The aim of this article is to share what values are described in the SEA-BES: CCRLS for Mathematics and Science (SEAMEO RECSAM, 2017) in relation to mathematics education. In the SEA-BES: CCRLS (2017) document, there are four meanings of values being discussed. The first one is the necessity of SEA-BES to provide the basis of specifying the style of descriptions, contents, and aims of SEA-BES. The second one is to describe the significance of mathematics, which provides the necessity of mathematics education. The third one elaborates the nature of mathematics which explains the framework for the objective of mathematics education. The fourth one explains the value of mathematics, which should be learned in mathematics classroom. This article explains those four meanings in SEA-BES: CCRLS and elaborates the fourth one for illustrating how students’ values will be developed through the teaching and learning of mathematics in the classroom.
The “SEAMEO Education Action Agenda (2016–2020)” puts forward seven priority areas to provide opportunities for the development of quality education to meet the needs of all ASEAN children. Agenda 7 — adopting a 21st-century curriculum — states to pursue a radical reform through systematic analysis of knowledge, skills, and values needed to effectively respond to changing global contexts, particularly to the ever-increasing complexity of the Southeast Asian economic, sociocultural, and political environment, developing teacher imbued with ASEAN ideals in building ASEAN community within 20 years (2015–2035). SEA-BES: CCRLS (2017) is the document developed for one of the projects in the action agenda #7 spearheaded by SEAMEO RECSAM. These issues concur the necessity for the development of a harmonious society in this competitive era. In these context, the SEA-BES the learning standards for mathematics and science are established under the following principles:

1. The standards are the common ground to develop the fullest potential and capabilities to acquire competency in the 21st century.

2. The standards are presumed for overcoming competitive society in this globalisation era and understanding others to create the ASEAN harmonious society under global citizenship.

3. The standards function as tools for analysing curriculum for the purpose of the project as stated in a to f:

   a. To be used as an analytical tool to support future development of a regional integrated curriculum necessary for ASEAN integration with emphasis on 21st-century skills

   b. To strengthen ASEAN collaboration on curriculum standards and learning assessment across different educational systems to respond effectively in the changing global context and complexity of ASEAN

   c. To promote in every member country the establishment of best practices to overcome differences in curriculum

   d. To produce systematic discussion process for the establishment of the regional integrated curriculum and assessment

   e. To be used as a platform for curriculum development and professional development for all stakeholders for developing teachers imbued with ASEAN ideals in building ASEAN community

   f. To serve as a platform for assessment such as the Southeast Asia Primary Learning Metrics (SEA-PLM)
The document developed consideration of the current status of the region as well as issues for curriculum reforms in the world. In keeping abreast with the global development, 21st-century skills has been emphasised in accordance to the term of competency defined by OECD (2005). The competency being defined encompasses the standards for successful living and maintenance for well-functioning societies. These goals are a continuously challenge to success and welfare in the economically competitive and dynamic society under globalisation. UNESCO (2015) sets the SDGs as the necessity for the development of every society and the sustainability of its social welfare.

Mathematics and science count as necessary forms of literacy in any field, including new unknown industries, which are expected by the STEM innovation, as well as general life well-being. Mathematics and science curriculum are tools for overcoming the challenges of diversities in Southeast Asia through developing competency for competitiveness and understanding others for creating a harmonious society. In these perspectives, the SEA-BES: CCRLS for Mathematics consider and describe three major components as for aims of education. First, for cultivating basic human characters through mathematical values, attitudes, and habits of mind. Second, for developing creative human capital where process skills are needed to be developed. Third, for cultivating well-qualified citizens through knowledge of mathematics. In the past mathematics education, third aim usually recognised as content of teaching by teachers. However first and second aims have been recognised in the several curriculum documents for mathematics more than several decades. For clarifying how these three components function within the mathematics framework, the nature of mathematics is initially discussed and to support how the aims of SEA-BES in mathematics are deduced. Subsequently, the format of the SEA-BES: CCRLS for Mathematics is described before the elaboration of every learning standard.

Excerpts of Descriptions for Values in SEA-BES: CCRLS for Mathematics

The following are excerpts for discussions of values to deduce the aims of mathematics education for SEA-BES: CCRLS for Mathematics.

Necessity to Learn Mathematics

Mathematics has been recognised as a necessary literacy for citizenship and not only for living economically, but also to establish a society with fluency of fruitful arguments and creations for better living. It has been taught as a basic language for all academic subjects using visual and logical-symbolic representations. Currently, mathematics also provides the necessary bases for STEM education. Beyond the limitation of STEM education, mathematics has been increasing its role for future prediction and designing with big data that produces innovation not only for technological product, but also for various business models. Mathematics is also an essential field of knowledge to establish common reasoning for sustainable development of society through viable argument in understanding each other and develop critical reasoning as the habits of mind.
Nature of Mathematics to Propose the SEA-BES: CCRLS for Mathematics Framework

