Higher Order Approximations for the Statistical Procedures

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A dissertation submitted to the Doctoral Program in Mathematics, the University of Tsukuba in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Science)

January, 2000

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Preface

In statistical inference, it is often useful to consider a higher order approximation in comparing statistical procedures like estimators and confidence intervals. In this thesis, using the higher order approximation, we show that some procedures are asymptotically better than others in various situations.

In Chapter 1, we consider the distribution of a sum of the dependent Bernouilli random variables. First it is shown that the distribution is obtained as a generalized binomial distribution determined by a two-state Markov chain, and its asymptotic distributions are also derived from the central limit theorem and the Edgeworth expansion. Further a distribution of the sum by the Bayesian approach is derived and its asymptotic distributions are provided. Numerical results are given. Consequently, the Edgeworth expansion is seen to be comparatively accurate.

In Chapter 2, for a one-parameter exponential family of discrete distributions, the higher order approximation of randomized confidence intervals derived from the optimum test is discussed. It is also seen from numerical results that the approximation is very useful in the case of Poisson, binomial distributions. Further, for the negative binomial and logarithmic series distributions, we obtain some approximations of the randomized confidence interval and give their numerical comparisons showing the higher order approximation to be useful in construction of the confidence interval.

In test for departure from normality, it is known that the kurtosis statistic \( b_2 \) plays an important part. In Chapter 3 we derive approximation formulae of a percentage point of the distribution of \( b_2 \) using the Cornish-Fisher expansion after a \( 1/k \)-th power transformation for \( k > 0 \). We also compare the values by the formula with those by the Pearson type curve of Pearson and Hartley ([PH76]). The values in case of \( k = 2 \) are consequently seen to be comparatively stable.

For a sum of not identically but independently distributed discrete random variables, its higher order large-deviation approximations are given in Chapter 4. They are compared with the normal and Edgeworth type approximations in various cases including the binomial and negative binomial ones. Consequently, the large-deviation approximations give sufficiently accurate results.