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One loop calculation of the SUSY Ward-Takahashi identity on the lattice with a Wilson fermion

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The one loop correction to the SUSY Ward-Takahashi identity is calculated on the lattice with a Wilson fermion. The supersymmetry on the lattice is broken explicitly by the gluino mass and the lattice artifact. We should fine-tune the parameters in the theory to the point given by the additive mass correction in order to eliminate the breaking effect of the lattice artifact. It is shown that the additive mass correction appearing from the SUSY Ward-Takahashi identity coincide with that from the axial $U(1)_R$ symmetry, as suggested by Curci and Veneziano. Two important symmetries of the super Yang-Mills theory can be recovered simultaneously in the continuum with a single fine-tuning. The operator mixing of the supercurrent is also investigated. We find that the supercurrent mixes only with a gauge-invariant current T_μ which is related to the gamma-trace anomaly.

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I. INTRODUCTION

There has been great progress in the nonperturbative understanding of the low-energy behavior of $N=1$ supersymmetric (SUSY) QCD [1]. The analysis is based on global symmetry and the holomorphy of the superpotential and we can derive the nonperturbative form of the superpotential. This is quite satisfactory when we investigate the vacuum structure of the theory. However, when it is required to understand the low-energy particle spectrum, including excited states or the influence of the Kähler potential, this method is insufficient and some other nonperturbative method is required.

The lattice regularization when applied to a supersymmetric theory breaks its supersymmetry explicitly. This is mainly caused by the artifact of the lattice regularization itself and the fermion problem on the lattice. However the ability of the lattice field theory to perform the path integration nonperturbatively with the Monte Carlo method is so fascinating that several efforts have been made to resolve the difficulty of SUSY on the lattice. These attempts are classified into two types. One is to realize a SUSY on the lattice which corresponds to the ordinary supersymmetry in the continuum limit [2–4]. Although this method is beautiful in construction and can extract a peculiar feature of the model due to supersymmetry before taking the continuum limit in principle, it is applicable only to the free Wess-Zumino model up to now. In order to treat the $N=1$ supersymmetric Yang-Mills (SYM) theory, whose component fields (gluon and gluino) are forced to stay at different places (links and sites) on lattice to keep the gauge symmetry, we need the second method. In this method we do not persist in the supersymmetry and discretize the theory straightforwardly making use of the well-known actions on the lattice. The SUSY is recovered only in the continuum [5]. This restoration of supersymmetry is not automatic and the discussion in Ref. [5] is as follows.

The SYM theory has two important global symmetries in the continuum. One is supersymmetry and the other is axial $U(1)_R$ symmetry which is broken by the anomaly. Both symmetries are broken explicitly when the theory is regularized on the lattice with the Wilson plaquette action for the gluon and the Wilson fermion action for the gluino. The source of this symmetry breaking is classified into the introduction of a gluino mass which cannot be forbidden in the Wilson fermion and the lattice artifacts of discretization. This explicit breaking effect of the lattice artifacts is given by irrelevant operators in the Ward-Takahashi (WT) identity and vanishes in the continuum at tree level. However, when a quantum correction comes into play this term usually produces an additive correction. It is required to fine-tune several parameters of the theory in order to recover the symmetry in the continuum. Although this fine-tuning should be performed independently for each symmetry, we have only one parameter (gluino mass) in $N=1$ SYM which we can freely tune. It was discussed by Curci and Veneziano [5] that both symmetries are restored simultaneously with a single fine-tuning of the gluino mass to the chiral $U(1)_R$ symmetric point. Several Monte Carlo studies of SYM theory have been done along this line [6–11] to reproduce the prediction of the low-energy effective theory [12,13].

In this paper we formulate the $N=1$ SYM theory on lattices with the Wilson plaquette and the Wilson fermion action according to Ref. [5]. We calculate the one loop correction to the Ward-Takahashi identity of both the SUSY and $U(1)_R$ symmetry perturbatively in the gauge-variant Green function. It is shown that the additive mass correction appearing from the SUSY WT identity coincides with that from the axial $U(1)_R$ symmetry as was suggested. This means that both symmetries of the super Yang-Mills theory can be recovered simultaneously in the continuum with a single fine-tuning of the gluino mass. We also investigate the mixing behavior of the supercurrent with the on-shell condition for gluino momentum and mass. The supercurrent mixes with the gauge-invariant operator T_μ as was predicted in Ref. [5]. This current is related to the gamma-trace anomaly of the supercurrent. An extra mixing with gauge-variant operators occurs too. This is because we used the gauge-variant Green function in our calculation. However, these extra mixings vanish by setting the renormalized gluino mass to zero together with the on-shell condition.

This paper is organized as follows. In Sec. II we introduce the lattice SYM action and the Feynman rules relevant for the one loop calculation. In Sec. III we define the super transformation on lattice and give the concerning SUSY WT identity. The WT identity for $U(1)_R$ symmetry is also given in this section. Sections IV and V are devoted to the calculation of quantum correction at one loop level for the axial and SUSY Ward-Takahashi identity. Our conclusion is summarized in Sec. VI.

The physical quantities are expressed in lattice units and the lattice spacing a is suppressed unless necessary. We take the SU(N_c) gauge group with the gauge coupling g, the generator T^a, and the structure constant f^{abc}. The normalization is given as $\mathrm{tr}(T^aT^b)=\frac{1}{2}\delta^{ab}$.

II. ACTION AND FEYNMAN RULE

The SYM theory is given as a minimally gauge coupled massless adjoint Majorana fermion system in the continuum. In this paper we adopt the following lattice regularization procedure [5]. The gauge part is given by a standard four-dimensional Wilson plaquette action

$$S_{\mathrm{gluon}}=\sum_n \sum_{\mu\nu} -\frac{1}{g^2}\mathrm{Re}\,\mathrm{tr}(U_{n,\mu}U_{n+\hat{\mu},\nu}U^\dagger_{n+\hat{\nu},\mu}U^\dagger_{n,\nu}). \tag{2.1}$$

The gluino part is given by the Wilson fermion,

$$\begin{aligned} S_{\mathrm{gluino}}=&\sum_n \mathrm{tr}\Bigg[\frac{1}{2}\bar{\psi}(n)(-r+\gamma_\mu)U_\mu(n)\psi(n+\mu)U^\dagger_\mu(n)\\ &+\frac{1}{2}\bar{\psi}(n+\hat{\mu})(-r-\gamma_\mu)U^\dagger_\mu(n)\psi(n)U_\mu(n)\\ &+(M+4r)\bar{\psi}(n)\psi(n)\Bigg]\\ =&\sum_n \mathrm{tr}[\bar{\psi}(n)(-r+\gamma_\mu)U_\mu(n)\psi(n+\mu)U^\dagger_\mu(n)\\ &+(M+4r)\bar{\psi}(n)\psi(n)], \end{aligned} \tag{2.2}$$

where gluino field $\psi=\psi^aT^a$ is the adjoint representation of the gauge group and satisfies the Majorana condition,

$$\psi=\psi^C=C\bar{\psi}^T, \quad \bar{\psi}=\bar{\psi}^C=\psi^T(-C^{-1}). \tag{2.3}$$

