CHAPTER 5

Concluding Remarks

In this final chapter we summarize the main results presented in this thesis. Some open problems for further investigation will be also suggested.

Much attention has recently been focused on the use of algebraic curves in information sciences such as algebraic-geometric codes, elliptic curve cryptography, factorizations of large numbers, and generating pseudorandom sequences. However the potential applications of algebraic curves in design theory are not well explored so far. In this thesis, we provided two new applications of algebraic curves in the construction of combinatorial designs and combinatorial arrays.

Balanced arrays, a natural and direct generalization of orthogonal arrays, have attracted great attention not only for their mathematical interest but also for their practical uses. Despite of the vast amount of energy spent, the existence and construction of balanced arrays are far from being settled completely. Block designs, cyclic codes, and other combinatorial configurations have been used as tools in the construction of balanced arrays. In this thesis, we ingeniously applied algebraic curves to construct balanced arrays. We introduced a new concept of a symmetric set of curves. We showed that a balanced array can be constructed from such a symmetric set of curves. When the genus of the base curve is 0, Riemann-Roch Theorem can give us such symmetric sets of curves. If the genus is greater than or equal to 1, although we have some construction methods such as the one by means of elliptic curves, there is in general no effective way to obtain symmetric sets of curves. Finding new symmetric sets of curves is an interesting and challenging problem.

Most of combinatorial designs have blocks which are subsets of the set of treatments. However this restriction is not necessary for every case. Balanced n-ary designs are one of the combinatorial designs with repeated treatments in blocks, that is, the blocks may be multi-subsets of the set of treatments. Balanced n-ary designs have close relations with balanced arrays. The existence of a balanced array with constant column sum can imply the existence of a regular balanced n-ary design. In this thesis, algebraic curves are also used to construct balanced n-ary designs. When the genus of the base curve is 0, an infinite series of regular balanced n-ary designs has been obtained by collecting curves which have linearly equivalent divisors. This approach is quite different from those known to the present author.

The possibility of constructing various types of balanced n-ary designs and balanced arrays may increase when the base curves have larger genus. However if the genus of the base curve becomes larger, it is difficult to collect curves which satisfy the 'balanced' condition.

Another possible modification of the construction methods is to use some curves as base curves, although in this thesis balanced arrays and balanced n-ary designs are only constructed from a fixed base curve. In fact, by regarding the multiplicities of points on varieties as entries of a matrix, instead of the intersection multiplicities of curves with a base curve, we may construct balanced arrays and balanced n-ary designs, and may generalize the construction methods for binary designs from finite geometries to work for n-ary designs. These considerations will be proceeded in the future research.