Chapter 4
Theories

4.1 New Theory on the X-ray Energy Response of Semiconductor Detectors

We have recently proposed a generalized theory on the output signals of semiconductor X-ray detectors [8,9,43]. In this section, the exact theoretical treatment and the extension of the above theory to the signal analyses of commonly utilized multichannel detectors are discussed so as to clarify the proposed physics essentials.

The theory predicts the enhancement of the output signals due to X-ray-produced charges in a field-free substrate region behind the depletion layer of a detector, while the signals predicted by the conventional theory [7] originate from the depletion layer alone. The essential point of our theory is the inclusion of such three-dimensional diffusion effects of X-ray-produced charges on the total signals. The importance of these X-ray-response studies is highlighted by the comparison of the significant difference in T_e deduced from the conventional theory and from our formula. The problem becomes much complicated when a multichannel semiconductor-detector array fabricated on one silicon wafer is employed for the purpose of X-ray tomographic reconstructions [2,4,44-46]. In this case, the three-dimensionally diffusing charges from the field-free substrate of an X-ray injected channel to the neighboring channels behave as "channel crosstalk".

At first, the characteristic physics principles of the effect of the three-dimensional thermal diffusion of X-ray-produced charges in a semiconductor field-free-substrate region are formulated. The three-dimensional diffusion equation for a charge flux ϕ created in a substrate by X rays is described as

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\phi(r)}{dr}-\frac{1}{L^2}\phi(r)=-\frac{s(d)}{D}, \qquad (4\text{-}1)$$

where the minority-carrier diffusion length L is written as $L^2=D\tau$; D and τ are a diffusion coefficient and the lifetime of the charge, respectively. The Einstein relation provides the equation of $D=\mu_m kT/e$; here, μ_m, k, T, and e are the mobility of the charge, the Boltzmann constant, a detector temperature in K, and the electronic charge, respectively. For minority carriers (electrons) in p-type silicon, μ_m is around 1300-1500 cm^2 V^{-1} s^{-1} at 300 K with an impurity doping of 10^{15}-10^{13} cm^{-3} [47,48]. These values lead to the constant D of 34-39 cm^2 s^{-1}. The value of L is then estimated to be 75 μm with τ of 1.7-1.4 μs [49-51]. If the characteristic time of temporal variation in X-ray radiation from plasmas is sufficiently longer than τ, and intense X-ray signals are observed in a current (pile-up) mode, then a quasi-steady state for the charge-diffusion process is treated, as seen in Eq. (4-1). The source charge s is created by incident X rays at a depth of d from the front surface of the field-free substrate. The distance from the source location P in Fig. 4-1 to the bottom surface of the depletion layer is denoted as r. A flow density of the charge $J(r)$ from P along r is defined as

$$J(r)=-D\frac{d\phi}{dr}=\frac{s}{4\pi r^2}\left(\frac{r}{L}+1\right)\exp\left(-\frac{r}{L}\right), \qquad (4\text{-}2)$$

where

$$s(d)=I_0\left(\frac{E}{\varepsilon}\right)\mu\rho\exp(-\mu\rho d). \qquad (4\text{-}3)$$

Here, I_0 is the X-ray intensity with an energy E at the front surface of the substrate; ε stands for the energy required to create an electron-hole pair. The values of μ and ρ denote the silicon mass-absorption coefficient and the mass density, respectively.

The total amount of the three-dimensionally diffusing minority carriers from the production point P (Fig. 4-1) to the depletion-layer surface contributes to a signal, and is described as the integral of div $\boldsymbol{J}$ for the substrate volume; this integral is rewritten by the surface integral $\int J\,dS$ surrounding the substrate. We then integrate over z from 0 to the thickness of the substrate $d_{\rm sub}$ to scan the source point P in the substrate along the X-ray path in the z direction (Fig. 4-1). Furthermore, the effect of the external bias circuit is described by a factor 2 multiplication to the "pure" diffusion effect. Thus, the amount of the overall diffusion charges $F_{\rm new}$ is totally described as

$$F_{new} = I_0 \frac{E}{\varepsilon} \mu\rho \frac{L}{\mu\rho L+1} \left\{ 1-\exp\left[-\left(\mu\rho + \frac{1}{L} \right) d_{sub} \right] \right\}. \tag{4-4}$$

The totally collected charges $F_{\rm total}$ created in both a depletion layer and a field-free substrate for a single-channel detector are thus described as

$$\begin{aligned} F_{total} = {} & I_{plas} \exp(-\mu_{dead}\rho_{dead}d_{dead}) \exp(-\mu_{elec}\rho_{elec}d_{elec}) \left(\frac{E}{\varepsilon} \right) \\ & \times [1-\exp(-\mu_{dep}\rho_{dep}d_{dep}) + \frac{\mu\rho L}{\mu\rho L+1} \\ & \times \left\{ 1-\exp\left[-\left(\mu\rho + \frac{1}{L} \right) d_{sub} \right] \right\} \exp(-\mu_{dep}\rho_{dep}d_{dep})]. \end{aligned} \tag{4-5}$$

Here, $I_{\rm plas}$ is the X-ray intensity from plasmas. The subscripts dead, elec, and dep denote the dead layer, the electrode, and the depletion layer, respectively.

