PROTECTION STRATEGIES FOR CRITICAL RETAIL FACILITIES: APPLYING INTERDICTION MEDIAN AND MAXIMAL COVERING PROBLEMS WITH FORTIFICATION

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PROTECTION STRATEGIES FOR CRITICAL RETAIL FACILITIES: APPLYING INTERDICTION MEDIAN AND MAXIMAL COVERING PROBLEMS WITH FORTIFICATION

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Abstract Closing facilities due to a lack of demand is an unavoidable trend in areas experiencing decreasing populations. Strategies to prevent critical facilities from closing may be based on the concepts of maximum benefit or minimum cost. With the purpose of determining efficient strategies, we focused on a median and a maximal covering problem that considers two interacting players: interdiction and fortification. This study aims to develop an interdiction covering problem with fortification and compare it with a median problem. We specified the facilities to be grocery stores because justifying their protection is difficult given that they are private businesses; however, such protection is crucial because they provide a critical public service. First, we simulated the formulation on a linear urban space to explain the general characteristics and performance of the models. Second, we employed the formulation on a practical dataset to consider the heterogeneity of urban spaces. The result shows that, for models in a uniform space, the peripheral area is prioritized for protection when a lower level of damage is expected, while the central area is prioritized when more damage is expected. Moreover, these general characteristics can be sensitive to the spatial distribution of facilities.

Keywords: Facility planning, discrete optimization, critical facilities, fortification

1. Introduction

In the particular case of the public facility location problem, unlike that of the private facility location problem, the objectives are more complicated to legitimize and quantify [10]. Hence, utilizing facility location solutions rooted in the well-known p-median location problem (a median method) [10] or the maximal covering location problem (a covering method) [3] for practical decision-making processes requires several additional considerations to be taken into account, such as attributes, criticalities and changes in the environment. Furthermore, in several occasions, situations in which the number of facilities decreases can also be considered assuming two different objectives: maximizing the decline in accessibility and minimizing the decrease in accessibility of facilities. While the former is similar to situations such as terrorist or military attacks, the latter can reflect the protection or restoration strategies of facilities from damage, closure, malfunction or financial difficulties. The latter scenario is considered to be more suitable in the planning of location strategies for public facilities. Higher accessibility can be described as a decrease in the sum of the weighted distance of users or more coverage of users within a certain distance from each facility.

This paper aims to explain how the decline/damage in accessibility can be minimized by protection strategies of critical facilities using two concepts of measures, a median and a covering method, focusing on the applicableness of each method in different situations. More specifically, although much of the literature on facility location modeling has been directed towards formulating new models and modifications to existing models [6] to find optimal solutions or observe the performance of formulations using existing or random data, the implications on a general urban space have rarely been considered in a way that is useful in the planning process. Thus, there is a need to provide a qualitative explanation of the generalized characteristics of the fortification problem with respect to how the model works in an urban area. In this sense, a linear urban space is used to present the general environment with a homogeneous demand. Evaluating the result of varying degrees of damage and fortification can offer ideas about which facilities can be vulnerable and should be protected to make the system more robust according to the expected amount of total damage and total cost of protection. In addition, a case study of the application of data from a real urban space is presented to determine the effects of a heterogeneous distribution of facilities and demands. These results are used to draw implications for the implementation of facility fortification in practical situations. Considering both the median and the covering problems will help decision makers to compare the results from each viewpoint they would take.

This paper consists of six chapters. Chapter 2 provides an overview of previous studies on optimization problems related to diminishing facilities and on protecting facilities as a countermeasure. Chapter 3 presents the formulation of the defined problems using a median method and a covering method. In Chapter 4, the features of a change in accessibility resulting from damage and fortification are elucidated on a linear urban space. Chapter 5 analyzes the facilities that should be protected based on the level of the demand relative to that of damage, using an actual geographical dataset, from the two different viewpoints. Chapter 6 provides practical considerations and suggestions on the performances of strategies from two different viewpoints regarding accessibility in an urban space.

2. Literature Review

The concept of fortification, or protection, stemmed from a large number of studies on the development of reliable systems [17]. A set of studies on reliability addressed disruptions, namely interdiction problems, for example, natural disasters or attack strategies. Church *et al.* [5] formulated a median and a covering problem, named RIM/RIC (r-interdiction median problem/r-interdiction covering problem), to identify critical facilities under the assumption that disruptions are maximized by "attacking," "sabotaging," or "interdicting" critical facilities. In short, RIM is defined as finding the subset of r facilities out of the p different supply locations that, when removed, yields the highest level of increase in weighted distance. The formulation uses the following notation:

i = index representing places of demand ($i \in N$)
j = index representing locations of existing facilities ($j \in F$)
a_i = a measure of demand
d_{ij} = the shortest distance between demand i and the facility at j
r = the number of facilities to be eliminated or interdicted
$T_{ij} = \{k \in F | k \neq j, d_{ik} > d_{ij}\}$.

Additionally, the following decision variables are considered:

x_{ij} = 1 if demand i is assigned to a facility at j, 0 otherwise
s_j = 1 if a facility at j is eliminated, 0 otherwise.

Then, RIM can be formulated as follows:

$$\text{maximize} \quad \sum_i a_i \sum_{j \in F} d_{ij} x_{ij} \tag{2.1}$$

$$\text{subject to} \quad \sum_{j \in F} x_{ij} = 1, \forall i \tag{2.2}$$

$$\sum_{j \in F} s_j = r \tag{2.3}$$

$$\sum_{k \in T_{ij}} x_{ik} \leq s_j, \forall i, \forall j. \tag{2.4}$$

Objective (2.1) is to maximize the sum of the weighted distances after the interdiction of r facilities. Constraint (2.2) keeps each demand assigned to one facility after interdiction. Constraint (2.3) means that the total number of interdictions is r facilities. Constraint (2.4) assigns each demand to the closest remaining facility.

RIC is defined as finding the subset of r facilities of the p different service locations that, when removed, maximizes the decline in coverage. To formulate RIC, consider the following notation:

N_i = the set of j that covers demand i

Also, the following decision variables are considered:

y_i = 1 if demand i is no longer covered, 0 otherwise

Then, RIC can be formulated as follows:

$$\text{maximize} \quad \sum_i a_i y_i \tag{2.5}$$

$$\text{subject to} \quad y_i \leq s_j, \forall j \in N_i, \forall i \tag{2.6}$$

$$\sum_{j \in F} s_j = r. \tag{2.7}$$

Objective (2.5) maximizes the total population no longer covered after interdiction. Constraint (2.6) provides the limitation that demand i is no longer covered only when each facility j that covers demand i has been removed. Constraint (2.7) is the same as constraint (2.3) of RIM.