For clarifying the framework in SEA-BES: CCRLS for Mathematics and by knowing the role of mathematics education, the humanistic and philosophical natures of mathematics are confirmed as follows. Humanistic nature of mathematics is explained by the attitudes of competitiveness and understanding of others. This is illustrated by mathematicians such as Blaise Pascal, Rene Descartes, Isaac Newton, and Gottfried Wilhelm Leibniz. For example, if you read the letter from Pascal to Pierre de Fermat, you will recognise the competitive attitude of Blaise Pascal to Fermat’s intelligence and seeking the way to understand his excellent finding on his (Pascal) Triangles. By reading Pascal’s *The Pensées*, you would recognise how Pascal denied Descartes’s geometry using algebra from the perspective of ancient Greece geometry. On the other hand, Descartes tried to overcome the difficulties of ancient geometry by algebra. If you read the letter from Descartes to Elizabeth, you would recognise how Descartes felt happy and appreciated the Royal Highness Elizabeth used his ideas of algebra in geometry. Despite being a princess, Elizabeth had been continuously learning mathematics in her life.

There were discussions on who developed calculus, whether it was Britain or Continent. On that context, Johann Bernoulli, a Continental mathematician, posed a question on the journal about the Brachistochrone problem, regarding locus of the point on circumference of the circle when it rotates on the line. No one replied and Bernoulli extended the deadline of the answer and asked Newton to reply. Newton answered it within a day. Finally, six contributions of the appropriate answer, including Newton and other Continental mathematicians, were accepted. All those stories show that mathematics embraces the humanistic nature of proficiency for competitiveness and to understand others in order to share ideas.

Philosophical nature of mathematics can be explained based on ontological and epistemological perspectives. On the ontological perspective, mathematics can be seen as a subject for universal understanding and common scientific language. The views of Plato and Aristotle are usually compared on this perspective. Plato believes that the existence of the world of “idea” and mathematics existed in the world of idea on Platonism. On this context, mathematical creation is usually explained by the word “discover,” which means taking out the “cover” from which it has already existed. At the moment of discovery, reasonableness, harmony, and beautifulness of mathematical system is usually felt. Aristotle tried to explain about reaching idea from the material to the form. This explains that abstract mathematics can be understood with concrete materials using terms such as “modelling,” “instruments,” “embodiment,” “metaphor,” and “change representation.” From both the ontological perspectives, mathematics can be understood and acquirable by everyone and, if acquired, it serves as a common scientific language which, is used to express in any subjects. Once the ideas are represented using the shared common language, it is possible that the world perceives the same view autonomously.
On the epistemological perspective, mathematics can be developed through processes, which are necessary to acquire mathematical values and ways of thinking. From this perspective, idealism and materialism are compared. On the context of Hegel, a member of German idealism, Imre Lakatos explained the development of mathematics through proof and refutation. On this context, mathematics is not fixed but an expandable system that can be restructured through a process of dialectic in constructing viable arguments. Plato also used dialectic for reaching ideas with the examples of mathematics. The origin of dialectic is known as the origin of indirect proof. In education today, dialectic is a part of critical thinking for creation. Parallel perspectives for mathematical developments are given by George Polya and Hans Freudenthal. For the discovery of mathematics, Polya explained mathematical problem solving processes with mathematical ideas and mathematical ways of thinking in general. Freudenthal enhanced the activity to reorganise mathematics by the term “mathematisation.”

Genetic epistemologist Jean Piaget established his theory of operations based on various theories, including the discussion of Freudenthal, and explained the mathematical development of operations by the term “reflective abstraction.” Reflection is also a necessary activity for mathematisation by Freudenthal. On materialism, under Vygotskyian perspective, intermediate tools such as language become the basis for reasoning in the mind. Under his theory, the high-quality mathematical thinking can be developed, depending on the high-quality communication in mathematics classrooms. A dialectical-critical discussion should be enhanced in the mathematics class. From both the epistemological perspectives, mathematics can be developed through the processes of communication, problem solving, and mathematisation, which include reorganisation of mathematics. Those processes are necessary to acquire mathematical values and ways of thinking through reflection.

**Aims of Mathematics Education in SEA-BES: CCRLS**

The aims of mathematics in SEA-BES: CCRLS for developing basic human characters, creative human capital, and well-qualified citizens in Southeast Asia for a harmonious society are as follows:

- Develop mathematical values, attitudes, and habits of mind for human character
- Develop mathematical thinking and able to engage in appropriate processes
- Acquire proficiency in mathematics contents and apply mathematics in appropriate situations

Framework for SEA-BES: CCRLS for Mathematics as shown in Figure 6 is developed based on the three components with discussions of the humanistic and philosophical nature of mathematics. This framework also depicts the concrete ideas of mathematics learning of the above aims.
At every given context on the curriculum, those three components are mutually related and explains various objectives for education. Those components under this framework are functioning to interpret the objective of every learning standard in the SEA-BES: CCRLS.