The theoretical analysis of the three-dimensional charge diffusion effect is investigated for a multichannel semiconductor-detector signals using our thermal diffusion model. The diffusing-charge output profile for a multichannel semiconductor-detector array is calculated (Fig. 4-2).

Figure 4-2(a) illustrates a schematic view of a multichannel semiconductor X-ray-detector array along with an illustration of diffusing charges from a point located in the field-free substrate, where the incident X rays are absorbed. X rays are injected at $x=y=0$.

The incident X rays are assumed to be absorbed at the point P. Charges s produced by X-ray absorption at P diffuse in every direction according to the three-dimensional diffusion equation. Charges created in the range of a diffusion length L from the depletion layer reach the bottom surface of the depletion layer [i.e., the upper surface of the field-free substrate].

In Fig. 4-1, charges diffusing from P to an area dS with a radius of y and a width of dy is defined as $dQ_{\mathrm{dif}}(y)$. Here, $dQ_{\mathrm{dif}}(y)$ is described as

$$\begin{aligned} dQ_{dif}(y) &= 2\pi y J(r) dy \\ &= y\frac{s}{2r^2}\left(\frac{r}{L}+1\right)\exp\left(-\frac{r}{L}\right)\frac{z}{r}dy. \end{aligned} \tag{4-6}$$

The integral for dQ_{dif} over z from 0 to the thickness of the substrate d_{sub} gives the total charge flux to an area dS, since the source charges are distributed along the X-ray path in the z direction. Then the signal profile obtained from the field-free substrate is rewritten for a unit area along the y directions; this normalized value $q_{\mathrm{dif}}(y)$ is described as

$$q_{dif}(y) = \frac{I_0}{4\pi}\frac{E}{\varepsilon}\mu\rho\int_{r=y}^{\sqrt{y^2+d_{sub}^2}}\frac{1}{r^2}\left(\frac{r}{L}+1\right)\exp\left(-\frac{r}{L}-\mu\rho\sqrt{r^2-y^2}\right)dr. \tag{4-7}$$

Multichannel detector array with each channel size of x_{det} and y_{det} [Fig. 4-2(a)] is widely employed for X-ray tomography diagnostics. In the case of

$x_{det} \gg L$, we integrate over x from $x_{det}/2$ to $-x_{det}/2$ to take account of the diffusing-charge distribution in the x direction; here, x_{det} is the total width of the detector. Consequently, for the configuration of Fig. 4-2(a), an output-signal profile from the multichannel-detector array for unit-intensity X rays is written as

$$J_{sx}(y) = \int_{x=-x_{det}/2}^{x_{det}/2} \frac{I_0}{4\pi}\frac{E}{\varepsilon}\mu\rho \int_{r=\sqrt{y^2+x^2}}^{\sqrt{y^2+x^2+d_{sub}^2}} \frac{1}{r^2}\left(\frac{r}{L}+1\right) \times \exp\left(-\frac{r}{L}-\mu\rho\sqrt{r^2-\left(y^2+x^2\right)}\right) dr \cdot dx. \tag{4-8}$$

Here, we define the distance r as $r^2=x^2+y^2+z^2$.

The curves in Fig. 4-2(b) show the calculated diffusion signals $J_{sx}(y)$ using our three-dimensional diffusion theory.

In addition, the summation of this diffusing-signal profile over the finite X-ray beam width in the y direction produces the total diffusion signals $F_d(y)$ for each detector channel labeled at the location of y.

$$F_d(y) = \int_{y=|y_{sou1}|}^{|y_{sou2}|} \int_{x=-x_{det}/2}^{x_{det}/2} \frac{I_0}{4\pi}\frac{E}{\varepsilon}\mu\rho \int_{r=\sqrt{(y+|y_{sou}|)^2+x^2}}^{\sqrt{(y+|y_{sou}|)^2+x^2+d_{sub}^2}} \frac{1}{r^2}\left(\frac{r}{L}+1\right) \times \exp\left[-\frac{r}{L}-\mu\rho\sqrt{r^2-\left[\left(y+|y_{sou}|\right)^2+x^2\right]}\right] dr \cdot dx \cdot dy_{sou}. \tag{4-9}$$

Here, the width of the incident X-ray beam y_{sou} is written as $y_{sou}=y_{sou1}-y_{sou2}$; y_{sou1} and y_{sou2} are the locations of both edges of the rectangular X-ray beam. Equation (4-9) provides a more convenient formula for experiments using a finite-sized X-ray beam.