As a related approach, Aksen *et al.* [1] suggested a capacitated model as an extended version of RIM. On the other hand, Miyagawa *et al.* [14] discussed the robustness of a system of regularly positioned facilities based on the accessibility of facilities on the assumption that both disruption/closures and new openings would occur.

Fortification models have been studied as a further step towards the identification of facilities that protect, fortify, or preserve the functionality of the system as much as possible in the occurrence of external disruptions (Aksen *et al.* [1]). Therefore, the problem seeks the next expected maximum damage to be minimized. Brown *et al.* [2] suggested the basic concept of applying bi-level and tri-level optimization models that feature an intelligent attacker and defender that have transparent information and whose actions are sequential with respect to each other. Church *et al.* [4] provided the basic formulation with a certain interdiction and fortification amount. Liberatore *et al.* [12] employed a fortification strategy against stochastic interdictions to make the problem more robust.

Chapter 3 explains the basic formulation of the r-interdiction median problem with fortification by Church *et al.* [4] and its modification into a covering problem.

3. Formulation of Protection Problems

If the defender-attacker model is viewed as a protection-closure problem, accessibility can be evaluated using (1) the minimum weighted distance for users (a median problem) and (2) the maximum number of users covered within a certain distance (a covering problem). The median problem considering protection was formulated in a previous study [4]. In this study, a protection-closure problem based on the covering problem will be formulated as an additional approach. The covering problem seeks the best strategy in terms of equality while the median problem is often used to pursue maximum effectiveness. This choice can be explained as follows. With the median problem, there is the possibility of a demand to be assigned to a significantly distant facility, which is not desirable, especially for a society with aged members contributing to demand. On the contrary, the covering problem tries to foreclose those extreme cases.

Particularly in the case of locating critical public services such as health care centers [15] or fire stations [16], the maximal covering problem is adopted and modified; in contrast, numerous applications using the median problem postulate that some sacrifice should be endured for the benefit of the majority (e.g. day care centers [11]). It is notable that, in the location problem for competing/private facilities, the median method minimizing average customers' travel cost to attract as many customers as possible [7] [8] [9]. Therefore, it is the decision maker's role to choose which goal the strategy should strive towards. In this regard, it is valuable to interpret the difference of the general form of results from different viewpoints. For instance, the neighborhood theory has often been adopted to plan new residential district with the neighborhood centers located to provide widely needed services within walking distance. Although walking distance has differing definitions, it is desirable to contemplate the threshold of walking distance of mobility-impaired people such as elderly or handicapped people. Consequently, using two different methods can support decision making processes in which different standpoints and subjects lie with according to cases.

3.1. r-interdiction median problem with fortification (RIMF)

According to the notation in RIM, a_i is the demand population $i (i \in N)$, d_{ij} is the distance to the nearest facility $j (j \in F)$ from i, and r is the number of facilities to be closed.

Here, consider the additional notations:

H = set of every possible interdiction pattern with r out of p existing facilities
h= index of a specific interdiction pattern $(h \in H)$
B_i^h = the set of interdicted facilities in pattern h located closer than d_i^h from demand i
q= number of facilities to be fortified/protected.

The closure/attack/interdiction is independent, composing a pattern $h (h \in H)$, and d_i^h is the distance to the nearest facility that is not closed with a pattern h from i. The sum of the weighted distances to the unclosed facility for a given closing pattern, WD_h, can be expressed as follows:

$$WD_h = \sum_i a_i d_i^h \tag{3.1}$$

where

$$B_i^h = \{j \in F | d_{ij} < d_i^h\}, \forall i \in N, \forall h \in H. \tag{3.2}$$

If q facilities are assumed to be protected or fortified, the decrease in the weighted distance attributable to protection is the following:

$$a_i(d_i^h - d_{ij}), \forall j \in B_i^h. \tag{3.3}$$

The r-interdiction median problem with fortification (RIMF), which minimizes the maximum increase in the weighted distance attributable to closure, is formulated as follows:

$$\text{minimize} \quad W \tag{3.4}$$

$$\text{subject to} \quad x_{ij}^h \leq z_j, \forall i \in N, \forall h \in H, \forall j \in B_i^h \tag{3.5}$$

$$\sum_{j \in B_i^h} x_{ij}^h \leq 1, \forall i \in N, \forall h \in H, |B_i^h| \geq 2 \tag{3.6}$$

$$W \geq \sum_{i \in N} a_i d_i^h - \sum_{i \in N} \sum_{j \in B_i^h} a_i (d_i^h - d_{ij}) x_{ij}^h, \forall h \in H \tag{3.7}$$

$$\sum_{j \in F} z_j = q \tag{3.8}$$

$$x_{ij}^h = \{0, 1\} \tag{3.9}$$

$$z_j = \{0, 1\}, \tag{3.10}$$

where x_{ij}^h is a decision variable with a value of 1 if demand i is assigned to fortified facility j in interdiction pattern h and 0 otherwise. z_j has the value of 1 if a facility j is fortified and 0 otherwise.

Constraint (3.5) forbids demand i to be assigned to interdicted facility $j \in B_i^h$ unless fortified. Constraint (3.6) states that a demand is assigned to at most one facility in the event that the closing pattern h occurs. This demand can be assigned to either the closest non-interdicted facility or the closest fortified facility. It is more efficient to reduce the size of the membership of B_i^h by calculating only for the case of $|B_i^h| \geq 2$, because when $|B_i^h| = 1$, x_{ij}^h is no longer needed as the only option for improving the weighted distance for the i with the given pattern h is to fortify site j. In that case, z_j becomes the substitute of x_{ij}^h in equation (3.7). Furthermore, when there is no fortification anticipated ($q = 0$), WD_h remains unchanged as equation (3.1). Constraint (3.7) calculates the weighted distance after fortification with interdiction pattern h. For each interdiction pattern, the change in the weighted distance attributable to fortification is calculated. For each demand i, W remains unchanged unless the closest facilities are interdicted with h. Constraint (3.8) ensures that, in total, only q facilities are fortified. Constraint (3.9) and Constraint (3.10) defines x_{ij}^h and z_j are binary.

3.2. r-interdiction covering problem with fortification (RICF)

Likewise, let y_{ij} represent a binary variable that has a value of 1 if facility j covers demand i. Given interdiction pattern h, y_i^h has the value of 1 when demand i is covered and 0 otherwise.

