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Dynamical Quark Effects on Light Quark Masses

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Light quark masses are calculated in lattice QCD with two degenerate flavors of dynamical quarks. The calculations are made with improved actions with lattice spacing $a = 0.22$–0.11 fm. In the continuum limit we find $m_{ud}^{\overline{\rm MS}}(2\ {\rm GeV}) = 3.44^{+0.14}_{-0.22}$ MeV using the π and ρ meson masses as physical input, and $m_s^{\overline{\rm MS}}(2\ {\rm GeV}) = 88^{+4}_{-6}$ MeV or 90^{+5}_{-11} MeV with the K or ϕ meson mass as additional input. The quoted errors represent statistical and systematic combined, the latter including those from continuum and chiral extrapolations, and from renormalization factors. Compared to quenched results, two flavors of dynamical quarks reduce quark masses by about 25%.

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Masses of light quarks belong to the most fundamental parameters of the standard model [1], and yet their precise values have been difficult to determine due to quark confinement. Lattice QCD provides a fundamental approach to overcome this problem [2] since it enables first principle calculations of hadron masses as functions of quark masses. This approach has progressed considerably recently [3–5] through high statistics calculations that have allowed the continuum limit to be taken for quark masses, and the development of nonperturbative renormalization techniques for a reliable conversion of lattice bare quark masses to those in the continuum.

These studies, however, have been carried out within the quenched approximation which ignores effects of sea quarks. A limitation, shown in Ref. [4], is that the strange quark mass cannot be consistently determined, with the values differing by 20% depending on the choice of meson mass taken for input. It has been suspected [2,3], furthermore, that dynamical sea quark effects sizably reduce the values of light quark masses.

Clearly, systematic full QCD studies incorporating dynamical sea quark effects are needed for progress in the determination of quark masses. A recent attempt has been reported in [6]. In this Letter, we present results of our investigation in which the u and d quarks, assumed degenerate, are simulated dynamically while the s quark is treated in the quenched approximation [7].

Full QCD simulations are computationally much more demanding than quenched simulations. This problem can be significantly eased by the use of improved actions. Because of reduced cutoff errors, they should allow continuum extrapolations from coarser lattices, and hence require smaller lattice sizes, and smaller computational costs, for simulations with the same physical lattice size.

We employ improved actions both for the gluon part and the quark part. The gluon action consists of 1×1 and 1×2 Wilson loops whose coefficients are determined by an approximate renormalization-group analysis [8]. For the quark part we choose the "clover" improvement of the Wilson action [9], adopting, for the clover coefficient, a mean-field value $c_{\rm SW} = P^{-3/4}$. We substitute the one-loop result $P = 1 - 0.1402g^2$ [8] for the plaquette P, which agrees within 8% with the values measured in our runs. The one-loop result of $c_{\rm SW}$ [10] is found to be close to our choice.

The improved action described here was tested in our preparatory full QCD study [11]. We found that scaling violation in hadron masses is small with this action already at $a^{-1} \approx 1$ GeV, as compared to $a^{-1} \gtrsim 2$ GeV needed for the standard plaquette and Wilson quark actions. We therefore aim at a continuum extrapolation from simulations made at $a^{-1} \approx 1$–2 GeV.

We make runs at three values of the coupling $\beta \equiv 6/g^2 = 1.8, 1.95, 2.1$ to cover this range, employing lattices of a similar physical spatial size $La \approx 2.5$ fm, as listed in Table I. For each β, gauge configurations are generated by the hybrid Monte Carlo algorithm at four values of the sea quark hopping parameter $\kappa_{\rm sea}$ corresponding to the pseudoscalar (PS) to vector (V) meson mass ratio of $M_{\rm PS}/M_{\rm V} \approx 0.8$, 0.75, 0.7, and 0.6. For each sea quark mass, we calculate hadron masses at five values of the valence quark hopping parameter $\kappa_{\rm val}$ corresponding to $M_{\rm PS}/M_{\rm V} \approx 0.8$, 0.75, 0.7, 0.6, and 0.5, taking unequal as well as equal quark mass cases. Masses are extracted from hadron propagators with the standard correlated χ^2

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TABLE I. Run parameters of two-flavor full QCD simulations. The scale a is set by $M_\rho = 768.4$ MeV.