The charge conjugation matrix is given as $C=\gamma_2\gamma_0$. We used this condition in the second equality of Eq. (2.2). Our γ matrix convention is as follows:

$$\gamma_i=\begin{pmatrix}0 & -i\sigma^i\\ i\sigma^i & 0\end{pmatrix}, \quad \gamma_4=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$

$$\gamma_5\equiv\gamma_1\gamma_2\gamma_3\gamma_4=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{2.4}$$

$$\sigma_{\mu\nu}=\frac{1}{2}[\gamma_\mu,\gamma_\nu], \tag{2.5}$$

$$\epsilon_{1234}=1. \tag{2.6}$$

Weak-coupling perturbation theory is developed by writing the link variable as $U_{x,\mu}=e^{igA_\mu(x+\hat{\mu}/2)}$ and expanding it in terms of gauge coupling g. We adopt a covariant gauge fixing with a gauge parameter α defined by

$$S_{\mathrm{GF}}=\sum_n \frac{1}{2\alpha}\left[\nabla_\mu A^a_\mu\left(n+\frac{1}{2}\hat{\mu}\right)\right]^2. \tag{2.7}$$

We set $\alpha=1$ in this paper. The ghosts do not contribute to our present calculation at one loop level. The gluon propagator can be written as

$$G^{ab}_{\mu\nu}(p)=\frac{1}{4\sin^2 p/2}\left[\delta_{\mu\nu}-(1-\alpha)\frac{4\sin p_\mu/2\sin p_\nu/2}{4\sin^2 p/2}\right]\delta_{ab}, \tag{2.8}$$

where $\sin^2 p/2=\Sigma_\mu \sin^2 p_\mu/2$.

The free gluino propagator is the same as that of the Dirac fermion on the lattice,

$$S^{ab}_{\mathrm{F}}(p)\equiv\langle\psi^a(p)\bar{\psi}^b(-p)\rangle=\frac{-i\Sigma_\mu\gamma_\mu\sin p_\mu+W(p)}{\Sigma_\mu\sin^2 p_\mu+W(p)^2}\delta_{ab}, \tag{2.9}$$

where

$$W(p)=M+r\sum_\mu (1-\cos p_\mu). \tag{2.10}$$

We set the Wilson parameter $r=1$ in this paper. A peculiar feature of the Majorana fermion is that the propagators which connect two ψ's or two $\bar{\psi}$'s give the nonzero contribution

$$\begin{aligned} \langle\psi^a(p)\psi^b(-p)\rangle&=S^{ab}_{\mathrm{F}}(p)(-C),\\ \langle\bar{\psi}^a(p)\bar{\psi}^b(-p)\rangle&=C^{-1}S^{ab}_{\mathrm{F}}(p). \end{aligned} \tag{2.11}$$

In order to calculate the one loop correction to the SUSY WT identity we need two kinds of gluon-gluino interaction vertices

$$\begin{aligned} V^{ab,c}_{1\mu}(k,p)=&-\frac{1}{2}gf^{abc}\bigg\{\gamma_\mu\cos\frac{1}{2}(-k_\mu+p_\mu)\\ &-ir\sin\frac{1}{2}(-k_\mu+p_\mu)\bigg\}, \end{aligned} \tag{2.12}$$

$$\begin{aligned} V^{ab,cd}_{2\mu\nu}(k,p)=&\frac{1}{8}g^2(f^{ace}f^{ebd}+f^{ade}f^{ebc})\bigg\{i\gamma_\mu\sin\frac{1}{2}\\ &\times(-k_\mu+p_\mu)-r\cos\frac{1}{2}(-k_\mu+p_\mu)\bigg\}\delta_{\mu\nu}, \end{aligned} \tag{2.13}$$

and three gluon self-interaction vertices,

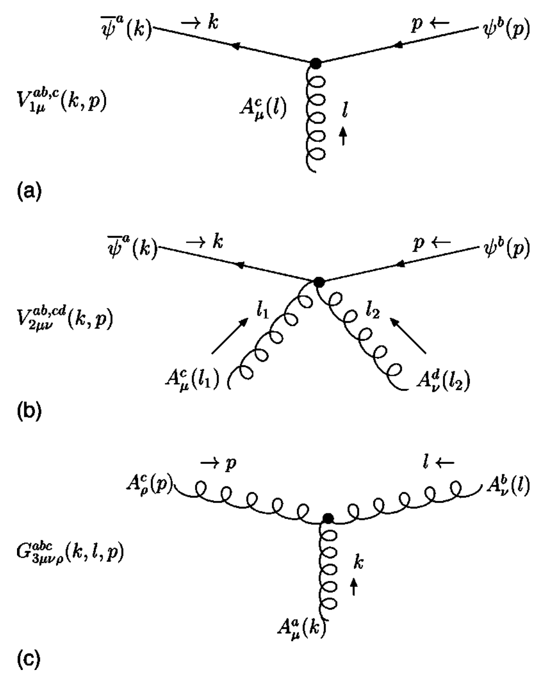

FIG. 1. Quark-gluon vertices needed for our one-loop calculations.

$$G^{abc}_{3\mu\nu\rho}(k,l,p)=\frac{1}{3}igf^{abc}\left(\delta_{\nu\rho}\cos\frac{k_\nu}{2}\sin\frac{1}{2}(p-l)_\mu + \delta_{\rho\mu}\cos\frac{l_\rho}{2}\sin\frac{1}{2}(k-p)_\nu+\delta_{\mu\nu} \times\cos\frac{p_\mu}{2}\sin\frac{1}{2}(l-k)_\rho\right). \tag{2.14}$$

Our assignments of momentum and color factors for the vertices are depicted in Fig. 1.

III. AXIAL AND SUSY WARD-TAKAHASHI IDENTITY ON LATTICES

The $U(1)_R$ transformation is given as an axial rotation of the gluino field,

$$\delta\psi=i\alpha\gamma_5\psi,\quad \delta\overline{\psi}=i\alpha\overline{\psi}\gamma_5 \tag{3.1}$$

with a rotation parameter α. The corresponding axial Ward-Takahashi identity is given by

$$\langle(\nabla_\mu j_{5\mu}(n))\mathcal{O}\rangle=2M\langle D_A(n)\mathcal{O}\rangle+\langle X_A(n)\mathcal{O}\rangle -\left\langle\frac{\delta\mathcal{O}}{\delta\alpha(n)}\right\rangle, \tag{3.2}$$

where the axial current $j_{5\mu}$ and the symmetry-breaking terms D_A, X_A are

$$j_{5\mu}(n)=\mathrm{tr}(\overline{\psi}(n)\gamma_\mu\gamma_5U_\mu(n)\psi(n+\mu)U^\dagger_\mu(n)), \tag{3.3}$$

$$D_A(n)=\mathrm{tr}(\overline{\psi}(n)\gamma_5\psi(n)), \tag{3.4}$$

$$X_A(n)=-r\sum_\mu \mathrm{tr}(\overline{\psi}(n)\gamma_5U_\mu(n)\psi(n+\mu)U^\dagger_\mu(n) +\overline{\psi}(n)\gamma_5U^\dagger_\mu(n-\mu)\psi(n-\mu)U_\mu(n-\mu) -2\overline{\psi}(n)\gamma_5\psi(n)). \tag{3.5}$$

∇_μ is a backward derivative, $\mathcal{O}$ is some operator, and $\alpha(n)$ is a localized transformation parameter. The trace is taken for the color indices only.