More concretely, when the total population covered WC_h after a certain interdiction pattern h can be calculated using summation (3.11), the calculation can be improved using fortification with increment of (3.12), where M_i is a set of facilities that cover demand i.

$$WC_h = \sum_i a_i y_i^h \tag{3.11}$$

$$a_i(y_{ij} - y_i^h), \forall j \in M_i, \forall i \in N \tag{3.12}$$

The r-interdiction covering problem with fortification (RICF), which maximizes the covered population, is formulated as follows:

$$\text{maximize} \quad V \tag{3.13}$$

$$\text{subject to} \quad x_{ij}^h \leq z_j, \forall i \in N, \forall h \in H, \forall j \in M_i \tag{3.14}$$

$$\sum_{j \in M_i} x_{ij}^h \leq 1, \forall i \in N, \forall h \in H, |M_i| \geq 2 \tag{3.15}$$

$$V \leq \sum_{i \in N} a_i y_i^h + \sum_{i \in N} \sum_{j \in M_i} a_i(y_{ij} - y_i^h) x_{ij}^h, \forall h \in H \tag{3.16}$$

$$(3.8), (3.9), (3.10). \tag{3.17}$$

Constraint (3.14), corresponding to (3.5) in RIMF, forces demand i to be assigned only to interdicted facilities that are fortified. Constraint (3.15) limits each assignment for a demand i to only one facility in the pattern h. Constraint (3.16) maximizes the coverage loss by the interdiction pattern h, which will be minimized in equation (3.13) by fortification. In equation (3.16), improvements in coverage by fortification can be only achieved when $|M_i|$ is larger than 1 (see Constraint (3.15)). It is on account that, when there is only facility j covering demand i in the given interdiction pattern h, the only option available for fortification that leads to improvement is to fortify j, and therefore, z_j will replace x_{ij}^h in equation (3.16). This corresponds to the explanation for equation (3.7) in section 3.1. The remaining constraints are equivalent to Constraint (3.8), (3.9) and (3.10), respectively.

4. Solutions to a Linear Urban Model

As mentioned, it is important to understand the basic characteristics of the values achieved by consideration of the formulated problems. A linear-shaped simplified urban form is employed for that purpose. Suppose the population is spread uniformly throughout a linear space with a length of 180 and facilities ($p = 9$) exist on this space at a regular interval of 20. Hereafter, explanation of the worst-case pattern is provided according to the location of r facilities to be closed.

First, considering the total distance in the median problem, two issues appear obvious: (i) the increment of the total distance is higher for closure of facilities on the fringes compared with those at the midsection and (ii) the increment increases with connected closures, which are interdictions of adjacent facilities at the same time. If we suppose the increment of the total distance from the closure of one facility to be 1 in the midsection, the increment from the closure of one facility in the fringes is 1.5 times larger. The increment also becomes 1.5

times per facility if two facilities in the midsection close. Furthermore, in the midsection, the increment increases to 2 times when three facilities close and 2.5 times when four facilities close. For connected closures on the fringes, the increment is 2.5 times when two facilities close, 3.5 times when three facilities close, and 4.5 times when four facilities close.

We calculated the results for both problems formulated in the previous chapter, RIMF and RICF, to verify the improvement in accessibility. Calculation results were generated using NUOPT 13.0 on an Intel i7 CPU (2.93GHz with 8GB of RAM). In general, execution time increases according to the size of r and q, respectively (see Table 1, Table 2, Table 3 and Table 4). The value of r plays a critical role in computation time in that it multiplies the size of H, $|H|$. For example,if the level of interdiction is $r = 2$ in Table 1, the number of combinations of $|H|$ is ${}_9C_2 = 36$ while it is ${}_9C_1 = 9$ when $r = 1$. In Table 3 and 4, $|H|$ is 276 when $r = 2$ while it is 16 when $r = 1$. It is also observed that calculation time begins to decrease after a certain level of q.

In response to these worst-case patterns, RIMF can make the system robust by taking into account prevention of increments to the total distance where possible (Table 1 and Figure 1). Figure 1 shows only one of the results of the protection patterns. The objective values are shown in Figure 3. According to these results, adopting the median method supports the following protection strategies: (i) when the number of closed facilities r is small, fortification should be performed on the fringes and (ii) if the number of facilities to be possibly fortified increases, fortification is best performing in regular segments to prevent connected closures. Furthermore, (iii) when closed facilities are the majority, fortification is conducted first on facilities in the midsection. Finally, (iv) a larger q is more effective for a larger r.

In contrast, using a covering method, suppose that the covering distance is 15 and the population is concentrated in the center of each segment with an interval distance of 10. The covering distance is set so that it can cover every population and it does not generate any unnecessary facility, which is longer than 10 and shorter than 20. The total population covered after the closure of facilities is calculated as the objective value. As a result, similar to the median method, two observations are made for the worst-case patterns: (i) the decrement of the objective value is larger with closures of facilities on the fringes compared with those in the midsections and (ii) the decrement becomes larger when the closed facilities are connected in the case of multiple interdictions. This situation is explained as follows: the covered population does not decrease when one facility in the midsection closes, but it does decrease for one segment when a facility on the fringes closes. Note that a loss of population occurs for one segment if two connected facilities in the midsection close. The loss becomes 4/3 times and 3/2 times as large for three and four connected closures in the midsection, respectively. For connected closures that occur on the fringes, the loss becomes 3/2 times for two facilities, 5/3 times for three facilities, and 7/4 times as large for four facilities, which indicates a sharper decrease than for the midsections.

To achieve optimal robust protection patterns against the worst-case interdictions, RICF was employed. Table 2 and Figure 2 show the results. Only one of the results is shown for interdiction on multiple facilities. The objective value of each q is shown in Figure 4.

