Anisotropy of vortex-liquid and vortex-solid phases in single crystals of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$: Violation of the scaling law

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The vortex-liquid and vortex-solid phases in single crystals of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ placed in tilted magnetic fields are studied by in-plane resistivity measurements using the Corbino geometry to avoid spurious surface-barrier effects. It was found that the anisotropy of the vortex-solid phase increases with temperature and exhibits a maximum at $T\approx 0.97T_c$. In contrast, the anisotropy of the vortex-liquid phase rises monotonically across the whole measured temperature range. The observed behavior is discussed in the context of dimensional crossover and thermal fluctuations of vortices in the strongly layered system.

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The anisotropic static and dynamical properties of superconductors have commonly been described by the three-dimensional Ginzburg-Landau (3DGL) theory with a single parameter of anisotropy $\gamma$ defined as a ratio $(m_a/m_ab)^{1/2}$ of the effective masses $m_a$ and $m_ab$ along the $c$ axis and in the $a$-$b$ plane, respectively. The anisotropy is related to the basic parameters of a superconductor as $\gamma = \xi_ab/\xi_c = \lambda_c/\lambda_ab = H_{c2l}/H_{c2}$, where $\xi_ab$ and $\xi_c$ are the in-plane and the out-of-plane coherence lengths, $\lambda_c$ and $\lambda_ab$ are the magnetic-field penetration depths along the $c$ axis and in the $a$-$b$ plane, while $H_{c2l}$ and $H_{c2}$ are the out-of-plane and the in-plane upper critical magnetic fields.

Based on 3DGL, Blatter et al. obtained the general scaling law, which can be applied for the description of different physical properties of anisotropic superconductors in oblique magnetic fields. For instance, the resistivity in a magnetic field $H$, tilted away from the $c$ axis for the angle $\theta$, depends only on the reduced field as

$$\rho(H, \theta) = \rho(H/\cos^2\theta + \sin^2\theta/\gamma^2).$$

(1)

The scaling law for the vortex lattice melting magnetic field was derived in a similar manner as

$$H_{c1}^m(\theta) = H_{c1}^m(\theta = 0)/\sqrt{\cos^2\theta + \sin^2\theta/\gamma^2}.$$  

(2)

In previous studies it was found that the scaling law describes well experimental data, such as resistivity, magnetization, and thermodynamic measurements in YBa$_2$Cu$_3$O$_{7-\delta}$ with anisotropy parameter $\gamma = 5 - 7$. Concerning the higher anisotropic layered systems, it was recognized earlier that the dissipation in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ depends practically only on the $c$-axis magnetic-field component except very close to the $a$-$b$ plane, which is in agreement with the 3DGL scaling law with high anisotropy parameter. However, it was reported that the resistivity data near the $a$-$b$ plane agree better with Thinkam’s thin film (hereafter 2DT) model than with the 3DGL theory. Later, it was also found that the measured anisotropy parameter for Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ thin films increases with temperature, according to the 2DT model, followed by an indication of the two- to three-dimensional (2D-3D) crossover near the transition temperature $T_c$. On the other hand, recent experimental studies of the first-order vortex-lattice melting transition (VLMT) have shown a linear decay of the out-of-plane melting field with an increasing in-plane magnetic field, even near the $c$ axis, in a strong contrast to the above-mentioned approaches. In a strict sense, it is reasonable to expect that the scaling law based on the 3DGL model may not be applicable to the extremely anisotropic layered system such as Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. In this material, the coherence length along the $c$ axis at low temperatures is smaller than the lattice parameters of the crystal, which means that the layerness of the material has to be considered. Thus, the 3DGL scaling approach, based on a continuous medium, is not valid due to the discreteness of the system, and therefore, it is not at all trivial that the expressions (1) and (2) are still valid.

Hence, there is still a question of how the anisotropy manifests itself in the different vortex phases in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ single crystals. In this paper, we report on the comparative study of the anisotropy of the vortex-solid and vortex-liquid phases in single crystals of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. The anisotropy of the first-order vortex-lattice melting transition increases with temperature in the range of $T < 0.97T_c$, whereas at higher temperatures, the VLMT anisotropy decreases. In contrast to the anisotropy of the VLMT, the anisotropy of the in-plane resistivity in the vortex-liquid phase increases monotonically with temperature in the whole measured range.