4.2 Thermal Barrier Potentials

The thermal barrier potential provides a thermal isolation between the electrons in the central cell and those in the plug cell. The thermal barrier potential not only reduces the heating power of electrons in the plug region but also enables the ion-confining potential to form with less density at the plug than in a conventional tandem mirror. The thermal barrier potential results from a pressure gradient between the central cell and the barrier cell, and then would comply with a Boltzmann law. Then, thermal barrier potential ϕ_b is described as

$$e\phi_b = T_{ec} \ln\left(\frac{n_c}{n_b}\right), \tag{4-10}$$

where T_{ec} is the temperature of the thermal electrons passing through the thermal barrier potentials including central cell, n_c and n_b are the electron density of the central cell and barrier cell.

The direct participation in potential depression is, however, the density of thermal electrons. The total electron density in the barrier cell is equal to the ion density in the barrier cell at the hydrogen plasma for the charge neutrality condition. Therefore, the accumulation of magnetically trapped hot electrons (density n_{bh}) in the barrier midplane is available to enhance the thermal barrier depth. Then, the Eq. (4-10) becomes

$$e\phi_b = T_{ec} \ln\left(\frac{n_c}{n_b - n_{bh}}\right). \tag{4-11}$$

This formula indicates the enhancement of ϕ_b clearly.

The above formula is assumed the full Maxwellian distribution for the thermal electrons. This assumption is not valid for the trapped thermal electrons in the barrier region, since distribution function of such electrons deform under the strong wave fields. In this case, Katanuma [52] suggests a modified Boltzmann law, which is described as

$$e\phi_b = T_{ec}\ln\left[\left(\frac{T_\perp}{T_e}\right)_{ef}\left(\frac{n_{mb}}{n_b - n_{bh}}\right)\right], \tag{4-12}$$

where $(T_\perp/T_e)_{ef}$ is fitting parameter dominantly determined by the rf field strength and n_{mb} is the density of barrier inner mirror throat.

4.3 Pastukhov's Theory

A problem of calculating the rate of end loss of particle from mirror system is one of calculating the rate of diffusion of electrons in velocity space from the trapped-particle regions of velocity space to loss boundaries [29].

The problem can be addressed by use of the Fokker-Planck equation:

$$\left(\frac{\partial f}{\partial t}\right)_C = \sum_\alpha \left[-\frac{\partial}{\partial v_i}\left(f_T \frac{\partial h_\alpha}{\partial v_i}\right) + \frac{1}{2}\frac{\partial^2}{\partial v_i \partial v_j}\left(f_T \frac{\partial^2 g_\alpha}{\partial v_i \partial v_j}\right)\right]\frac{4\pi n_\alpha q_T^2 q_\alpha^2}{m_T^2}\ln \Lambda,$$

$$g_\alpha(\mathbf{v}) = \int f_\alpha(\mathbf{v}')|\mathbf{v}-\mathbf{v}'|d\mathbf{v}'$$

$$h_\alpha(\mathbf{v}) = \frac{m_T}{\mu_\alpha}\int \frac{f_\alpha(\mathbf{v}')}{|\mathbf{v}-\mathbf{v}'|}d\mathbf{v}' \qquad (4\text{-}13)$$

$$\mu_\alpha = \frac{m_T m_\alpha}{m_T + m_\alpha}$$

$$\Lambda = \frac{3}{2}\frac{kT_\alpha}{Z_\alpha^2 e^2}\left(\frac{kT_e}{\pi n_e e^2}\right)^{1/2}$$

where the functions, g_α and h_α, are usually called Rosenbluth potentials and subscripts α and T denote field particle and test particle, respectively.

Pastukhov's theory [53-56] is approximate analytical solutions of the Fokker-Planck equation. Assumptions to derive the solutions is as follows:

(1) The distribution function of trapped electron is Maxwellian except for the region, $v > v_\alpha$ (v_α is electron thermal velocity).

(2) Rosenbluth potentials can be linearized using the expansion of high velocity limit (for electrons).

