Chapter 3
X-ray Measurements

3.1 X-ray Radiation from Plasmas

An X-ray diagnostic is one of the most important methods to investigate high temperature plasmas. X-ray radiation, which arises from bremsstrahlung, recombination, line radiation and two photons decay, gives information on electron density, electron temperature, fraction of impurity abundance and other properties [35-38].

Bremsstrahlung arises from Coulomb scattering of free electron with ions [39-41]. When the electron velocity distribution function is Maxwellian and ions of charge and density are Z_i and n_i, respectively, X-ray emissivity per unit volume and unit energy interval, as a function of photon energy k' is

$$\frac{dI_{ff}(k')}{dk'} \propto \frac{n_e}{\sqrt{kT_e}} \exp\left(-\frac{k'}{kT_e}\right) \sum_i n_i Z_i^2 g_{ff}, \qquad (3\text{-}1)$$

where k is Boltzmann constant and g_{ff} is the free-free Gaunt factor.

Recombination radiation is emitted when a free electron is trapped by an ion into a shell of principal quantum number n. The X-ray intensity by recombination radiation for Maxwellian electrons is given by

$$\frac{dI_{fb}(k')}{dk'} \propto \frac{n_e}{\sqrt{kT_e}} \sum_i \sum_n n_i Z_i^2 \frac{2E_n}{nkT_e} \exp\left(\frac{E_n - k'}{kT_e}\right) g_{fb}, \qquad (3\text{-}2)$$

$$(E_n < k')$$

where E_n is the electron binding energy at the shell of n and g_{fb} is the free-bound Gaunt factor.

The line intensity from level l to m is given as

$$J_{lm} = n(l)A_{lm}, \tag{3-3}$$

where $n(l)$ is population of level l, which is calculated from excitation and recombination rates, and A_{lm} is radiative transition probability from level l to m.

Two photons are emitted with a continuous spectrum from metastable $2S$ state of hydrogenic and helium-like ions. The X-ray intensity can be written as

$$\frac{dI_{2\gamma}(k')}{dk'} \propto P(2S-1S)\frac{k'^2(E_T-k')}{E_T^3}, \tag{3-4}$$

where E_T is the energy difference between $2S$ and $1S$ states and $P(2S\text{-}1S)$ is the photon emission rate in the two photon processes.

Total X-ray intensity $dI(k')/dk'$ is calculated by summation over Eqs. (3-1)-(3-4).

3.2 Pulse Height Analyses

The warm electron temperature T_{ew} is measured by soft X-ray pulse height analyzer (PHA) the line of sight of which lies in the midplane of the central cell region. A silicon lithium (Si(Li)) detector with a diameter of 0.4 cm and a depletion layer of 0.27 cm is used to measure the photon spectrum from 0.7 to 55 keV with a 0.008 mm beryllium window. A movable stainless steel collimator, 2.0 cm thick, with apertures of 5 mm, 10 mm and race track type of 10 mm wide and 36 mm long, and a fixed lead collimator 9.8 cm thick are placed in the vacuum chamber. The main aim of the silicon lithium detector is a detailed analysis of T_{ew}. To avoid exposure to background stray photons, the detector is shielded coaxially by 4.5 cm lead. Suppression of the background radiation is verified by plugging the front apertures of the detectors. The detected photon signals are amplified with a shaping time of 2 μs and analyzed in 4096 channels. The channels are divided into 64 or 32 blocks, depending on the duration of the successive sample periods (1 or 2 ms). The temporal evolution of the X-ray spectra is determined by carefully choosing several discharges with similar plasma parameters and adding up the spectra with the help of a data processing computer.

3.3 The X-ray Absorption Methods

Radial profiles of the X-rays are measured with a multi-anode Chevron type channel plate (50 channel MCP, Hamamatsu Photonics Type F1943-22MX) and a novel matrix-type semiconductor detector in the central cell region. The MCP has a tandem configuration and is coated with normal electrode materials (Fe and Cr), whose channel diameter is 15 μm, channel pitch 19 μm, length to diameter ratio 40, channel bias angle 13°, and open area ratio 0.57. The soft X-ray radiation is imaged on the MCP through a 1.9 cm thick lead collimator (aperture of 3 mm, spatial resolution in the plasma: 2.1 cm perpendicular to the line of sight in the central cell regions). Lead shields of 0.4 cm and 1.8 cm thickness are placed between the collimator and the MCP. Vacuum vessel of soft iron 1 cm thick is used as magnetic shield.

The observed X-ray signal is written as follows,

$$I_{SX} = \int_0^\infty \frac{dI(k')}{dk'} \exp(-\mu(k')\rho d)\eta(k')dk' , \qquad (3\text{-}5)$$

where μ, ρ, d, and $\eta(k')$ are the mass absorption coefficient, density, thickness of the absorber and the detection efficiency of the detectors, respectively.