The improvement of accessibility by one additional protection (increase of q) appears larger when the size of damage (r) increases. Compared to Figure 3, the third fortification, the improvement of accessibility by one additional protection appears larger when r increases. Observe that there are marginal gains in improvement for both RIMF and RICF according to the level of q. This means that, after a certain amount of protection, no substantial improvement is expected; therefore, the level of protection is excessive. The results

Table 1: Results of RIMF (a linear space)

Existing facilities p	Interdicted facilities r	Interdiction pattern $\|H\|$	Remaining facilities $p-r$	Fortified facilities q	Optimal fortification patterns z	Total distance WD	Execution time sec
9	0	-	9	-	-	90	-
	1	9	8	0		120	0.02
				1	(*)	120	0.01
				2	(1,9)	110	0.03
				3	(1,*,9)	110	0.05
				4	(1,*,*,9)	110	0.01
				5	(1,*,*,*,9)	110	0.01
				6	(1,*,*,*,*,9)	110	0.03
				7	(1,*,*,*,*,*,9)	110	0.02
				8	(1,*,*,*,*,*,*,9)	110	0.02
	2	36	7	0		190	0.05
				1	(*)	190	0.02
				2	(1,9)	150	0.13
				3	(1,*,9)	150	0.36
				4	(1,*,*,9),(2,4,6,8)	150	0.53
				5	(1,3,5,7,9)	130	0.52
				6	(1,3,5,7,9,*)	130	0.34
				7	(1,3,5,7,9,*,*)	130	0.34
	3	84	6	0		300	0.02
				1	(*)	300	0.05
				2	(2,8)	210	0.99
				3	(2,5,8)	180	13.42
				4	(2,4,6,8),(1,4,6,9)	170	10.47
				5	(1,3,5,7,9)	150	13.42
				6	(1,3,5,7,9,*)	150	5.06
	4	126	5	0		450	0.03
				1	(*)	450	0.33
				2	(3,7)	290	37.80
				3	(2,5,8)	210	104.61
				4	(2,4,6,8)	190	99.98
				5	(1,3,5,7,9)	170	140.98
	5	126	5	0		640	0.08
				1	(5)	480	1.00
				2	(3,7)	310	43.11
				3	(2,5,8)	240	514.92
				4	(2,4,6,8)	210	404.16
	6	84	3	0		870	0.06
				1	(5)	550	11.97
				2	(3,7)	350	160.36
				3	(2,5,8)	270	628.25
	7	36	2	0		1140	0.03
				1	(5)	660	18.48
				2	(3,7)	410	161.95
	8	9	1	0		1450	0.02
				1	(5)	810	15.22

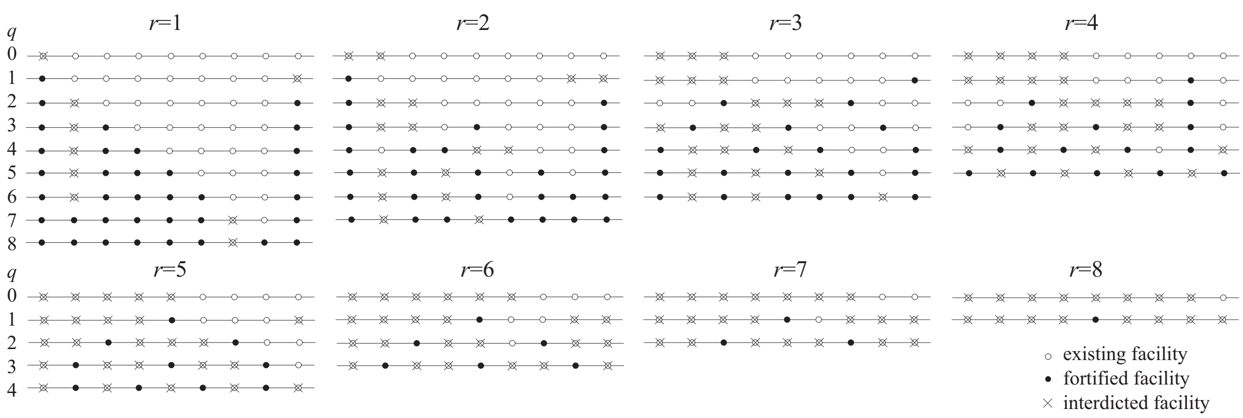

Figure 1: Results of RIMF in a linear urban space ($p = 9$)

Table 2: Results of RICF (a linear space)

Existing facilities p	Interdicted facilities r	Interdiction pattern $\|H\|$	Remaining facilities $p-r$	Fortified facilities q	Optimal fortification patterns z	Demand covered WC	Execution time sec
9	0	-	9	-	-	18	-
	1	9	8	0		17	0.02
				1	(1),(9)	17	0.03
				2	(1,9)	18	0.05
				3	(1,*,9)	18	0.02
				4	(1,*,*,9)	18	0.02
				5	(1,*,*,*,9)	18	0.08
				6	(1,*,*,*,*,9)	18	0.01
				7	(1,*,*,*,*,*,9)	18	0.01
				8	(1,*,*,*,*,*,*,9)	18	0.02
	2	36	7	0		15	0.01
				1	(*)	15	0.14
				2	(1,8),(1,9),(2,9)	16	0.41
				3	(1,*,9)	16	0.55
				4	(1,*,*,9)	16	1.67
				5	(1,3,5,7,9)	18	0.05
				6	(1,3,5,7,9,*)	18	0.05
				7	(1,+.+,+,+,9)	18	0.05
	3	84	6	0		13	0.09
				1	(*)	13	0.53
				2	(1,8),(1,9),(2,8),(2,9),$\cdots$	14	2.70
				3	(1,4,7),(2,5,8),(3,6,9)	15	4.20
				4	(1,4,6,8),(1,4,6,9), (2,4,6,8),(2,4,6,9)	16	5.69
				5	(1,3,5,7,9)	18	0.22
				6	(1,3,5,7,9,*)	18	0.22
	4	126	5	0		11	0.05
				1	(*)	11	1.09
				2	(1,9),(2,6),(3,7),(4,8),$\cdots$	12	13.53
				3	(2,5,8)	14	26.16
				4	(2,4,6,8)	16	44.23
				5	(1,3,5,7,9)	18	0.75
	5	126	5	0		9	0.05
				1	(5)	10	1.05
				2	(2,6),(3,7),(4,8)	11	49.92
				3	(2,5,8)	13	153.86
				4	(2,4,6,8)	16	10.88
	6	84	3	0		7	0.05
				1	(4),(5),(6)	8	2.41
				2	(3,6),(3,7),(4,7)	10	159.06
				3	(2,4,6),(2,5,8),(4,6,8)	12	285.39
	7	36	2	0		5	0.03
				1	(3),(4),(5),(6),(7)	6	1.86
				2	(2,5),(2,6),(2,7),(3,6),(3,7), (3,8),(4,7),(4,8),(5,8)	8	44.31
	8	9	1	0		3	0.02
				1	(2),(3),(4),(5),(6),(7),(8)	4	0.44