β	$g^2_{\overline{\mathrm{MS}}}$ $(\mu = 1/a)$	$L^3 \times T$	c_{SW}	a (fm)	La (fm)	$m_{\mathrm{PS}}/m_{\mathrm{V}}$ for sea quarks:No. of Traj.
1.80	3.168	$12^3 \times 24$	1.60	0.215(2)	2.58(3)	0.807(1):6250 0.753(1):5000 0.694(2):7000 0.547(4):5250
1.95	2.816	$16^3 \times 32$	1.53	0.155(2)	2.48(3)	0.804(1):7000 0.752(1):7000 0.690(1):7000 0.582(3):5000
2.10	2.565	$24^3 \times 48$	1.47	0.108(1)	2.58(3)	0.806(1):4000 0.755(2):4000 0.691(3):4000 0.576(3):4000

fit. Errors are estimated with the jackknife procedure with a bin size of 50 trajectories, derived from an autocorrelation study.

For the Wilson-type quark action including the clover action, different definitions of quark masses lead to results that differ at finite lattice spacing due to explicit breaking of chiral symmetry. We employ three definitions in the present work, checking consistency among them for reliability of results: (i) the quark mass defined using the axial vector Ward identity $\nabla_\mu A_\mu^{\mathrm{imp}}(x) = 2m_q^{\mathrm{AWI}} P(x)$, with P the pseudoscalar density and $A_\mu^{\mathrm{imp}} = A_\mu + c_A \nabla_\mu P$ the axial vector current improved to $O(a)$; (ii) another possibility, suggested by the vector Ward identity and naturally appearing in perturbative analyses, reads $m_q^{\mathrm{VWI}} a = (1/\kappa - 1/\kappa_c)/2$, where κ_c represents the critical hopping parameter at which the PS meson mass vanishes $M_{\mathrm{PS}}(\kappa_{\mathrm{val}} = \kappa_{\mathrm{sea}} = \kappa_c) = 0$; (iii) a third possibility, suggested in [12] and denoted by $m_q^{\mathrm{VWI,PQ}}$, replaces κ_c in (ii) by the "partially quenched" critical value κ_c^{PQ} where the PS meson mass vanishes as a function of the valence hopping parameter κ_{val} when κ_{sea} for sea quark is fixed to the physical point of u and d quark, $M_{\mathrm{PS}}(\kappa_{\mathrm{val}} = \kappa_c^{\mathrm{PQ}}; \kappa_{\mathrm{sea}} = \kappa_{ud}) = 0$.

We express the PS meson mass M_{PS}^2 in terms of quark masses by a general quadratic ansatz of the form,

$$M_{\mathrm{PS}}^2 a^2 = b_s m_{\mathrm{sea}} a + b_v m_{\mathrm{val}} a + c_s (m_{\mathrm{sea}} a)^2 + c_v (m_{\mathrm{val}} a)^2 + c_{sv} m_{\mathrm{sea}} a m_{\mathrm{val}} a \,. \qquad (1)$$

Here $m_{\mathrm{val}} = (m_{\mathrm{val},1} + m_{\mathrm{val},2})/2$ with $m_{\mathrm{val},i}$ $(i = 1, 2)$ the bare mass of the valence quark and antiquark of the PS meson, and m_{sea} the mass of sea quark. We mention that details vary depending on the definitions, e.g., terms depending only on m_{sea} are absent for the case of AWI mass since M_{PS} is zero for vanishing $m_{\mathrm{val}}^{\mathrm{AWI}}$, and a cross term $\propto m_{\mathrm{val},1} a m_{\mathrm{val},2} a$ is found to be necessary in the case of VWI mass.