Mathematical Values, Attitudes, and Habits of the Mind

For cultivating basic human characters, values, attitudes, and habits of mind are essentials to be developed through mathematics. Values are the basis for setting objectives and making decisions for future directions. Attitudes are mind-sets for attempting to pursue undertakings. Habits of mind are necessary for soft skills that contribute to living harmoniously in the society.
Mathematical values, mathematical attitudes, and mathematical habits of mind are simultaneously developed and inculcated through the learning of the content. Essential examples of values, attitudes, and habits of mind are given in Figure 6. On seeking values in mathematics, characteristics such as generalisable and expandable ideas are usually recognised as strong ideas. In explaining why such a particular proving is necessary in mathematics is usually related to seeking for reasonableness. Harmony and beautifulness can also be thought as mathematical values that are not only described in relation to mathematical arts, but also in the science of patterns and mathematical systems. Usefulness and simplicity are used in selection of mathematical ideas and procedures.

SEA-BES: CCRLS is written by four content strands and one process-humanity strand and in every strand, standards are described by using verb and adjective to embed the process and value into content of teaching.

**Teaching of Thinking, Process, Value, and Attitude in Mathematics Classroom**

In traditional manner of teaching mathematics, drills and exercises are used for teaching the contents. However the ways to teach thinking, processes, values, and attitudes in mathematics are not clearly stated like the teaching of mathematics content. SEA-BES: CCRLS (2017) illustrates that mathematical thinking, process, value, and attitude can be learned by students when teachers provide appropriate contexts in relation to specific content of teaching. Isoda (2012) illustrated that appropriate contexts are given by the task sequence in curriculum and student learning are usually distinguished by the terminologies such as acquisition of contents, reflection of processes, and appreciation of values.

![Figure 7: Learning mathematics in the appropriate context](image-url)
Isoda (2012 and 2016) illustrated the task sequence from task 1 (37 x 3) to task 2 (15,873 x 7) for explaining how to develop mathematical thinking on Figure 6 framework (see the lecture video series by SEAMEO QITEM (https://www.youtube.com/watch?v=oCp2HijnVSk).

![Figure 21: Illustration from task 1 to task 2](image)

Source: M. Isoda and S. Katagiri (2016)
It begins from $37 \times 3 = 111$ (here, it is called “tentatively doggy number” because of Japanese sounds for barking) and through only questioning, “What do you want to do next?”, the activity continues by learners’ replies to this simple open question. If learners recognised $37 \times 6 = 222$, which is referred as “love numbers” (Japanese sounds for kissing to kids), then it is assumed that learners are able to find repeated digit patterns; firstly, finding the alternative natural number pattern, secondly, integrating different patterns and finally as one pattern. In the process, learners also find the conditions when every pattern can work and write the propositions by if-then format. These are the outcomes of mathematics and can be categorised as “content” in Figure 6.

Acquisitions of these content are enhanced by the exercise in task 2 as $15,873 \times 7$. The sequence from task 1 to task 2 is an application of learning. Although, the activity in task 1 is led by the teacher, students are able to challenge task 2 as long as the students could follow the explanation in task 1, which is written in the box of Figure 8. Content knowledge can be acquired by doing exercises. However, thinking, processes, values, and attitudes are difficult to be learned if teachers do not provide enough time for students to think by themselves on task 1.

In the activity elaborated above, the teacher only gave the question, “What do you want to do next?,” subsequently providing the time to think and communicate and listening to students’ ideas and any answers for that open question. Students are able to think whatever they can imagine, set their directions and representation for thinking, finding and sharing every pattern by themselves. Their reasoning can be expressed by the terminologies in Figure 6 such as “set” on mathematical ideas, “inductive, deductive and analogical thinking,” “extension and integration,” and “generalisation and specialisation” on mathematical thinking; “generality and expandability,” “reasonableness and harmony,” and “beautifulness” on mathematical values; and “pose question and develop explanation” and “change representation for meaningful explanation” on mathematical attitude. In the process of task 1, the question, “What do you want to do next?,” is used to direct the thinking and reasoning of students whom, by themselves, would not be aware of using these terminologies on Figure 6 in the process. However, at the moment when they find $111,111$ in task 2, they are able to reflect on and appreciate the whole process in the box of Figure 8 and recognise the various activities, efforts, and challenges done during the process of task 1 as the set of sequential activity for task 2. At the moment, if students felt that task 1 and task 2 might be the same activity, it meant that they are able to reflect and appreciate the process of task 1. At this juncture, the task sequence from task 1 to task 2 can be seen as a context on Figure 7 and the teacher is able to teach various terminologies in Figure 6 on this context.

Japanese textbooks well-embedded these type of task sequences for Figure 6 under Japanese curriculum on the name of “problem-solving approach.” Some of SEAMEO countries already published in their editions or in process (Maitree and Isoda, 2010).
The SEA-BES: CCRLS for Mathematics curriculum has been developed based on the fundamentals of the nature of mathematics. With that concrete grounding, the values of mathematics prevail in the framework. This would ultimately enable the attainment of the aims of SEA-BES: CCRLS, particularly in promoting mathematical thinking and values in teaching and learning of mathematics in the classroom. The reference for SEA-BES: CCRLS itself was written in Managao, Ahmad, and Isoda (2017). The development process of SEA-BES: CCRLS was written in Montecillo, Teh, and Isoda (2018), which illustrated the curriculum development process as the elaborative and dialectic process of educational values.

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