(3) Low velocity sources is assumed for sustaining steady state operation.

(4) Loss rate is small enough to neglect the term of time variation in the Fokker-Planck equation compared to the scattering term near the loss boundary.

(5) The boundary condition of f=0 and the maximum of confining potential occurs at mirror throats.

(6) Fictitious "negative source" is assumed to establish approximately the condition f=0 at the loss boundary.

(7) Large mirror ratio is assumed.

(8) The time of particle scattering into the loss cone exceeds the transit time along the trap.

(9) The square well approximation is adopted.

Assumptions of (1)-(4) are satisfied on our experiment. An assumption of (5) is different in our configuration. This situation could cause the deviation of factor. Assumptions of (6)-(8) are required to solve the bounce averaged Fokker-Planck equation. A square well approximation is adopted in our calculation because the electrostatic potential profile along axis is unknown.

The solution of the distribution function,

$$
\begin{aligned}
\tilde{f} &= \frac{e^{-x^2}}{\pi^{3/2}} + \pi\int_a^\infty q e^{-\xi^2}\tilde{F}\xi^2 d\xi \\
&\approx \frac{e^{-x^2}}{\pi^{3/2}} \\
&- \frac{qa^2 x e^{-x^2}}{4Z_e\mu} \\
&\times \ln\left\{\frac{\dfrac{e^{a^2}}{\sqrt{2a^2}} + \mu\dfrac{e^{x^2}}{\sqrt{2x^2}} + \left[\dfrac{1-\mu^2}{Z_e}e^{2x^2} + \left(\dfrac{e^{a^2}}{\sqrt{2a^2}} + \mu\dfrac{e^{x^2}}{\sqrt{2x^2}}\right)^2\right]^{1/2}}{\dfrac{e^{a^2}}{\sqrt{2a^2}} - \mu\dfrac{e^{x^2}}{\sqrt{2x^2}} + \left[\dfrac{1-\mu^2}{Z_e}e^{2x^2} + \left(\dfrac{e^{a^2}}{\sqrt{2a^2}} - \mu\dfrac{e^{x^2}}{\sqrt{2x^2}}\right)^2\right]^{1/2}}\right\},
\end{aligned}
\tag{4-14}
$$

$$
W^2 \approx 1 + 1/RZ_e,\ \exp a^2 = W\exp\left(x_0^2\right),\ q \approx \frac{4Z_e}{\pi^{3/2}x_0^3\ln\left((W+1)/(W-1)\right)},
$$

is shown in Fig. 4-3. The contour plot shows that deeply trapped electrons form isotropic Maxwellian. Also fictitious "negative source" is

$$Q(x,\mu) = -q\exp(-x^2)\delta(1-\mu^2)\theta(x-a), \tag{4-15}$$

where θ is step function. The particle and energy loss rates are determined by integrating the "negative" source function over appropriately weighted phase volume elements ($x^2dxd\mu$ or $x^4dxd\mu$). The particle loss rate is expressed in

$$\frac{\tau_{ee}}{n}\frac{dn}{dt} = -\frac{4}{\sqrt{\pi}}Z_e\frac{T_e}{e\phi}\frac{1}{G(Rz_e)}\exp\left(-\frac{e\phi}{T_e}\right)I\left(\frac{T_e}{e\phi}\right). \tag{4-16}$$

$$G(x) = (1+1/x)^{1/2}\ln\left(\frac{(1+1/x)^{1/2}+1}{(1+1/x)^{1/2}-1}\right)$$

$$I(x) = 1+\frac{1}{2}\sqrt{\pi x}\exp\left(\frac{1}{x}\right)\left[1-erf(1/\sqrt{x})\right]$$

Also the energy loss rate is expressed in

$$\begin{aligned}\frac{3}{2}\frac{d(nT_e)}{dt} &= e\phi\left(\frac{1}{I(T_e/e\phi)}+\frac{3}{2}\frac{T_e}{e\phi}\right)\frac{dn}{dt} \\ &= -e\phi\left(\frac{1}{I(T_e/e\phi)}+\frac{3}{2}\frac{T_e}{e\phi}\right)\frac{4nZ_e}{\tau_{ee}\sqrt{\pi}G(Rz_e)}\frac{T_e}{e\phi}\exp\left(-\frac{e\phi}{T_e}\right)I\left(\frac{T_e}{e\phi}\right).\end{aligned} \tag{4-17}$$

Note that the main contribution to the loss of electrons from the trap is made by the region directly adjacent to the vertex of the hyperboloid.