The vector meson mass M_{V} is written in a similar manner, adopting, however, PS meson masses as independent variables. We fit data with the formula

$$M_{\mathrm{V}} a = A + B_s \mu_{\mathrm{sea}} a + B_v \mu_{\mathrm{val}} a + C_s \mu_{\mathrm{sea}}^2 a^2 + C_v \mu_{\mathrm{val}}^2 a^2 + C_{sv} \mu_{\mathrm{sea}} a \mu_{\mathrm{val}} a \,. \qquad (2)$$

Here $\mu_{\mathrm{val}} = (\mu_1 + \mu_2)/2$ represents the average of PS meson mass squared $\mu_i = M_{\mathrm{PS}}^2(\kappa_{\mathrm{val},i}, \kappa_{\mathrm{val},i}; \kappa_{\mathrm{sea}})$ made of a degenerate quark-antiquark pair of the valence hopping parameter $\kappa_{\mathrm{val},i}$, and $\mu_{\mathrm{sea}} = M_{\mathrm{PS}}^2(\kappa_{\mathrm{sea}}, \kappa_{\mathrm{sea}}; \kappa_{\mathrm{sea}})$.

Fitting our hadron mass data with (1) and (2) we find reasonable results with χ^2/N_{DF} in the range 0.6–2.3 (except for (1) for VWI quark mass at $\beta = 1.8$ for which $\chi^2/N_{\mathrm{DF}} = 4.0$. We then determine the bare lattice value of the average u and d quark mass $m_{ud} a$ by fixing the valence and sea quark masses to be degenerate in (1) and (2), and requiring the experimental value for the ratio $M_\pi/M_\rho = 0.1757$. For the s quark mass $m_s a$ we use either $M_K/M_\rho = 0.6477$ or $M_\phi/M_\rho = 1.3267$ while keeping the sea quark mass m_{sea} at the value m_{ud} determined above. The lattice scale a^{-1} is set using $M_\rho = 0.7684$ GeV as input.

We convert bare quark masses calculated above to renormalized quark masses in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme at $\mu = 1/a$ through $m_R^{\mathrm{VWI}} a = Z_m(g^2) m_q^{\mathrm{VWI}} a$ and $m_R^{\mathrm{AWI}} a = [Z_A(g^2)/Z_P(g^2)] m_q^{\mathrm{AWI}} a$. In these relations, the $O(a)$ improvement terms with the coefficients b_m, b_A, and b_P are also included. For renormalization factors and improvement coefficients, including that for c_A, one-loop perturbative values for massless quark [13] are used. For the coupling constant we adopt a mean-field improved value in the $\overline{\mathrm{MS}}$ scheme appropriate for the renormalization-group-improved gluon action (see Table I for numerical values): $g_{\overline{\mathrm{MS}}}^{-2}(1/a) = (3.648 W_{1\times1} - 2.648 W_{1\times2})\beta/6 - 0.1006 + 0.03149 N_f$ where measured values extrapolated to zero sea quark mass are substituted for the Wilson loops. The results for quark masses are run from $\mu = 1/a$ to $\mu = 2$ GeV using the three-loop beta function for $N_f = 2$ [14]. Numerical values of quark masses at each β are given in Table II.

TABLE II. Renormalized quark masses (in MeV) in the $\overline{\mathrm{MS}}$ scheme at $\mu = 2$ GeV for each β, and in the continuum obtained by linear fits in a (for these fits χ^2/N_{DF} is also given). All errors are statistical.

β	m_{ud}^{VWI}	$m_{ud}^{\mathrm{VWI,PQ}}$	m_{ud}^{AWI}	$m_s^{\mathrm{VWI}}(K)$	$m_s^{\mathrm{VWI,PQ}}(K)$	$m_s^{\mathrm{AWI}}(K)$	$m_s^{\mathrm{VWI}}(\phi)$	$m_s^{\mathrm{VWI,PQ}}(\phi)$	$m_s^{\mathrm{AWI}}(\phi)$
1.8	2.277(27)	4.183(42)	3.322(37)	102.92(92)	104.54(93)	88.0(1.0)	129.1(2.2)	130.7(2.2)	113.9(2.4)
1.95	2.489(38)	4.064(43)	3.321(38)	100.65(98)	102.08(99)	87.2(1.0)	123.1(1.7)	124.5(1.7)	109.8(1.7)
2.1	2.966(55)	3.816(47)	3.344(46)	95.6(1.1)	96.4(1.1)	87.0(1.2)	108.0(2.2)	108.8(2.2)	100.0(2.2)
$a \to 0$	3.47(10)	3.50(10)	3.36(9)	89.4(2.3)	89.5(2.3)	85.8(2.4)	90.1(4.9)	90.3(4.9)	88.1(4.9)
χ^2/N_{DF}	10.8	2.4	0.07	2.1	2.7	0.03	6.0	6.5	2.4