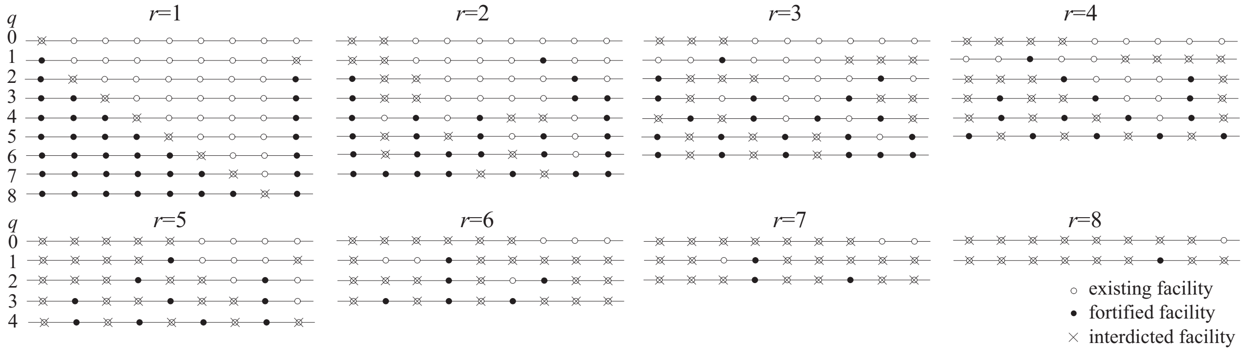

Figure 2: Results of RICF in a linear urban space ($p = 9$)

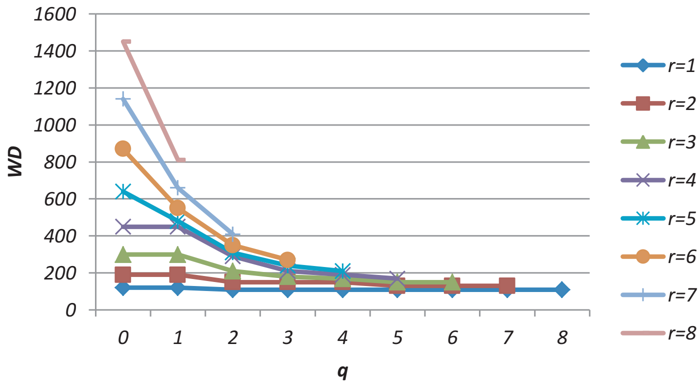

Figure 3: Total weighted distance in the results of RIMF

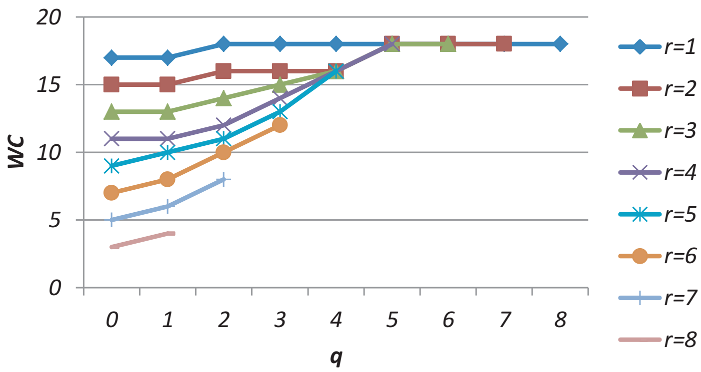

Figure 4: Total population covered in the results of RICF

correspond to those for RIMF and indicate that (i) the protection is best conducted on the fringes if r is small, (ii) the protection becomes more effectively used as fortification at regular intervals if q increases to prevent connected closures, (iii) when the value of r is greater than half of the total number of facilities, the midsection is assigned to be protected first to prevent a large connected closure on the fringes, and (iv) a larger q is more effective when r increases. Although the results seem the same qualitatively, the patterns appear different as compared to those of RIMF. To summarize, it was attempted to determine, in a general sense, in which location the protection should be provided to avoid the worst damage with expected size of damage using a linear urban model. Notwithstanding the generality of the linear model, effects due to non-uniformity of distribution cannot be taken into account.

5. Optimal Protection Patterns for Closures in a New-town

In the previous section, the fundamental function of the formulated problems was elucidated given the assumption of a uniform urban space. However, because real urban space is not homogenized, agglomeration or distribution patterns of facilities and demands can affect the optimization result. In the following, applications of RIMF and RICF are implemented as examples of planning strategies for *shopping difficulties*, based on a real urban spatial dataset. Shopping difficulties, also called “food desert” problems, have become more obvious with a decreasing and aging population, especially in suburban areas where urban density is declining. For the location problem related to providing fresh groceries daily, two surrogate measures are considered for optimization of overall accessibility: convenience and equality as discussed. In this context, grocery stores are retail facilities that function as public infrastructure. If we consider that grocery shopping is crucial for a dignified daily life, those stores ought to be provided at a minimum level.

Towards this end, hereafter, discussions focus on practical results by applying RIMF and RICF to supermarkets (SMs) and neighborhood centers (NCs) in a new-town that is experiencing an increase in its elderly population. The area of our case study is Senri New-town, Sakai, Osaka prefecture. The average age of the population is around 45 years and the elderly population comprises 23.7% of the total population, both of which are higher than the average for the country (in 2005). In particular, the area poses shopping difficulties for elderly people on account of closures of neighborhood centers which consist of small retail providers [13]. In addition, elderly people are disadvantaged compared to others in traveling long distance in general, and thus they prefer using neighborhood centers that are easily accessible rather than supermarkets. Therefore, in this case, protection is considered an inducement policy or a subsidy provided to retail stores in the declining area. Because of the focus on relieving risks of shopping difficulties for elderly people, only the elderly population is used in this study.