On the other hand there are several choices for the definition of the supertransformation on the lattice. The restriction is only to recover the proper form in the continuum limit. Adding to this condition we require the supertransformation to commute with the parity transformation on the lattice as in the continuum,

$$\psi(\vec{x},t)\to\psi^{\mathcal{P}}(\vec{x}^{\mathcal{P}},t)=\gamma_0\psi(-\vec{x},t),$$
$$\overline{\psi}(\vec{x},t)\to\overline{\psi}^{\mathcal{P}}(\vec{x}^{\mathcal{P}},t)=\overline{\psi}(-\vec{x},t)\gamma_0, \tag{3.6}$$

$$U_0(\vec{x},t)\to U_0^{\mathcal{P}}(\vec{x}^{\mathcal{P}},t)=U_0(-\vec{x},t),$$
$$U_k(\vec{x},t)\to U_k^{\mathcal{P}}(\vec{x}^{\mathcal{P}},t)=U_k^\dagger(-\vec{x}-\hat{k},t). \tag{3.7}$$

In this paper we adopt the following definition to satisfy the above conditions:

$$\delta_\xi U_\mu(n)=ig\overline{\xi}\gamma_\mu\frac{1}{2}[\psi(n)U_\mu(n)+U_\mu(n)\psi(n+\mu)], \tag{3.8}$$

$$\delta_\xi U^\dagger_\mu(n)=-ig\overline{\xi}\gamma_\mu\frac{1}{2}(U^\dagger_\mu(n)\psi(n)+\psi(n+\mu)U^\dagger_\mu(n)), \tag{3.9}$$

$$\delta_\xi\psi(n)=-\frac{1}{2}\sigma_{\mu\nu}\xi P_{\mu\nu}(n), \tag{3.10}$$

$$\delta_\xi\overline{\psi}(n)=\frac{1}{2}\overline{\xi}\sigma_{\mu\nu}P_{\mu\nu}(n), \tag{3.11}$$

where ξ, $\overline{\xi}$ are fermionic transformation parameters satisfying the Majorana condition. For the field strength $P_{\mu\nu}$ we employ the definition with clover plaquette,

$$P_{\mu\nu}(n)=\frac{1}{4}\sum_{i=1}^{4}\frac{1}{2ig}(U_{i\mu\nu}(n)-U^\dagger_{i\mu\nu}(n)), \tag{3.12}$$

$$U_{1\mu\nu}(n)=U_\mu(n)U_\nu(n+\hat{\mu})U^\dagger_\mu(n+\hat{\nu})U^\dagger_\nu(n), \tag{3.13}$$

$$U_{2\mu\nu}(n)=U_\nu(n)U^\dagger_\mu(n-\hat{\mu}+\hat{\nu})U^\dagger_\nu(n-\hat{\mu}), \tag{3.14}$$

$$U_{3\mu\nu}(n)=U^\dagger_\mu(n-\hat{\mu})U^\dagger_\nu(n-\hat{\mu}-\hat{\nu}) \times U_\mu(n-\hat{\mu}-\hat{\nu})U_\nu(n-\hat{\nu}), \tag{3.15}$$

$$U_{4\mu\nu}(n)=U_\nu^\dagger(n-\hat{\nu})U_\mu(n-\hat{\nu})U_\nu(n+\hat{\mu}-\hat{\nu})U_\mu^\dagger(n). \tag{3.16}$$

This definition is slightly different from the original one [5].

By transforming the vacuum expectation value of some operator

$$\langle\mathcal{O}\rangle=\int dUd\psi\mathcal{O}e^{-S_{\rm gluon}-S_{\rm gluino}} \tag{3.17}$$

with a localized transformation parameter we find the SUSY WT identity on the lattice,

$$\langle(\nabla_\mu S_\mu(n))\mathcal{O}\rangle=M\langle D_S(n)\mathcal{O}\rangle+\langle X_S(n)\mathcal{O}\rangle-\left\langle\frac{\delta\mathcal{O}}{\delta\bar{\xi}(n)}\right\rangle, \tag{3.18}$$

where the supercurrent S_μ and the gluino mass term D_S becomes

$$S_\mu(n)=-\frac{1}{2}\sum_{\rho\sigma}\sigma_{\rho\sigma}\gamma_\mu{\rm tr}(P_{\rho\sigma}(n)U_\mu(n)\psi(n+\hat{\mu})U_\mu^\dagger(n)$$
$$+P_{\rho\sigma}(n+\mu)U_\mu^\dagger(n)\psi(n)U_\mu(n)), \tag{3.19}$$

$$D_S(n)=\sum_{\rho\sigma}\sigma_{\rho\sigma}{\rm tr}(P_{\rho\sigma}(n)\psi(n)). \tag{3.20}$$

The explicit SUSY breaking term X_S is given by a sum of four terms

$$X_S(n)=X_S^{(1)}(n)+X_S^{(2)}(n)+X_S^{(3)}(n)+X_S^{(4)}(n) \tag{3.21}$$

with

$$X_S^{(1)}(n)=\sum_{\mu\rho\sigma}r\sigma_{\rho\sigma}\,{\rm tr}\left\{P_{\rho\sigma}(n)\left[\psi(n)-\frac{1}{2}(U_\mu(n)\psi(n+\mu)U_\mu^\dagger(n))-\frac{1}{2}(U_\mu^\dagger(n-\mu))\psi(n-\mu)U_\mu(n-\mu)\right]\right\}, \tag{3.22}$$

$$X_S^{(2)}(n)=\sum_{\mu\nu\rho\sigma}\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\gamma_\nu\gamma_5\,{\rm tr}(U_\mu^\dagger(n)\psi(n)$$
$$\times U_\mu(n)P_{\rho\sigma}(n+\mu)-U_\mu(n-\mu)\psi(n)$$
$$\times U_\mu^\dagger(n-\mu)P_{\rho\sigma}(n-\mu)), \tag{3.23}$$