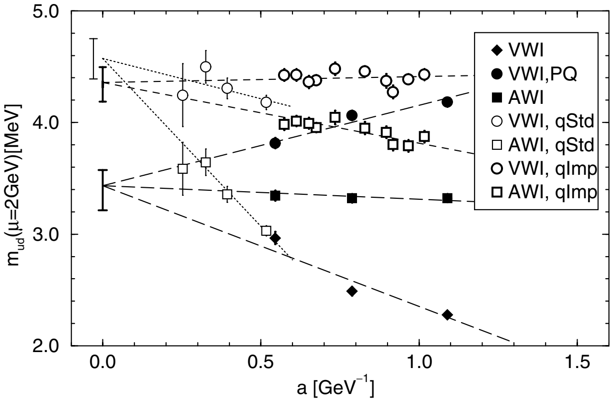

FIG. 1. Continuum extrapolation of the average up and down quark mass m_{ud} for full QCD (filled symbols) and quenched QCD (thick open symbols) obtained with the improved action, and quenched results with the standard action (thin open symbols). Lines show combined fits linear in a.

In Fig. 1 we show our two-flavor full QCD results for $m_{ud}^{\overline{\mathrm{MS}}}(2\text{ GeV})$ with filled symbols. The values for the three definitions, while differing sizably at finite lattice spacings [6], tend to converge toward the continuum limit. A similar trend has been seen in the quenched data for the standard action [4], reproduced in Fig. 1 with thin open symbols.

For our choice of the improved action, the scaling violation starts at $O[g^2_{\overline{\mathrm{MS}}}(\mu)a]$ for the quark masses at the scale $\mu = 2$ GeV. We therefore make a continuum extrapolation linear in a. The results are given in Table II. The masses for the three definitions are consistent with each other at two-sigma level of statistics. Hence we carry out a combined linear fit, as shown in Fig. 1 by dashed lines, obtaining $m_{ud} = 3.44(9)$ with $\chi^2/N_{\mathrm{DF}} = 2.9$ where the error is only statistical. The systematic error of the continuum extrapolation is estimated from the spread of values obtained by separate fits of data for the three definitions. The fractional error thus calculated is given in Table III.

This table lists our estimate for two more systematic errors that we need to incorporate. One is an uncertainty due to chiral extrapolations. We estimate this error from the change of the combined linear fit in the continuum limit when the quadratic term μ^2 in the vector mass formula (2) is replaced by $\mu^{3/2}$ or cubic terms m^3 are included in the PS mass formula (1). Another is the error due to the use of one-loop perturbative values for the renormalization factors. As nonperturbative values are not yet available, we estimate the effect of higher order contributions by recalculating masses while either shifting the matching scale from $\mu = 1/a$ to $\mu = \pi/a$ or using an alternative definition of coupling given by $g^{-2}_{\overline{\mathrm{MS}}}(1/a) = W_{1\times1}\beta/6 + 0.2402 + 0.03149N_f$ using only the plaquette.

TABLE III. Contributions to total error in continuum limit.

	Stat.	Chiral	Z factor	Cont. Ext.
m_{ud}	+2.6%	+1.2%	+2.3%	+1.7%
	−2.6%	−2.3%	−5.0%	−2.3%
m_s (K input)	+2.4%	+1.6%	+2.2%	+1.4%
	−2.4%	−2.2%	−5.6%	−2.8%
m_s (ϕ input)	+4.8%	+1.5%	+1.7%	+0.9%
	−4.8%	−7.6%	−6.9%	−1.6%

Combining the statistical error and the systematic errors listed in Table II by quadrature to obtain the total error, we find for our final value,

$$m_{ud}^{\overline{\mathrm{MS}}}(2\text{ GeV}) = 3.44^{+0.14}_{-0.22}\text{ MeV}. \quad (3)$$

Our full QCD result for the average u-d quark mass is considerably lower than our previous quenched result for the standard action given in [4] as 4.57(18) MeV, where the error is only statistical. In order to confirm that the decrease is a dynamical sea quark effect, we carry out a quenched simulation using the same improved gluon and quark actions as for the full QCD runs. This simulation is made at 10 values of β chosen so that the string tension matches that of two-flavor full QCD for each simulated value of sea quark mass and for the chiral limit at $\beta = 1.95$ and 2.1.