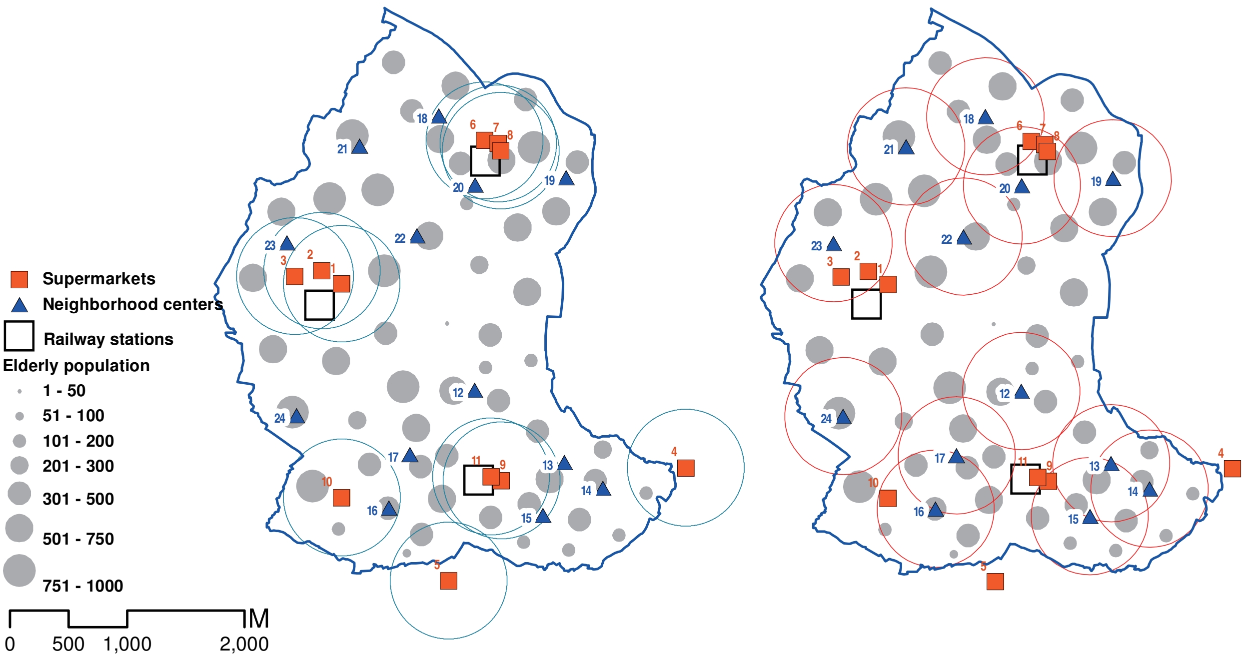

Figure 5: Locations and covering population of supermarkets (left) and neighborhood centers (right)

Figure 5 shows the locations of grocery stores in this area. The stores include 11 supermarkets and 13 neighborhood centers ($p = 24$). The supermarkets include those outside of the boundary of Senri New-town within a 500 meter buffer, as people inside can use the stores located in outside area and vice versa. Supermarkets are numbered from 1 to 11, and neighborhood centers are numbered from 12 to 24. Supermarkets are mostly concentrated near three railway stations, which are designated as regional centers by planning, whereas neighborhood centers are distributed to provide equal cover for the local demand, where possible. Here, the optimal protection patterns are calculated for the cases with number of interdictions r equal to 1 and 2. A larger p results in a considerably larger computational complexity because the combination of patterns of r increases exponentially.

Table 3 shows the results of the protection patterns calculated using the median method and Table 4 shows the results using the covering method. On the optimal fortification pattern, supermarkets are indicated in bold. The decrement of total distance column shows how much of an improvement in accessibility could be achieved by the corresponding fortification pattern. Execution time is the elapsed number of seconds spent on calculating the corresponding problem. The next interdiction pattern indicates the interdiction pattern on facilities to occur after the optimal fortification. The choice of covering distance here resembles to the approach shown in Chapter 4. That is, considering that planned neighborhoods and regional centers are 16 districts in total, the minimum distance to cover the entire area (1,160 ha) is approximately 270 meters. Therefore, the covering distance of each facility in the model is fixed at 500 meters, which is widely adopted as the approximate maximum walking distance of elderly people and larger than 270 meters. Geographic patterns of fortification and interdiction in RIMF and RICF are depicted in Figure 6 and Figure 7, respectively. Accessibilities, represented by the total weighted distance in RIMF and by the total population covered in RICF, are shown in Figure 8 and Figure 9, respectively, according to the number of closures and protections. Although neighborhood centers have a higher priority in both case, it is notable that, in RICF, which assumes that accessibility is equal when using a facility within a certain distance, a supermarket located solely in the fringe area (see SM 10) is designated to be protected at a higher priority. Note that SM 4 does not have priority for protection as it has little demand covered, even though it is in the fringe area in terms of location.

Table 3: Fortification results of RIMF

♯ interdicted facilities r	♯ fortified facilities q	Optimal fortification pattern **bold** = supermarket	Total distance $WD/1,000$	Decrement of total distance /1,000	Execution time sec	Interdiction pattern
1	0	-	9,235.4	-	-	12
	1	12	9,223.1	-122.6	0.01	24
	2	12,24	8,986.4	-236.7	0.01	21
	3	12,21,24	8,849.7	-136.7	0.02	18
	4	12,18,21,24	8,773.7	-75.9	0.01	17
	5	12,17,18,21,24	8,771.2	-2.5	0.01	19
	6	12,17,18,19,21,24	8,725.4	-45.8	0.01	10
	7	**10**,12,17,18,19,21,24	8,718.2	-7.2	0.01	22
	8	**10**,12,17,18,19,21,22,24	8,616.8	-101.4	0.01	16
	9	**10**,12,16,17,18,19,21,22,24	8,566.0	-50.7	0.01	23
	10	**10**,12,16,17,18,19,21,22,23,24	8,547.1	-18.9	0.01	13
	11	**10**,12,13,16,17,18,19,21,22,23,24	8,522.3	-24.9	0.00	14
	12	**10**,12,13,14,16,17,18,19,21,22,23,24	8,516.1	-6.2	0.01	15
	13	**10**,12,13,14,15,16,17,18,19,21,22,23,24	8,509.7	-6.4	0.01	20
2	0	-	10,078.0	-	-	12,24
	1	12	9,830.4	-247.6	0.14	10,24
	2	12,24	9,822.2	-8.2	2.49	18,21
	3	12,21,24	9,242.9	-579.3	2.08	17,18
	4	12,18,21,24	9,164.5	-78.4	4.23	17,19
	5	12,17,18,21,24	9,116.2	-48.3	4.72	20,22
	6	12,17,18,20,21,24	9,116.2	-0.1	11.69	10,19
	7	**10**,12,17,18,21,22,24	9,007.5	-108.6	20.60	16,19
	8	**10**,12,17,18,19,21,22,24	8,888.3	-119.2	19.46	13,14
	9	**10**,12,13,17,18,19,21,22,24	8,802.3	-86.0	49.02	16,23
	10	**10**,12,13,16,17,18,19,21,22,24	8,746.0	-56.3	17.45	14,15
	11	**10**,12,13,15,16,17,18,19,21,22,24	8,713.4	-32.6	2.17	3,23
	12	**10**,12,13,14,16,17,18,19,21,22,23,24	8,645.3	-68.1	4.86	15,20
	13	**10**,12,13,14,15,16,17,18,19,21,22,23,24	8,599.2	-46.1	51.00	1,20