$$X_S^{(3)}(n)=\sum_{\mu\nu}\gamma_\nu\,{\rm tr}(U_\mu(n-\mu)\psi(n)U_\mu^\dagger(n-\mu)H_{\mu\nu}(n-\mu)$$
$$+U_\mu^\dagger(n)\psi(n)U_\mu(n)H_{\mu\nu}(n+\mu)$$
$$-2\psi(n)H_{\mu\nu}(n)), \tag{3.24}$$

$$X_S^{(4)}(n)=-\frac{1}{2}ig\sum_\mu\{[\gamma_\mu\psi^a(n)]\overline{\psi^b}(n)(-r+\gamma_\mu)$$
$$\times\psi^c(n+\mu){\rm tr}([T^a,T^b]U_\mu(n)T^cU_\mu^\dagger(n))$$
$$+[\gamma_\mu\psi^a(n)]\overline{\psi^b}(n)(r+\gamma_\mu)\psi^c(n-\mu)$$
$$\times{\rm tr}([T^a,T^b]U_\mu^\dagger(n-\mu)T^cU_\mu(n-\mu))\}, \tag{3.25}$$

where $H_{\mu\nu}$ is given by a subtraction between the clover leafs,

$$H_{\mu\nu}(n)=\frac{1}{8ig}[(U_{1,\mu\nu}(n)-U_{1,\mu\nu}^\dagger(n))+(U_{4,\mu\nu}(n)$$
$$-U_{4,\mu\nu}^\dagger(n))-(U_{2,\mu\nu}(n)-U_{2,\mu\nu}^\dagger(n))$$
$$-(U_{3,\mu\nu}(n)-U_{3,\mu\nu}^\dagger(n))]. \tag{3.26}$$

Here we notice that $X_S^{(2)}$, $X_S^{(3)}$ come from the pure gauge part and $X_S^{(1)}$, $X_S^{(4)}$ are originated from the gluino action. Especially $X_S^{(1)}$ is given by transforming the gluino field in the Wilson term.

At tree level the breaking terms due to the Wilson fermion $X_S^{(1)}$, $X_S^{(4)}$ represent the $\mathcal{O}(a)$ irrelevant operator and those from the plaquette action $X_S^{(2)}$, $X_S^{(3)}$ represent the $\mathcal{O}(a^2)$ operator. The SUSY WT identity is recovered in the continuum with the gluino mass set to zero since all the irrelevant operators vanish and the lattice supercurrent $S_\mu(n)$ gives the continuum form

$$S_\mu(n)\to-\sum_{\rho\sigma}\sigma_{\rho\sigma}\gamma_\mu\,{\rm tr}(F_{\rho\sigma}\psi). \tag{3.27}$$

However, at one loop order every term gives the finite contribution and the symmetry restoration becomes nontrivial.

Derivation of the Feynman rules for the supercurrent and breaking terms is a straightforward but tedious task and the results are very complicated. We omitted them in this paper.

IV. ONE LOOP CORRECTION TO AXIAL WARD-TAKAHASHI IDENTITY

In this section we calculate the one loop correction to the axial Ward-Takahashi identity. We consider the WT identity with $\mathcal{O}=\psi(y)\overline{\psi}(z)$:

$$0=\langle(\nabla_\mu j_{5\mu}(n))\psi(y)\overline{\psi}(z)\rangle-2M\langle D_A(n)\psi(y)\overline{\psi}(z)\rangle$$
$$-\langle X_A(n)\psi(y)\overline{\psi}(z)\rangle+\delta_{n,y}\gamma_5\langle\psi(n)\overline{\psi}(z)\rangle$$
$$+\delta_{n,z}\langle\psi(y)\overline{\psi}(n)\rangle\gamma_5. \tag{4.1}$$

We calculate the quantum correction to each Green functions in the identity. Although our fermion is Majorana the one loop correction becomes the same as that for the Dirac fermion system [14,15] except for the color factor. One loop contributions to the Green functions $\langle(\nabla_\mu j_{5\mu})\psi\overline{\psi}\rangle$ and

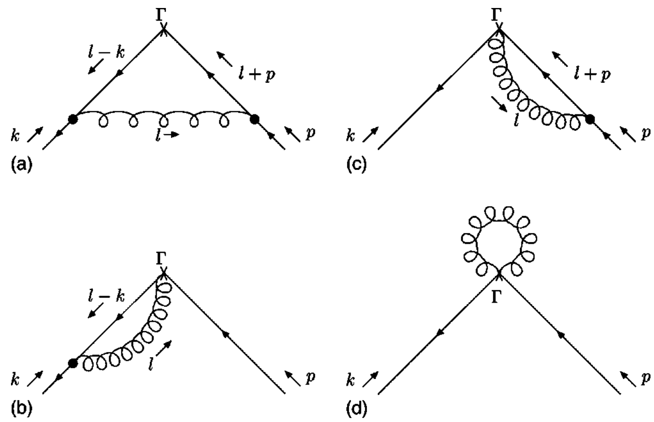

FIG. 2. One loop diagrams which contribute to the operator vertex correction in the axial Ward-Takahashi identity. $\Gamma = j_{5\mu}, D_A, X_A$ for the first diagram and $\Gamma = j_{5\mu}, X_A$ for the remaining three.

$\langle X_A \psi \bar{\psi} \rangle$ are given by the four diagrams in Fig. 2. One loop level full Green functions become

$$\langle (\nabla_\mu j_{5\mu}) \psi(k) \bar{\psi}(p) \rangle^{\mathrm{full}} = \frac{Z_2}{-i\not{k} + Z_m^{-1} M} i(k+p)_\mu T_A \gamma_\mu \gamma_5 \times \frac{Z_2}{i\not{p} + Z_m^{-1} M}, \tag{4.2}$$

$$\langle X_A \psi(k) \bar{\psi}(p) \rangle^{\mathrm{full}} = \frac{Z_2}{-i\not{k} + Z_m^{-1} M} [i(k+p)_\mu X_a \gamma_\mu \gamma_5 + (X_m M + X_0)\gamma_5] \frac{Z_2}{i\not{p} + Z_m^{-1} M}. \tag{4.3}$$

The one loop correction to $\langle D_A \psi \bar{\psi} \rangle$ is given by the first diagram in Fig. 2,

$$\langle D_A \psi(k) \bar{\psi}(p) \rangle^{\mathrm{full}} = \frac{Z_2}{-i\not{k} + Z_m^{-1} M} T_P \gamma_5 \frac{Z_2}{i\not{p} + Z_m^{-1} M}. \tag{4.4}$$