The MCP and semiconductor detectors are calibrated with synchrotron radiation from the storage ring at the National Laboratory for High Energy Physics (KEK). Figure 3-1 shows X-ray intensity as a function of Maxwellian electron temperature with various absorbers, which is calculated from Eq. (3-5). T_e is estimated from the ratio of X-ray intensities with different absorbers. In Fig. 3-2, the calculated intensities with various T_e are normalized by the intensities with the polypropylene combined with the 1.5 μm thick polyester absorber as a function of the thickness of the polyester. The observed data are fitted on these curves, and then the best fit curve gives the value of T_e. Also, another method of the T_e evaluation is made using the temporal evolution of X-ray intensities of Isx. Under the assumption of a low Z condition or a constant value of Z, Isx is normalized by n_e and n_i, and then compared with the calculated curves in Fig. 3-1, since these curves are normalized by unit values of n_e, n_i and Z. Thus these depend on T_e alone.

The observed X-ray signals are digitized every 20 μs and then transferred to the data processing computer.

3. 4 Tomographic Reconstructions of X-ray Images

The data analyses to reconstruct a two-dimensional X-ray emissivity in the plasma cross section are carried out by using a computer tomography technique. We have used the Cormack inversion [42], and its procedure including the response characteristics of the detectors is briefly summarized as follows: The X-ray brightness $f(\kappa,\phi)$ measured by each channel of the detectors is a line integral of the X-ray emissivity $g(r,\chi)$ along the viewing chord L. Here, (r,χ) represent polar coordinates, κ is the chord radius and ϕ is the angle between the normal to the chord L and the axis given by χ=0. The brightness $f(\kappa,\phi)$ is written as,

$$f(\kappa,\phi)=\left(\frac{s_c s_d \cos\chi_c \cos\chi_d}{4\pi d^2}\right)\int_L g(r,\chi)dl, \tag{3-6}$$

where s_c and s_d are the area of the aperture and the effective area of the detectors, respectively. In order to make a precise evaluation of the detector viewing region the quantity in the brackets of Eq. (3-6) is calculated in detail. The length, d, is the distance from the collimator to the detectors; χ_c is the angle between the normal to the aperture cross section, l_c, and the chord L; χ_d is the angle between the normal to each detection channel surface of the detectors, l_d, and the chord L. In our experimental setup, the 50 channel MCPs and the novel matrix-type X-ray semiconductor detector are assembled in a flat surface, and hence the line l_c, is parallel to the lines l_d. Thus, χ_d equals to χ_c. Therefore, we can rewrite Eq. (3-6) to Eq. (3-7),

$$f(\kappa,\phi)=\left(\frac{s_c s_d (\cos\chi_c)^4}{4\pi d_0^2}\right)\int_L g(r,\chi)dl, \tag{3-7}$$

where d_0 is the distance from the aperture to the center of the detectors, as shown in Fig. 3-3.

Here, it is noted that, in order to obtain the values of $g(r,\chi)$ in Eq. (3-7), corrections for the detected currents are made by using the calibration data of the detectors as well as the X-ray absorber characteristics.

Next, $f(\kappa,\phi)$ and $g(r,\chi)$ are expanded in Fourier harmonics in terms of ϕ and χ;

$$f(\kappa,\phi)=\sum_{m=0}^{\infty}\left\{f_m^c(\kappa)\cos m\phi+f_m^s(\kappa)\sin m\phi\right\}, \tag{3-8}$$

$$g(r,\chi)=\sum_{m=0}^{\infty}\left\{g_m^c(r)\cos m\chi+g_m^s(r)\sin m\chi\right\}, \tag{3-9}$$

where $f_m(\kappa)$ and $g_m(r)$ are the expansion coefficients, and for simplicity, the constant value on the right hand side of Eq. (3-7) is replaced by unity. If $f_m(\kappa)$ is expanded in the following form,

$$f_m^{c,s}(\chi)=2\sum_{l=0}^{\infty}a_m^{(c,s)l}\sin\{(m+2l+1)\cos^{-1}\kappa\}, \tag{3-10}$$

then the function $g_m(r)$ is

$$g_m^{c,s}(r)=\sum_{l=0}^{\infty}(m+2l+1)a_m^{(c,s)l}R_m^l(r), \tag{3-11}$$

and

$$R_m^l(r)=\sum_{n=0}^{l}\frac{(-1)^n(m+2l-n)!\,r^{m+2l-2n}}{n!(m+l-n)(1-n)!}, \tag{3-12}$$

where $R^l_m(r)$ is the Zernicke polynomial. In our data analyses, we take into account angular harmonics of m=0, $cos\chi$, $sin\chi$ and $cos2\chi$ as well as radial harmonics from l=0 to L_0, L_1^c, L_1^s and L_2^c, respectively. The coefficients of $a_m^{(c,s)l}$ are determined by the least squares fit, that is, the following value is minimized:

$$
\begin{aligned}
\sum_{i=1}^{N}[u_i &- 2\sum_{l=0}^{L_0} a_0{}^l \sin\{(2l+1)\cos^{-1}\kappa_i\} \\
&- 2\sum_{l=0}^{L_1^c} a_1^{c}{}^l \cos\phi_i \sin\{(2l+1)\cos^{-1}\kappa_i\} \\
&- 2\sum_{l=0}^{L_1^s} a_1^{s}{}^l \sin\phi_i \sin\{(2l+1)\cos^{-1}\kappa_i\} \\
&- 2\sum_{l=0}^{L_2^c} a_2^{c}{}^l \cos 2\phi_i \sin\{(2l+3)\cos^{-1}\kappa_i\}]^2,
\end{aligned}
\qquad (3\text{-}13)
$$

where u_i is i-th data and N is a number of detectors.