The in-plane resistivity measurements were performed for three as-grown single crystals of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ with transition temperatures $T_{c1} = 90.3$ K, $T_{c1} = 86.0$ K, and $T_{c2} = 90.0$ K for the samples s1, s2, and s3, respectively. In order to probe the true bulk properties, i.e., to avoid the surface-barrier effects that occur in conventional strip geometry, we have used the Corbino electrical contact configuration [see inset in Fig. 1(a)]. The diameters of the Corbino discs were $D_{I} = 1.9$ mm, $D_{II} = 1.95$ mm, and $D_{III} = 2.7$ mm while the thickness was $t = 20 \mu$m for all samples. The resistance has been measured by using the ac lock-in technique at a low frequency of 37 Hz. The magnetic field, generated up to 70 kOe by a superconducting split...
magnet, was rotated by a fine goniometer with angular resolution of 0.01°, which is better than the mosaicity of the crystals (≈0.03°).

A typical set of the resistance curves is presented in Fig. 1(a) for $T = 86.6$ K. The sharp resistance drop, attributed to the VLMT, is clearly detected across the wide angular range ($\theta < 89.86°$). Using the resistance criteria of $R = 10^{-6}$ Ω, which is set somewhat below the resistivity kink, and $R = 10^{-3}$ Ω, which is well above the VLMT anomaly, we construct the phase diagram of the VLMT and the equiresistance contour for the vortex liquid in the $H_{c2}(T)$ plane [Fig. 1(b,c)]. The VLMT phase line $H_{c2}(T)$ [Fig. 1(b)] exhibits the peculiar stepwise shape. Namely, starting from the $c$ axis, the out-of-plane component of the VLMT magnetic field, $H_{c2}^{\text{melt}}$, decays linearly with increasing $H_{ab}$. As the magnetic field further approaches the $a$-$b$ plane, the linear dependence $H_{c2}^{\text{melt}}(H_{ab})$ sharply transforms to the plateau [inset in Fig. 1(b)], which is attributed to the trapping of pancake vortices by Josephson vortices. Very close to the $a$-$b$ plane ($\theta \approx 89.94°$), the first-order phase transition [filled symbols in Fig. 1(b)] changes to the continuous resistance transition (open symbols), which exhibits the cusp in the $H_{c2}(H_{ab})$ phase plane. In strong contrast to the linear decay of $H_{c2}^{\text{melt}}(H_{ab})$ around the $c$ axis, the equiresistance line $H_{c2}^{\text{e}}(H_{ab})$ [Fig. 1(c)] in the vortex-liquid phase demonstrates a smooth quadratic behavior. Near the $a$-$b$ plane, the equiresistance contour still has a tail, as if it traces the VLMT cusp. The cusplike shape of both the VLMT line and the equiresistance contour, pronounced near the $a$-$b$ plane, is possibly associated with the intrinsic pinning which, surprisingly, seems to be active even in the vortex-liquid phase, quite far from the VLMT.

We have to mention that our measurements have been done in the $H$-$T$ region where strong fluctuations are expected. However, according to the theoretical analysis, the fluctuations of the amplitude of the order parameter are relevant only for high magnetic fields $H_{c2} \gtrsim H_{c2}(T) = 50 (1 - T/T_c) \text{kOe}$. Thus, fluctuations in the measured field range are related mostly to the fluctuations of the phase of the order parameter induced by the thermal fluctuations of the positions of vortices, which means that the commonly accepted concept of vortex-solid and vortex-liquid phase is applicable. The next question is how to estimate the anisotropy of the vortex-liquid phase and the VLMT. The smooth quadratic dependence $H_{c2}^{\text{e}}(H_{ab})$, can be derived from the general symmetry law $\rho(H_{c2},H_{ab}) = \varphi(H_{c2}^{\text{e}} - H_{ab}^{\text{e}})$, assuming that the resistivity is an analytical function of $H_{ab}$ in the vortex liquid. The last assumption seems to be natural deep in the structureless vortex-liquid phase, where phase transitions are not expected. The simple analysis gives the angular dependence $H_{c2}^{\text{e}}(H_{ab}) = H_{c2}^{\text{e}}(0) - H_{ab}^{\text{e}}[H_{c2}^{\text{e}}(0)/\gamma_{\text{phen}}^{2}]$ for the limited angular range of $0° < \theta < 90° - 57.3°/\gamma_{\text{phen}}^{2}$, with phenomenological anisotropy parameter $\gamma_{\text{phen}}(T,H_{c2})$. This equation successfully fits our data at 86.6 K in the vortex-liquid phase except very close to the $a$-$b$ plane with $\gamma_{\text{phen}} = 200$ [dotted line in Fig. 1(c)]. It is worth noting that the phenomenological angular dependence $H_{c2}^{\text{e}}(H_{ab})$ coincides with the angular dependence of the upper critical field $H_{c2}(\theta)$ for thin superconducting films obtained by Tinkham [11] as $H_{c2}(\theta) = H_{c2}(0) \cos \theta H_{c2}(0) + H_{c2}(\theta) \sin^{2}(\theta) \varphi_{\text{DGL}}^{2} H_{c2}(0) = 1$ with the field independent anisotropy parameter $\gamma_{\text{DGL}}^{2} = 1/\sqrt{1 - T/T_c}$. On the other hand, to describe the equiresistance contours in the vortex-liquid phase, it is convenient to apply the more common DGL theory. Following the DGL theory, the vortex-liquid phase, it is convenient to apply the more common DGL model. Following the DGL model, the vortex-liquid phase, it is convenient to apply the more common DGL model. Following the DGL model, the vortex-liquid phase, it is convenient to apply the more common DGL model.
FIG. 2. The temperature dependence of $\gamma_L$, extracted by fitting the equiresistance lines to an ellipse, for samples s1 and s2. The dashed line corresponds to the dependence $\gamma_L=28[1-(T/T_c)]^{1/2}$. Inset represents the dotted curves showing the $R(H)$ data obtained at $T=89.5$ K at various magnetic-field orientations with respect to the $c$ axis (from left, $\theta=66^\circ$, $74^\circ$, $80^\circ$, $84^\circ$, $86.5^\circ$, $88.05^\circ$, $88.75^\circ$, $89.2^\circ$, $89.39^\circ$, $89.6^\circ$). The solid curves represent scaled resistance curves according to Eq. (1) with $\gamma=\gamma_L=350$.

