 \), was calculated evidently on the basis of the kinetic theory for free molecular gases, that is the Hertz-Knudsen theory for evaporation. It is a rather simple procedure to convert Atkins' condensation coefficient \( f \) to the correct condensation coefficient on the basis of the kinetic theory which rigorously takes molecular collisions into account as follows:

The evaporation mass flux \( J_m \) is expressed in the form of Wiechert's expression[32] as,

\[
J_m = \frac{L_{MM}}{\rho_{0,v} T_0} \left[ \left( \frac{dP}{dT} \right)_{vpc} \Delta T_W - \Delta P_v \right] + \frac{L_{ME} - L_{MM} h_{0,v}}{T_0^2} (\Delta T_W - \Delta T_v) ,
\]

(5.1)

where \( \Delta T_W, \Delta T_v \) and \( \Delta P_v \) are the deviations of the liquid surface temperature, the vapor temperature and the vapor pressure from the equilibrium values, and \( \rho_{0,v} \) and \( h_{0,v} \) are the equilibrium values of the vapor density and the vapor enthalpy per unit mass. \( L_{MM} \) and \( L_{ME} \) are the Onsager coefficients[4][59] that are calculated on the basis of the kinetic theory. The quantity \( \left( \frac{dP}{dT} \right)_{vpc} \), the slope of the saturated vapor pressure curve, is given by the Clausius-Clapeyron relation of Eq. (4.14) as;

\[
P_0 + \left( \frac{dP}{dT} \right)_{vpc} \Delta T_W = P_0 \exp \left( \frac{L_0}{RT_0} \frac{\Delta T_W}{T_W} \right) = P_W .
\]

(5.2)

Here \( L_0 \) is the latent heat of evaporation of He II per unit mass. Substituting Eq. (5.2) into Eq. (5.1) and evaluating the Onsager coefficients with the aid of some kinetic approaches, the evaporation mass flux \( J_m \) can be calculated. The Onsager coefficients evaluated on the basis of the
Herz-Knudsen theory yields the following evaporation mass flux

\[ J_{mH-K} = f \rho_{0,v} \left( \frac{1}{\sqrt{2\pi RT_0}} \right)^{1/2} \times \left[ \frac{1}{\rho_{0,v}} \left\{ \exp \left( \frac{L_0}{RT_0} \left( \frac{\Delta T_W}{T_W} \right) \right) - \Delta P_v \right\} - \frac{R}{2} (\Delta T_W - \Delta T_v) \right] , \quad (5.3) \]

and that based on the Sone-Onishi theory[5] is given as

\[ J_{mS-O} = \frac{2.1973 \alpha_c}{2.1973 - \alpha_c} \rho_{0,v} \left( \frac{1}{\sqrt{2\pi RT_0}} \right)^{1/2} \times \left[ \frac{1}{\rho_{0,v}} \left\{ \exp \left( \frac{L_0}{RT_0} \left( \frac{\Delta T_W}{T_W} \right) \right) - \Delta P_v \right\} - 0.4287R (\Delta T_W - \Delta T_v) \right] . \quad (5.4) \]

The conversion formula is derived by equating the two expressions, Eqs. (5.3) and (5.4).

\[ J_{mH-K} = J_{mS-O}. \quad (5.5) \]

As in Eqs. (5.3) and (5.4) the second terms in the square brackets, \( 0.5R(\Delta T_W - \Delta T_v) \) and \( 0.4287R(\Delta T_W - \Delta T_v) \), are sufficiently smaller than the first ones, they can be neglected. Consequently one has the conversion formula between \( f \) and \( \alpha_c \) as

\[ \alpha_c = \frac{2.1973 f}{f + 2.1973} . \quad (5.6) \]

The similar conversion formula was presented in the paper by Hunter and Osborne[21], where the data on the basis of the Herz-Knudsen theory is converted into that on the basis of the HO theory. The formula, Eq. (5.6), gives the condensation coefficient \( \alpha_c \) of 0.69 for \( f = 1 \) at the temperature about 1.2K as reported in Atkins et al.[20]. This value is found to be slightly smaller than our experimental result at the temperature 1.28K by 0.08. This discrepancy may be attributed to experimental error. The possible experimental error in the Atkins result was also pointed out by Hunter and Osborne[21]. It was reported by Pound[13] and Chekmarev[14] that for general substances the condensation coefficient is almost unity in the free molecular situation, but it becomes smaller than unity in the continuum situation. It may be concluded that in the continuum situation the
condensation coefficient of He II will be larger than those of general substances which are smaller than 0.5, though they depend on substances. It is because the super thermal conduction contributes to enhance condensation of impinging molecules to He II as mentioned above.

5.3 Concluding Remarks

The condensation coefficient of He II is obtained from the direct comparison of experimental results with the most reliable kinetic theory analysis results in the temperature range between 2.16 K and 1.28 K, and following conclusions are drawn.

1. The detail of temperature dependence of the condensation coefficient of He II is presented.

2. It is confirmed that the condensation coefficient considerably decreases as the temperature approaches to the λ point because the excellent heat transfer mechanism, the so called super thermal conduction deteriorates in the temperature range.

3. It is found that the condensation coefficient exhibits a tendency to weakly increase as the temperature lowers. This tendency is consistent with the well known fact in quantum evaporation experiments that the striking coefficient is unity at temperatures as low as 0.03 K.

4. It is found that the effect of condensation coefficient change must be taken into account for the estimation of the theoretical pressure rise in the evaporation wave as the rise of free surface temperature at the near λ temperature.