For $r = 1$ (Table 3), the results of RIMF show that (i) stores expected to be interdicted

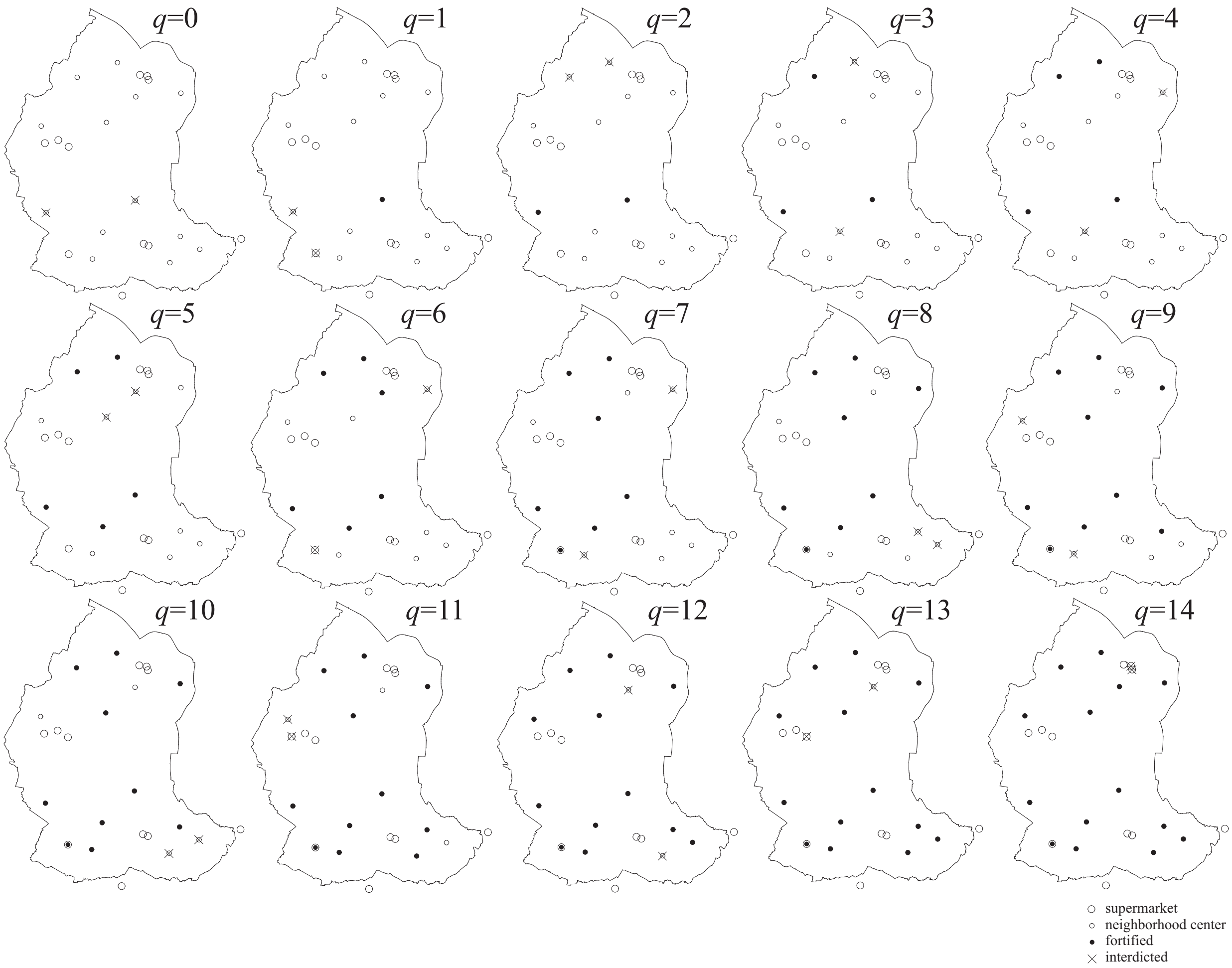

Figure 6: Results of RIMF ($r = 2$)

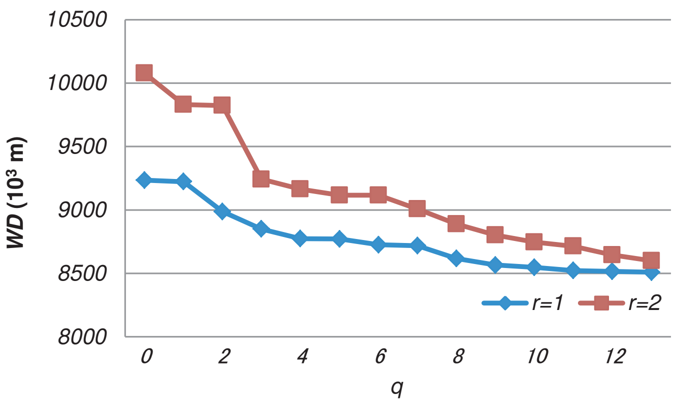

Figure 7: Accessibility in RIMF based on the number of protected facilities and interdicted facilities