FIG. 3. (a) The phase diagrams of the vortex melting transition plotted in the phase plane ($H_c/H^*_c$-$H_{ab}/H^*_{ab}$) at different temperatures for s1. The left and right insets display the temperature dependence of $H^*_c$ and $H^*_{ab}$, respectively [for definition of the fields $H^*_c$ and $H^*_{ab}$, see Fig. 1(b)]. (b) The temperature dependence of the VLMT anisotropy defined as $\gamma_{mel}=H^*_c/H^*_{ab}$, obtained for three samples. The solid line is the empirical equation for $\gamma_{mel}(T)$ discussed in the inset. Inset represents the calculated temperature dependence of the anisotropy in the frame of the model (Ref. 26).

The temperature dependence of the characteristic fields $H^*_c$ and $H^*_{ab}$ exhibit different curvatures [see insets in Fig. 3(a)], which result in a maximum of the temperature dependence of the VLMT anisotropy $\gamma_{mel}(T)$ at $T\approx 0.97T_c$, as it is seen in Fig. 3(b). The obtained temperature dependence of $\gamma_{mel}$ may be approximated by the empirical equation $\gamma_{mel}(T) = \gamma_0(1-T/T_{c1})^\alpha(1-T/T_{c2})^\beta$ [the solid line in Fig. 3(b)] with parameters $T_{c1}=89.4$ K, $T_{c2}=90.7$ K, $\alpha=1.08$, and $\beta=0.6$, obtained by fitting the fields $H^*_c(T)=H^*_c(0)(1-T/T_{c1})^\alpha$ and $H^*_{ab}(T)=H^*_{ab}(0)(1-T/T_{c2})^\beta$ for the sample s1. The two characteristic temperatures ($T_{c1}$, $T_{c2}$) in the above empirical formula, may appear due to the fact that the thermodynamic vortex fluctuations have different influence on the interlayer and the intralayer superconductivity in the tilted magnetic fields. The similar maximum of $\gamma(T)$, accompanied by the disappearance of the cusp in the dependence $H_{onset}(\theta)$ ($R(H_{onset})=0$) near the $a$-$b$ plane, was observed earlier in thin films of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ although there was neither indication of the VLMT nor clear evidence which vortex phase was probed. The phenomenon was interpreted as a crossover from 2D to 3D behavior, which occurs when the coherence length $\xi_c$ exceeds the distance between CuO$_2$ layers. In agreement with this scenario, the cusp in the $H_c$-$H_{ab}$ phase diagram, associated with the continuous resistivity transition, also disappears around the temperature [see Fig. 3(a)] at which $\gamma_{mel}(T)$ is at a maximum. However, based on the 2D-3D crossover, it is impossible to explain why the anisotropy of the vortex-liquid phase still...
increases at temperatures $T > 0.97T_c$ where $\gamma_{mech}(T)$ decreases. Moreover, the 2D-3D scenario proposed in (Ref. 13) describes only the temperature and angular dependence of the upper critical field $H_{c2}(\theta, T)$, which, strictly speaking, cannot be applied to the VLMT and the equiresistance contours.