Converting the loss rates to confinement times provides a convenient performance benchmark for tandem mirror operation. These conversions are normally based on a modification of left hand side of Eq. (4-16) and Eq. (4-17) replaced by $-n/\tau_p$ and $3nT/2\tau_E$, respectively.

Particle confinement time is expressed in

$$\tau_p = \frac{\sqrt{\pi}}{4}\tau_{ee}\left(\frac{e\phi}{T_e}\right)\exp\left(\frac{e\phi}{T_e}\right)\frac{G(R)}{I(T_e / e\phi)}. \qquad (4\text{-}18)$$

Also energy confinement time is

$$\tau_E = \frac{1}{\frac{2}{3}\frac{e\phi}{T_e} \Big/ I\left(\frac{T_e}{e\phi}\right) + 1}\tau_p. \qquad (4\text{-}19)$$

Some problems stated as follows.

(i) The real loss boundary is more complicated than the theory. The maximum of the confining potentials are not located at mirror throat. (See Fig. 4-4(c))

(ii) The passing particles is existed in real situation.

(iii) There is a possibility for electron in tandem mirror that finding the loss rates in a magnetic field is not approximatable by a single square well.

Answers are as follows.

(i) Pastukhov distribution function of R=1.23 shown in Fig. 4-5 agrees well with real loss boundary unexpectedly.

(ii) Passing particle is the warm electron in our case. Amount of the electron is known to be small. Hence, the electron can not cause additional large amount of collision induced axial losses.

(iii) Special case of the double square well, two square wells in series, is described in Ref [54]. The particle loss rate and energy loss rate are approximately reduced by factor. This cause no significant deviation.

4.4 Strong ECH Theory

The plug potential is created with fundamental ECH near the 1 T layer on the outer slope of the end-mirror magnetic field. Then plug electrons feel the axial force F_z

$$F_z = e\frac{\partial \Phi}{\partial z} - \mu\frac{\partial B}{\partial z}, \tag{4-20}$$

where μ is the magnetic moment. The plug electrons are push out of there since their magnetic moment increase by perpendicular heating by the plug ECH. As a result, an axial potential distribution is formed, if ion density is invariable, so as to keep the axial force being equal to zero for the charge neutrality condition.

In the strong ECH case [57], Cohen derives the expression for the potential difference ϕ_{pb} between the plug and barrier regions of a tandem mirror thermal barrier cell when ECH is strong enough to dominate over collisions for all electrons expect passing electrons which traverse the entire system. In this theory, it is assumed that passing electrons have much shorter transit time in the barrier cell than collisions by ECH; thus, their distribution function is the appropriate potion of a Maxwellian f_I with temperature T_{ec} and density n_c. Electron velocity space at the plug is shown in Fig. 4-6; passing electrons correspond to region I in Fig. 4-6. Furthermore, ECH at the plug is assumed to dominate for the electrostatically trapped electrons which do not reach the resonant heating zone near the barrier (region III), but heating at the barrier dominates for the particles in region II. Distribution functions under the strong ECH limit condition are constant along ECH characteristics. Then the characteristics for region II is ellipses concentric in the plug velocity space. In the absence of explicit sources and sinks, weak residual diffusion across characteristics (for example, due to collisional scattering) ensures that in region III, distribution is constant with a value set by continuity at the boundary with region I and II.

The relation between the electrostatically trapped plug density n_{pt} and the "thermal" barrier density n_m is given by integrating distribution function in region III and solving for ϕ_{pb}. One obtains

$$\phi_{pb} = T_{ec}\left(\frac{3}{4}\pi^{1/2}\frac{R_{pb}-1}{R_{pb}}\frac{n_{pt}}{n_m}\exp\{\phi_b/[T_{ec}(R_{mb}-1)]\}\right)^{2/3}, \qquad (4\text{-}21)$$

where R_{pb} and R_{mb} are the ratio of the magnetic field at the plug to the barrier and that at the mirror throat to the barrier. Substituting the magnetic field strength in GAMMA 10, $\phi_{pb}=\phi_b+\phi_c$ and $n_m = n_c\exp(-\phi_b/T_{ec})$ with n_c the central cell density, the Eq. (4-21) is transformed

$$\phi_c = T_{ec}\left[0.665\frac{n_{pt}}{n_c}\exp\left(1.19\frac{\phi_b}{T_{ec}}\right)\right]^{2/3} - \phi_b, \qquad (4\text{-}22)$$

where n_{pt} can be interpreted as the total plug electron density n_p, since the densities of magnetically trapped and passing electrons at the plug are smaller than n_{pt}. The characterized semiconductor ion detector is utilized for the observations of end-loss ions in the GAMMA 10 tandem mirror.