In this paper we adopt the lattice scheme [16] and evaluate the renormalization factors. This renormalization scheme is given by a Taylor expansion around zero external momentum and quark mass $k=p=M=0$ and a zero momentum subtraction. We start by rewriting the quantum correction as

$$T_A(k,p,M) = [T_A(k,p,M) - T_A^{\mathrm{cont}}(k,p,M)] + T_A^{\mathrm{cont}}(k,p,M), \tag{4.5}$$

where T_A^{cont} is a continuum form of quantum correction integrated in the lattice loop momentum between $-\pi/a$ and π/a. This T_A^{cont} is introduced to treat the infrared divergence of the massless theory which appears in T_A at $k=p=M=0$. Since T_A^{cont} has the same IR singularity, the IR divergence is subtracted in the first term of Eq. (4.5). We can evaluate it by a simple Taylor expansion around $(k,p,M)=(0,0,0)$. Meanwhile the second term should be calculated carefully with some IR cutoff. In this paper we introduce a gluon mass λ into the gluon propagator inside the loop as an IR cutoff.[1] We can evaluate T_A^{cont} quite simply with a Taylor expansion in terms of (k,p,M) keeping λ finite,

$$T_A^{\mathrm{cont}}(k,p,M;\lambda) = T_A^{\mathrm{cont}}(0;\lambda) + k_\mu \frac{\partial T_A^{\mathrm{cont}}(0;\lambda)}{\partial k_\mu} + \cdots, \tag{4.6}$$

where $T_A^{\mathrm{cont}}(0;\lambda)$ contributes to the renormalization of the operator and the remaining terms are $\mathcal{O}(a)$ errors.

In this scheme the vertex corrections are given as follows:

$$T_A = 1 + \frac{g^2}{16\pi^2} N_c \left[-\log\frac{(\lambda a)^2}{\pi^2} - 6.977 \right], \tag{4.7}$$

$$T_P = 1 + \frac{g^2}{16\pi^2} N_c \left[-4\log\frac{(\lambda a)^2}{\pi^2} + 2.585 \right], \tag{4.8}$$

$$X_a = \frac{g^2}{16\pi^2} N_c (8.664), \tag{4.9}$$

$$X_m = \frac{g^2}{16\pi^2} N_c (-19.285), \tag{4.10}$$

$$X_0 = \frac{g^2}{16\pi^2} N_c (102.8694). \tag{4.11}$$

Numerical errors of the finite parts in this section are in the last digit written. The gluino wave function and mass renormalization factors are evaluated by the quantum correction to the gluino propagator,

$$Z_2 = 1 + \frac{g^2}{16\pi^2} N_c \left[\log\frac{(\lambda a)^2}{\pi^2} + 15.641 \right], \tag{4.12}$$

$$Z_m = 1 + \frac{g^2}{16\pi^2} N_c \left[3\log\frac{(\lambda a)^2}{\pi^2} - 8.584 \right]. \tag{4.13}$$

We sum up the one loop level Green functions in the form of the WT identity in order to see the mixing behavior of the operators.

[1]The IR cutoff λ is canceled if we move to the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme [16,17]. The renormalization factors which connect the operators evaluated on lattice to that of the $\overline{\mathrm{MS}}$ scheme are independent of the gluon mass λ and are consistent with those calculated with other IR cutoff-like external momentum schemes [18] at the one loop level.

$$G_A=\langle(\nabla_\mu j_{5\mu})\psi(k)\overline{\psi}(p)\rangle^{\text{full}}-2M\langle D_A\psi(k)\overline{\psi}(p)\rangle^{\text{full}}-\langle X_A\psi(k)\overline{\psi}(p)\rangle^{\text{full}}$$
$$=\frac{Z_2}{-i\not{k}+Z_m^{-1}M}\left[i(k+p)_\mu(T_A-X_a)\gamma_\mu\gamma_5-2\left(M\left(T_P+\frac{X_m}{2}\right)+\frac{X_0}{2}\right)\gamma_5\right]\frac{Z_2}{i\not{p}+Z_m^{-1}M}$$
$$=Z_2(Z_A^{-1}\langle(\partial_\mu j_{5\mu})\psi(k)\overline{\psi}(p)\rangle_R-Z_2X_a\langle(\partial_\mu j_{5\mu})\psi(k)\overline{\psi}(p)\rangle_R-2M_R\langle D_A\psi(k)\overline{\psi}(p)\rangle_R), \quad (4.14)$$

where suffix R means the renormalized quantity and X_a represents a mixing between the explicit breaking term and the axial vector current. Here we notice that X_a is an $\mathcal{O}(g^2)$ quantity and we will make use of the relation $Z_2X_a=X_a+\mathcal{O}(g^4)$ in the following discussion.

The gluino mass is renormalized as

$$M_R=Z_m^{-1}(M-\Sigma_0), \quad (4.15)$$

where Σ_0 is an additive mass correction

$$\Sigma_0=-\frac{X_0}{2}=\frac{g^2}{16\pi^2}N_c(-51.4347), \quad (4.16)$$

which gives the critical hopping parameter corresponding to the chiral symmetric point

$$K_c=\frac{1}{8}\left(1-\frac{\Sigma_0}{4}\right). \quad (4.17)$$

The multiplicative mass renormalization factor evaluated from the WT identity agrees with that from the gluino propagator

$$Z_m^{-1}=Z_2\left(T_P+\frac{X_m}{2}\right)=1-\frac{g^2}{16\pi^2}N_c\left[3\log\frac{(\lambda a)^2}{\pi^2}-8.584\right]. \quad (4.18)$$

The renormalization factor of the axial vector current is given with the relation

$$(j_{5\mu})_R=Z_A(j_{5\mu})_{\text{bare}} \quad (4.19)$$

and becomes

$$Z_A^{-1}=Z_2T_A=1+\frac{g^2}{16\pi^2}N_c(8.664). \quad (4.20)$$

As is easily seen, contribution Z_A from the bare axial vector current and X_a from the explicit breaking term cancel each other

$$Z_A^{-1}-X_a=1.000, \quad (4.21)$$

and a proper form of the axial WT identity is recovered automatically for the renormalized quantities [14]

$$0=\langle(\partial_\mu j_{5\mu}(n))\psi(y)\overline{\psi}(z)\rangle_R-2M_R\langle D_A(n)\psi(y)\overline{\psi}(z)\rangle_R$$
$$+\delta_{n,y}\gamma_5\langle\psi(n)\overline{\psi}(z)\rangle_R+\delta_{n,z}\langle\psi(y)\overline{\psi}(n)\rangle_R\gamma_5. \quad (4.22)$$

V. ONE LOOP CORRECTION TO SUSY WARD-TAKAHASHI IDENTITY

We consider the following SUSY WT identity with $\mathcal{O}=A_\alpha(y)\psi(z)$:

$$0=\langle(\nabla_\mu S_\mu(n))A_\alpha(y)\overline{\psi}(z)\rangle-M\langle D_S(n)A_\alpha(y)\overline{\psi}(z)\rangle$$
$$-\langle X_S(n)A_\alpha(y)\overline{\psi}(z)\rangle+\delta_{n,y}\gamma_\alpha\langle\psi(y)\overline{\psi}(z)\rangle$$
$$+\delta_{n,z}\frac{1}{2}\sigma_{\rho\sigma}\langle A_\alpha(y)P_{\rho\sigma}(z)\rangle. \quad (5.1)$$

Although these Green functions are gauge variant we can derive gauge-independent quantities such as the additive mass correction correctly. One loop corrections to the Green functions $\langle(\nabla_\mu S_\mu)A_\alpha\overline{\psi}\rangle$, $\langle D_SA_\alpha\overline{\psi}\rangle$, and $\langle X_S^{(i)}A_\alpha\overline{\psi}\rangle$ with $i=1,2,3$ are given by the diagrams in Fig. 3. The one loop correction to the explicit breaking term with $i=4$ is given by the two diagrams in Fig. 4.