Another possible origin of the temperature dependence of the anisotropy could be related to the thermal fluctuations of vortices, which suppresses the Josephson coupling between the CuO$_2$ layers more efficiently than the superconductivity within the layers. As a result, $\lambda_0(T)$ increases faster than $\lambda_{ab}(T)$ and, thus, the anisotropy $\gamma=\lambda_0/\lambda_{ab}$ rises with increasing temperature. According to the model proposed for the magnetic field parallel to the c axis, the temperature dependence of the anisotropy is determined by the equation $\gamma^2/\gamma_0^2=\exp(\delta(T, T_c)\gamma^2/\gamma_0^2)$, where $\delta = \pi T H_s \lambda_{ab}^2(T)/2\Phi_0$ with $\gamma_0 = \gamma(T=0)$. In order to take account the temperature dependence of the VLMT and equiresistance magnetic fields, the empirical equation $H_\gamma = H_\gamma^0(1-T/T_c)$ is used. Since $\gamma$ is a monotonic function of $\delta(T, T_c)$, the temperature dependence of $\gamma$ has a maximum approaching $T_c$, if $\alpha > 1$, otherwise $\gamma$ monotonically increases with $T$. The inset in Fig. 3(b) displays the temperature dependence of $\gamma$ obtained by using the above equations with $\gamma_0 = 300$ and $\alpha = 1.08$, corresponding to the measured VLMT [inset in Fig. 3(b)], and $\alpha = 1$, as assumed for the vortex-liquid phase. Nevertheless, the calculated value of the magnetic field $H_0 = 29$ kOe disagrees with the experimental value of the c-axis field component, but is close to the total applied magnetic field. The discrepancy could be related to the presence of the Josephson vortices that have not been considered in the model. The pancake vortex sublattice interacts with the Josephson vortex sublattice in the crossing lattice structure (set in the tilted magnetic fields) providing the additional mechanism of the renormalization of the anisotropy in the vortex-solid phase.19,20

In summary, in contrast to the prevailing belief, we have found clear experimental evidence that neither the 3DGL scaling law nor the 2D Tinkham’s model consistently describe the resistivity in the vortex-liquid phase as well as the behavior of the vortex-lattice melting transition, which can qualitatively be accounted for by the thermal fluctuations of vortices.

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17 The radial current distribution results in the circular motion of pancake vortices away from the surface (for Josephson vortex surface barrier is not relevant). Besides, such a current distribution excludes the situation with force-free configuration, since the magnetic field cannot be parallel to the driving current in the whole sample. For more detailed discussion related to the Cobino geometry, see Ref. 15.
21 It is worth noting that the $H_c-H_{ab}$ phase diagram of the VLMT does not practically depend on the resistance criterion if the resistance level is taken below the kink anomaly. On the other hand, the field of the continuous resistance transition varies strongly with the resistance criterion, and a question concerning how the detected resistivity transition is related to the vortex freezing phase transition near the $a$-$b$ plane remains still unclear (for more detailed discussion, see Ref. 25).
24 In a wide angular range $90°-57.3°/\gamma > \theta$, the equation $R(H_c,H_{ab})=R(H_c,0)+\nu(H_c,0)H_{ab}^2$ with $\nu(H_c,0)=1/2(\partial^2 R(H_c,H_{ab})/\partial H_{ab}^2)|_{H_{ab}=0}$ can be used, since a linear term does not follow the symmetry restrictions. Thus, $H_{c0}^0(H_{ab})$ should satisfy the equation $R(H_c^0(H_{ab}),0)+\nu H_{ab}^0=R(H_c^0(0))$, with $R(H_c^0(0))=R^*$ and $R^*=10^{-3}$ Ω. The approximate solution of the equation, presented in the text, yields the phenomenological anisotropy $\gamma_{phen}(T,H_c(0)) = \sqrt{(\partial R(H_c,0)/\partial H_c)|_{H_c=0}-\nu_{ab}^2(H_c^0(0))}$. This equation can be used for $H_c > H_c^0(0)$.
27 We use the equation obtained in Ref. 26 for the Josephson coupling prevailing over the electromagnetic one, which is valid if $\gamma S < \lambda_{ab}(T)$. The last condition is satisfied at $T > T^*$, with $T^*/T_c = 1 - \lambda_{ab}^2(T=0)/\gamma^2 S^2 = 0.8$ for $\lambda_{ab}(T=0) = 2000$ Å, $S = 15$ Å, and $\gamma = 300$. 