Analyses leading from hadron masses to quark masses parallel those for full QCD. In particular we employ polynomial chiral expansions of the form (1), (2), except that terms referring to sea quark masses are dropped. As a cross check we also make an analysis parallel to the one in [4], employing quenched chiral perturbation theory formulas, and obtain consistent results.

We plot results of this quenched analysis with thick open symbols in Fig. 1. Good consistency is observed between the continuum values for the standard and improved actions. We also note that scaling violations are visibly reduced for the latter. Making a combined linear extrapolation we obtain in the continuum limit $m_{ud}^{\overline{\mathrm{MS}}}(2\text{ GeV}) = 4.36^{+0.14}_{-0.17}$ MeV, where the error is estimated in a similar way as for full QCD. From this analysis we conclude that the effect of two dynamical quarks is to decrease $m_{ud}^{\overline{\mathrm{MS}}}(2\text{ GeV})$ by about 25%.

In Fig. 2 we show results for the strange quark mass $m_s^{\overline{\mathrm{MS}}}(2\text{ GeV})$ determined from either the K meson mass or the ϕ meson mass. Using K^* instead of ϕ gives the same results within 1%. The strange quark is heavy enough so that the difference between κ_c and κ_c^{PQ} has only small effects on the VWI masses in full QCD. Employing combined linear continuum extrapolations in a (with $\chi^2/N_{\mathrm{DF}} = 1.3$ and 3.0, respectively), and estimating the error in the same way as for m_{ud} (see Tables II and III for details), we obtain

$$m_s^{\overline{\mathrm{MS}}}(2\,\text{GeV}) = 88^{+4}_{-6}\text{ MeV} \qquad M_K\text{ input} \quad (4)$$

$$= 90^{+5}_{-11}\text{ MeV} \qquad M_\phi\text{ input}. \quad (5)$$

With (3) this gives $m_s^{\overline{\mathrm{MS}}}/m_{ud}^{\overline{\mathrm{MS}}} = 26(2)$ to be compared with 24.4(1.5) [15] computed from chiral perturbation theory to one loop.

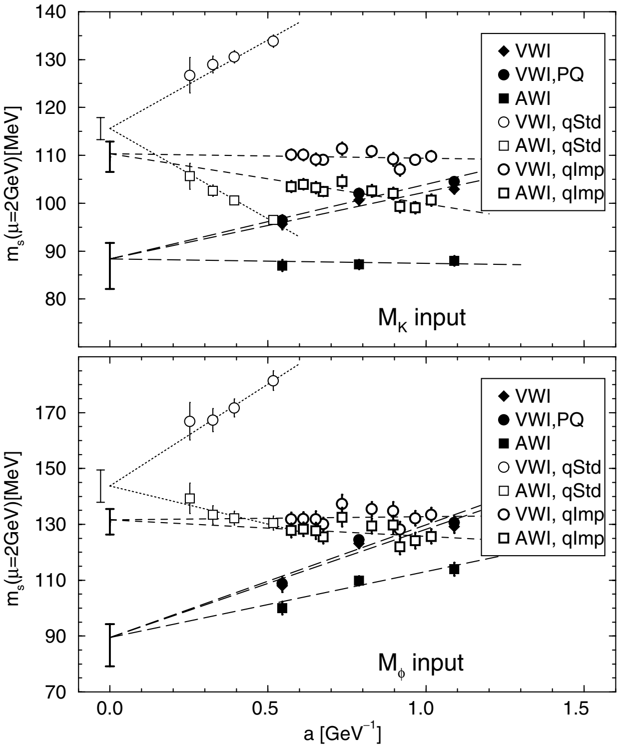

FIG. 2. Continuum extrapolation of the strange quark mass m_s. Symbols have the same meaning as in Fig. 1.