As in the previous section we adopt the lattice scheme [16] and introduce the gluon mass λ here again in order to settle the IR divergence of the massless theory at vanishing external momentum. At the one loop level all the IR divergence comes from the first four diagrams of Fig. 3 and can be regularized with this scheme. The fermion loops in Fig. 4 do not have any IR divergence.

In order to investigate the full Green function we need to evaluate the wave function renormalization factors of the gluino and gluon fields. The Z_2 for the gluino field is treatable in the lattice scheme with gluon mass λ. However, Z_3 of the gluon field has a contribution from the fermion and ghost loops and all the IR divergence at vanishing external momentum cannot be regularized with gluon mass λ. The lattice scheme [16] is not appropriate to renormalize the whole operators in the SUSY WT identity. Meanwhile the additive correction to the gluino mass and the mixing coefficient of the supercurrent with extra current T_μ are free from the IR divergence at the one loop level and need not be multiplied with the wave renormalization factor Z_2Z_3 since they are $\mathcal{O}(g^2)$ values. They can be investigated in any renormalization schemes appropriately at the one loop level. In this paper we will adopt the lattice scheme for simplicity and con-

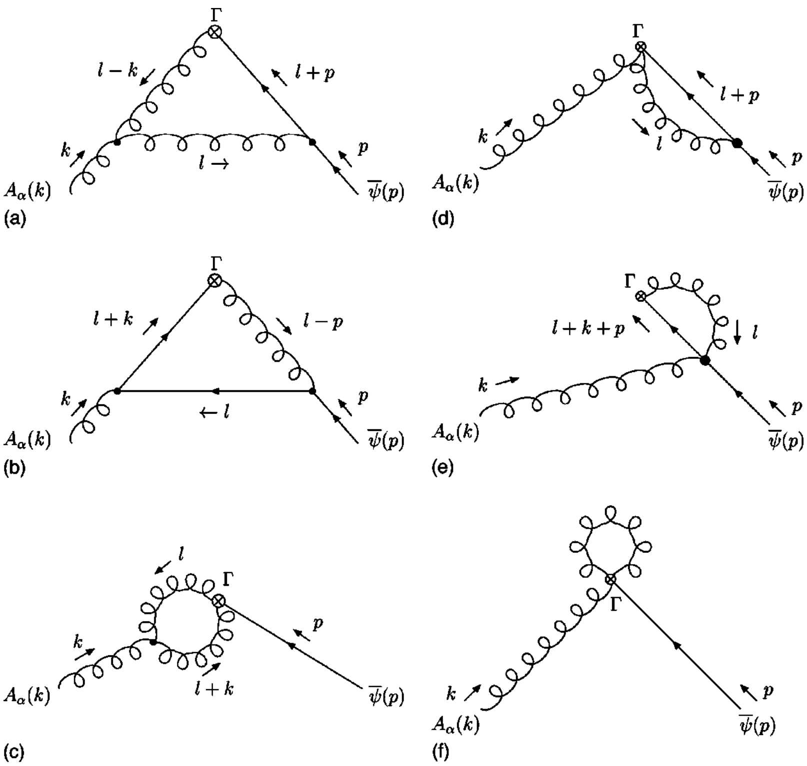

FIG. 3. One loop diagrams which contribute to the operator vertex correction in the SUSY WT identity. $\Gamma = S_\mu, D_S, X_S^{(1-3)}$.

centrate on these problems of additive mass correction and the operator mixing and evaluate the concerning vertex corrections only.

We calculate the one loop contribution imposing the on-shell condition to the external gluino momentum,

$$\Gamma(i\not{p}+M_R)=0, \tag{5.2}$$

where Γ represents some operator vertex. This is applicable to the bare mass inside the one loop correction by making

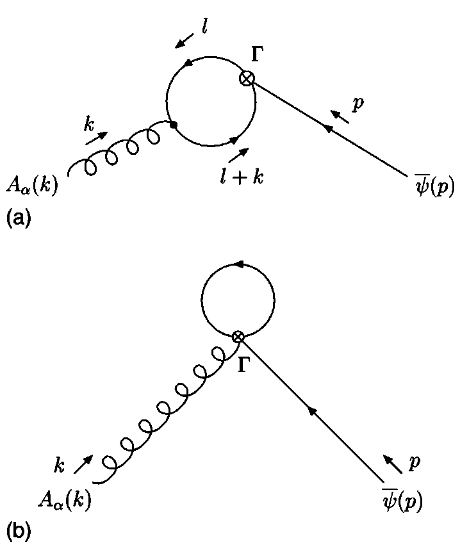

FIG. 4. One loop diagrams which contribute to the explicit SUSY breaking term; $\Gamma = X_S^{(4)}$.

use of the relation $g^2M = g^2M_R + \mathcal{O}(g^4)$. The one loop level vertex correction to the Green functions $\langle(\nabla_\mu S_\mu)A_\alpha\overline{\psi}\rangle$ and $\langle D_S A_\alpha\overline{\psi}\rangle$ are

$$\langle(\nabla_\mu S_\mu)A_\alpha(k)\overline{\psi}(p)\rangle_1 = \frac{1}{k^2}(k_\mu+p_\mu)(k_\nu\sigma_{\nu\alpha}\gamma_\mu T_S^{(S)} + (\delta_{\mu\alpha}\not{k} - k_\mu\gamma_\alpha)T_T^{(S)})\frac{1}{i\not{p}+M}, \tag{5.3}$$

$$\langle D_S A_\alpha(k)\overline{\psi}(p)\rangle_1 = \frac{1}{k^2}(iMk_\mu\sigma_{\mu\alpha}T_D^{(D)} + p_\alpha\not{p}T_{GP}^{(D)} + p^2\gamma_\alpha T_{GM}^{(D)})\frac{1}{i\not{p}+M}, \tag{5.4}$$

where

$$T_S^{(S)} = \frac{g^2}{16\pi^2}N_c\left(-27.874(1) + \frac{2\pi^2}{N_c^2}\right), \tag{5.5}$$

$$T_T^{(S)} = \frac{g^2}{16\pi^2}N_c[6.372(1)], \tag{5.6}$$

$$T_D^{(D)} = \frac{g^2}{16\pi^2}N_c\left(-3\ \ln\frac{(\lambda a)^2}{\pi^2} - 23.6453(7) + \frac{2\pi^2}{N_c^2}\right), \tag{5.7}$$

TABLE I. Contribution to additive mass correction from each $X_S^{(i)}$ at $r=1$. 80% of the contribution is given by $X_S^{(1)}$ and $X_S^{(4)}$.