Similar analyses for quenched QCD with improved actions lead to the results $m_s^{\overline{\rm MS}}(2\ \text{GeV}) = 110^{+3}_{-4}$ MeV (M_K input) and 132^{+4}_{-6} MeV (M_ϕ input). For comparison we also quote the values 116(3) MeV (M_K input) and 144(6) MeV obtained with the standard action [4].

The quenched values for the standard and improved actions are mutually consistent for each choice of the input. This confirms the existence of a systematic uncertainty of 20%–30% in the value of $m_s^{\overline{\rm MS}}$ in quenched QCD [4].

One of our important results for the strange quark mass is that this uncertainty disappears within an error of 10% by the inclusion of two flavors of sea quarks. The consistency reflects a closer agreement of the $K - K^*$ and $K - \phi$ mass splittings with experiment in our two-flavor QCD results compared to the quenched case [7].

Another important result is that dynamical quark effects reduce the value of $m_s^{\overline{\rm MS}}$ significantly, from the range 110–140 MeV in quenched QCD to 90 MeV for two-flavor full QCD. It will be interesting to see whether the inclusion of dynamical effects of the strange quark itself would decrease the value of $m_s^{\overline{\rm MS}}$ even further. This is an important issue to settle as our two-flavor results are already close to the lower bounds estimated from the positivity of spectral functions [16].

Clearly, establishing the values of light quark masses incorporating three flavors of dynamical quarks will be one of the main tasks of future lattice QCD calculations.

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[1] S. Weinberg, Trans. N.Y. Acad. Sci. **38**, 185 (1977).
[2] Early studies were reviewed in A. Ukawa, Nucl. Phys. (Proc. Suppl.) **B30**, 3 (1993). For a recent review, see R. Gupta and T. Bhattacharya, Nucl. Phys. (Proc. Suppl.) **B63**, 95 (1998); Phys. Rev. D **55**, 7203 (1997).
[3] B. J. Gough *et al.,* Phys. Rev. Lett. **79**, 1622 (1997).
[4] CP-PACS Collaboration, S. Aoki *et al.,* Phys. Rev. Lett. **84**, 238 (2000); Nucl. Phys. (Proc. Suppl.) **B73**, 189 (1999).
[5] JLQCD Collaboration, S. Aoki *et al.,* Phys. Rev. Lett. **82**, 4392 (1999); D. Becirevic *et al.,* Phys. Lett. B **444**, 401 (1998); V. Giménez *et al.,* Nucl. Phys. **B540**, 472 (1999); S. Capitani *et al.,* Nucl. Phys. **B544**, 669 (1999); J. Garden *et al.,* Nucl. Phys. **B571**, 237 (2000); M. Göckeler *et al.,* Phys. Rev. D **62**, 054504 (2000).
[6] SESAM Collaboration, N. Eicker *et al.,* Phys. Lett. B **407**, 290 (1997).
[7] For preliminary reports, see, R. Burkhalter, Nucl. Phys. (Proc. Suppl.) **B73**, 3 (1999); CP-PACS Collaboration, S. Aoki *et al.,* Nucl. Phys. (Proc. Suppl.) **B73**, 192 (1999).
[8] Y. Iwasaki, University of Tsukuba Report No. UTHEP-118 (1983).
[9] B. Sheikholeslami and R. Wohlert, Nucl. Phys. **B259**, 572 (1985).
[10] S. Aoki *et al.,* Nucl. Phys. **B540**, 501 (1999).
[11] CP-PACS Collaboration, S. Aoki *et al.,* Phys. Rev. D **60**, 114 508 (1999).
[12] R. Gupta and T. Bhattacharya, in Ref. [2].
[13] S. Aoki *et al.,* Phys. Rev. D **58**, 074505 (1998); Y. Taniguchi and A. Ukawa, *ibid.* **58**, 114 503 (1998).
[14] O. V. Tarasov *et al.,* Phys. Lett. B **93**, 429 (1980).
[15] H. Leutwyler, Phys. Lett. B **378**, 313 (1996).
[16] For a recent overview, see S. Narison, Nucl. Phys. (Proc. Suppl.) **B86**, 242 (2000).