X_S^i	$X_S^{(1)}$	$X_S^{(2)}$	$X_S^{(3)}$	$X_S^{(4)}$
$T_D^{\rm add}(i)$	42.2083(5)	7.32191(2)	2.85525(4)	−0.95093(2)

$$T_{GP}^{(D)}=\frac{g^2}{16\pi^2}N_c[1.000(1)], \tag{5.8}$$

$$T_{GM}^{(D)}=\frac{g^2}{16\pi^2}N_c[-1.000(1)]. \tag{5.9}$$

The finite term proportional to $1/N_c$ emerges from the last tadpole diagram in Fig. 3. In the above the finite part of the loop correction is evaluated by performing the loop integrals with the Monte Carlo routine VEGAS in double precision. We employ 20 sets of 10^6 points for integration. Errors are estimated from the variation of integrated values over the sets. We eliminated the terms when their coefficients become smaller than the numerical errors. Here we remind the reader that the logarithmic divergence appears only in the term concerning the mass renormalization. The one loop contribution to the explicit breaking term $\langle X_S A_\alpha \overline{\psi}\rangle$ is given by

$$\begin{aligned}\langle X_S A_\alpha(k)\overline{\psi}(p)\rangle_1=&\frac{1}{k^2}[(k+p)_\mu(k_\nu\sigma_{\nu\alpha}\gamma_\mu T_S^{(X)}\\&+(\delta_{\mu\alpha}\not{k}-k_\mu\gamma_\alpha)T_T^{(X)})+ik_\mu\sigma_{\mu\alpha}\\&\times(T_D^{\rm add}+MT_D^{(X)})+k_\alpha\not{p}T_{GK}^{(X)}+p^2\gamma_\alpha T_{GM}^{(X)}]\\&\times\frac{1}{i\not{p}+M},\end{aligned} \tag{5.10}$$

where

$$T_S^{(X)}=\frac{g^2}{16\pi^2}N_c[-7.775(1)], \tag{5.11}$$

$$T_T^{(X)}=\frac{g^2}{16\pi^2}N_c[3.3716(7)], \tag{5.12}$$

$$T_D^{\rm add.}=\frac{g^2}{16\pi^2}N_c[51.4345(5)], \tag{5.13}$$

$$T_D^{(X)}=\frac{g^2}{16\pi^2}N_c[11.130(1)], \tag{5.14}$$

$$T_{GK}^{(X)}=\frac{g^2}{16\pi^2}N_c[-2.000(1)], \tag{5.15}$$

$$T_{GM}^{(X)}=\frac{g^2}{16\pi^2}N_c[-1.842(2)]. \tag{5.16}$$

Here we should notice that the one loop correction to the SUSY explicit breaking term X_S produces an additive mass correction given by Eq. (5.13). This additive correction coincides with that from the axial Ward-Takahashi identity (4.16) within a numerical error. This fact confirms the prediction of Ref. [5] that the supersymmetry is recovered at the same fine-tuning point as the axial $U(1)_R$ symmetry. We have a comment on the origin of this additive correction. The SUSY explicit breaking term X_S can be classified into four parts $X_S^{(i)}$. $X_S^{(1)}$ is given by supertransforming the gluino fields of the Wilson term, $X_S^{(4)}$ appears from the gluino action by transforming the link variable in the covariant derivative. $X_S^{(2)}$, $X_S^{(3)}$ are originated from the gluon plaquette action. Since the additive mass in the axial WT identity (4.16) is due to the axial symmetry-breaking term given by rotating the Wilson term, it might have been expected that the additive correction in the SUSY WT identity comes only from $X_S^{(1)}$, $X_S^{(4)}$ which are directly related to the Wilson term. However, $X_S^{(1)}$, $X_S^{(4)}$ produces only 80% of $T_D^{\rm add}$ and remaining 20% is a contribution from $X_S^{(2)}$, $X_S^{(3)}$. Contributions from each $X_S^{(i)}$ is given in Table I. The Wilson parameter dependence of $T_D^{\rm add}$ is given nontrivially inside the diagram multiplying $X_S^{(2)}$, $X_S^{(3)}$ with the Wilson parameter in the gluino propagator and the interaction vertex. We depicted the r dependence of Σ_0 and $T_D^{\rm add}$ in Table II, which are in good agreement for every r within a numerical error.

Summing up all the vertex contributions we can investigate the mixing behavior.

$$\begin{aligned}G_S^{(1)}=&\langle(\nabla_\mu S_\mu)A_\alpha(k)\overline{\psi}(p)\rangle_1-M\langle D_S A_\alpha(k)\overline{\psi}(p)\rangle_1\\&-\langle X_S A_\alpha(k)\overline{\psi}(p)\rangle_1\\=&\frac{1}{k^2}[(k_\mu+p_\mu)(k_\nu\sigma_{\nu\alpha}\gamma_\mu(T_S^{(S)}-T_S^{(X)})\\&+(\delta_{\mu\alpha}\not{k}-k_\mu\gamma_\alpha)T_T^{(1)})-ik_\mu\sigma_{\mu\alpha}(MT_D^{(1)}+T_D^{\rm add})\\&-k_\alpha\not{p}T_{GK}^{(X)}-p_\alpha\not{p}T_{GP}^{(D)}-p^2\gamma_\alpha T_{GM}]\frac{1}{i\not{p}+M},\end{aligned} \tag{5.17}$$

TABLE II. Wilson parameter dependence of the additive mass correction from axial and SUSY WT identity. Both results are in good agreement for every r within a numerical error.

r	0	0.2	0.4	0.6	0.8	1.0
$T_D^{\rm add}$	0	19.793(8)	30.707(8)	38.286(7)	44.964(8)	51.4345(5)
Σ_0	0	19.791(1)	30.695(2)	38.283(3)	44.960(4)	51.4346(1)

where

$$T_T^{(1)} = T_T^{(S)} - T_T^{(X)} = \frac{g^2}{16\pi^2} N_c [3.000(2)], \tag{5.18}$$

$$T_D^{(1)} = T_D^{(D)} + T_D^{(X)} = \frac{g^2}{16\pi^2} N_c \left(-3 \ln \frac{(\lambda a)^2}{\pi^2} - 12.515(2) + \frac{2\pi^2}{N_c^2} \right), \tag{5.19}$$

$$T_{GM} = T_{GM}^{(D)} + T_{GM}^{(X)} = \frac{g^2}{16\pi^2} N_c [-2.842(3)]. \tag{5.20}$$

We consider the mixing property of the operators by making use of the following conditions for the renormalized Green functions at small momentum:

$$\langle (\partial_\mu S_\mu) A_\alpha(k) \bar{\psi}(p) \rangle_R = \frac{1}{k^2} (k_\mu + p_\mu) k_\nu \sigma_{\nu\alpha} \gamma_\mu \frac{1}{i\not{p} + M}, \tag{5.21}$$

$$\langle (\partial_\mu T_\mu) A_\alpha(k) \bar{\psi}(p) \rangle_R = \frac{1}{k^2} (k_\mu + p_\mu) (\delta_{\mu\alpha} \not{k} - k_\mu \gamma_\alpha) \frac{1}{i\not{p} + M}, \tag{5.22}$$

$$\langle D_S A_\alpha(k) \bar{\psi}(p) \rangle_R = \frac{1}{k^2} M i k_\nu \sigma_{\nu\alpha} \frac{1}{i\not{p} + M}, \tag{5.23}$$

$$\langle (\partial_\mu A_\mu \not{\partial} \psi) A_\alpha(k) \bar{\psi}(p) \rangle_R = -\frac{1}{k^2} k_\alpha \not{p} \frac{1}{i\not{p} + M}, \tag{5.24}$$

$$\langle (A_\mu \partial_\mu \not{\partial} \psi) A_\alpha(k) \bar{\psi}(p) \rangle_R = -\frac{1}{k^2} p_\alpha \not{p} \frac{1}{i\not{p} + M}, \tag{5.25}$$

$$\langle (\not{A} \partial_\mu \partial_\mu \psi) A_\alpha(k) \bar{\psi}(p) \rangle_R = -\frac{1}{k^2} p^2 \gamma_\alpha \frac{1}{i\not{p} + M}. \tag{5.26}$$

Here we introduce a gauge-invariant fermionic current with dimension 7/2

$$T_\mu(n) = 2 \,\mathrm{tr}[P_{\mu\nu}(n) \gamma_\nu \psi(n)]. \tag{5.27}$$

We can easily see that the first term in Eq. (5.17) with $T_S^{(S)}$ contributes to the multiplicative normalization factor of the supercurrent. The second term with $T_T^{(1)}$ represents the mixing with a gauge-invariant current T_μ as was discussed in Ref. [5]. Mixing with T_μ is also reported in the continuum theory with the dimensional regularization [19]. This term is related to the gamma-trace anomaly corresponding to the super conformal symmetry breaking. Its coefficient $T_T^{(1)}$ should be identical with the one loop level β function of the $N=1$ SYM theory,

$$T_T^{(1)} = \frac{g^2}{16\pi^2} 3N_c = -\frac{\beta_{\text{1-loop}}}{g} \tag{5.28}$$

as was required for the gamma-trace anomaly [12]. This condition is satisfied in our calculation (5.18). T_D^{add} in the third term gives the additive mass correction, which coincides with Σ_0 from the axial WT identity (4.16). $T_D^{(1)}$ contributes to the multiplicative mass renormalization factor.

The remaining three terms are mixing with gauge-variant operators. These mixings are because we adopted gauge-variant operator $\mathcal{O} = A_\alpha(y)\psi(z)$ in Eq. (5.1) and a fixed gauge in the perturbative calculation. However, these extra mixings disappear if we impose an on-shell condition to the gluino momentum and set the renormalized gluino mass to zero

$$i\not{p} = -M_R = 0. \tag{5.29}$$

At last we have a comment on the renormalization of the whole operators in the SUSY WT identity. In order to derive the multiplicative renormalization factors of the supercurrent and gluino mass we need an appropriate renormalization scheme like [20], in which the IR divergence of the fermion and ghost loop is treatable. We have the same form of the vertex correction as in Eqs. (5.3), (5.4), (5.10) even if we change the renormalization scheme. Once the gluino and gluon wave-function renormalization factors Z_2, Z_3 are given the supercurrent is renormalized as

$$(S_\mu)_{\mathrm{bare}} = Z_S^{-1} (S_\mu)_{\mathrm{R}} + T_T^{(S)} (T_\mu)_{\mathrm{R}} \tag{5.30}$$

with

$$Z_S^{-1} = \sqrt{Z_3 Z_2} (1 + T_S^{(S)}). \tag{5.31}$$

If we sum up all the contributions to the SUSY WT identity

$$G_S = \langle (\nabla_\mu S_\mu) A_\alpha(k) \bar{\psi}(p) \rangle^{\mathrm{full}} - M \langle D_S A_\alpha(y) \bar{\psi}(z) \rangle^{\mathrm{full}} - \langle X_S A_\alpha(k) \bar{\psi}(p) \rangle^{\mathrm{full}}, \tag{5.32}$$

the one loop corrections to the supercurrent and the explicit breaking term shall cancel out

$$Z_S^{-1} - T_S^{(X)} = 1, \tag{5.33}$$

and the proper form of the SUSY WT identity with gamma trace anomaly term is supposed to be recovered for the renormalized quantities.

VI. CONCLUSION

In this paper we regularize the supersymmetric Yang-Mills theory on the lattice with the Wilson plaquette action for the gluon and the Wilson fermion for the gluino. In this regularization the supersymmetry and axial $U(1)_R$ symmetry of the continuum SYM theory are broken explicitly. However, both of the symmetries can be recovered in the continuum by fine-tuning the mass parameter.

In order to see this restoration process we calculated the one loop correction to the SUSY Ward-Takahashi identity perturbatively. It is shown that the additive mass correction (the critical mass) given by the SUSY Ward-Takahashi identity coincides with that from the axial WT identity. This means the SUSY and the $U(1)_R$ symmetry can be restored simultaneously in the continuum limit with a single fine-tuning of the gluino mass. Since tuning to the chiral symmetric point is a well-known subject in computer simulation, there would have been no technical difficulty in dealing with the SYM system on the lattice even nonperturbatively if the axial $U(1)_R$ symmetry had no anomaly. The chiral symmetric point cannot be given by the vanishing pion mass for the anomalous $U(1)_R$ symmetry of SYM and an alternative method is needed. Application of the vacuum degeneracy of residual Z_{2N_c} symmetry due to gluino condensation seems to be hopeful [10].

The supercurrent on the lattice mixes nontrivially with T_μ. If we can extract this gamma-trace anomaly part nonperturbatively from the SUSY WT identity, we may be able to evaluate the exact β function of the $N=1$ SYM theory.

A peculiar point in our calculation is that the SUSY breaking term given by supertransforming the plaquette action also contributes to the additive mass correction. Therefore when we consider using the domain-wall fermion as a gluino part, it is nontrivial to see the disappearance of the additive mass correction because the domain-wall fermion system contains the Wilson term in its action before integrating out the unphysical heavy modes. The negative unity Wilson parameter remains in the gluino-gluon interaction vertex. This is a fascinating future problem.

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