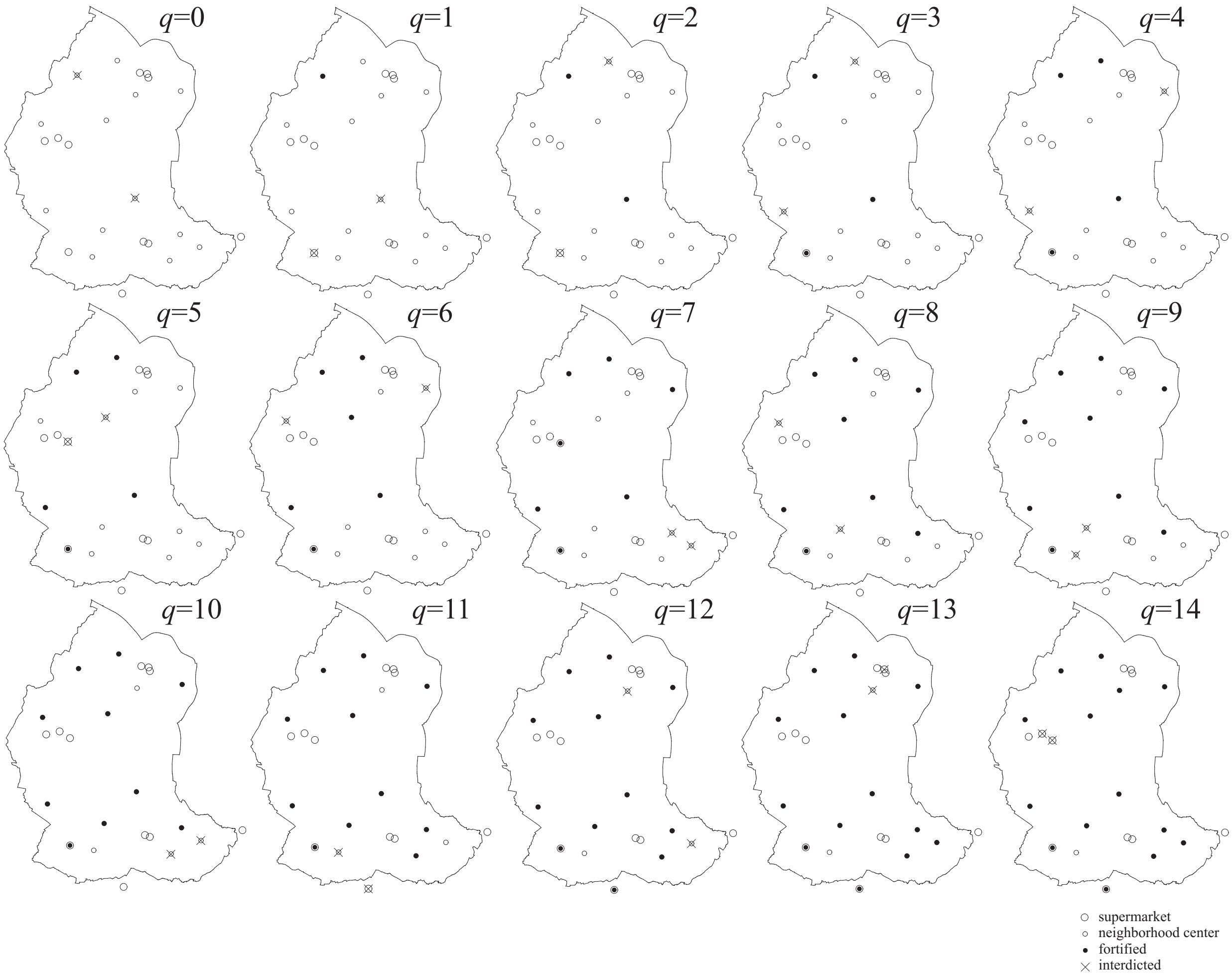

Figure 8: Results of RICF ($r = 2$)

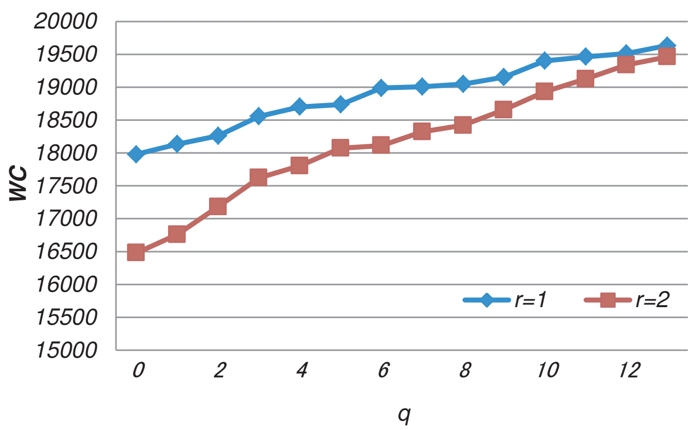

Figure 9: Accessibility in RICF based on the number of protected facilities and interdicted facilities

Table 4: Fortification results of RICF

♯ interdicted facilities r	♯ fortified facilities q	Optimal fortification pattern **bold** = supermarket	Demand covered WC	Increment of demand covered	Execution time sec	Interdiction pattern
1	0	-	17,980	-	-	21
	1	21	18,133	153	0.19	12
	2	12,21	18,260	127	0.25	10
	3	**10**,12,21	18,557	297	0.19	18
	4	**10**,12,18,21	18,700	143	0.23	24
	5	**10**,12,18,21,24	18,738	38	0.25	19
	6	**10**,12,18,19,21,24	18,989	251	0.23	22
	7	**10**,12,18,19,21,22,24	19,009	20	0.23	23
	8	**10**,12,18,19,21,22,23,24	19,047	38	0.23	17
	9	**10**,12,17,18,19,21,22,23,24	19,155	108	0.25	13
	10	**10**,12,13,17,18,19,21,22,23,24	19,400	245	0.30	15
	11	**10**,12,13,15,17,18,19,21,22,23,24	19,461	61	0.23	20
	12	**10**,12,13,15,17,18,19,20,21,22,23,24	19,512	51	0.20	14
	13	**10**,12,13,14,15,17,18,19,20,21,22,23,24	19,634	122	0.16	6
2	0	-	16,479	-	-	12,21
	1	21	16,759	280	19.08	10,21
	2	12,21	17,183	424	23.17	10,18
	3	**10**,12,21	17,623	440	27.04	18,24
	4	**10**,12,18,21	17,804	181	27.05	19,24
	5	**10**,12,18,21,24	18,072	268	28.92	1,22
	6	**10**,12,18,21,22,24	18,113	41	33.87	19,23
	7	**1**,**10**,12,18,19,21,24	18,321	208	41.04	13,14
	8	**10**,12,13,18,19,21,22,24	18,422	101	51.40	17,23
	9	**10**,12,13,18,19,21,22,23,24	18,657	235	58.27	16,17
	10	**10**,12,13,17,18,19,21,22,23,24	18,931	274	75.53	14,15
	11	**10**,12,13,15,17,18,19,21,22,23,24	19,126	195	62.79	5,16
	12	**5**,**10**,12,13,15,17,18,19,21,22,23,24	19,339	213	44.05	14,20
	13	**5**,**10**,12,13,14,15,17,18,19,21,22,23,24	19,461	122	24.59	7,20

must be protected in the next fortification step and (ii) stores that are supposed to be protected in the previous fortification step must also be included in the next fortification step. Neighborhood center (NC) 12 has the highest priority for fortification, and its location features a high population and no competitive stores within a short distance. As the number of fortifying stores q increases from $q = 1$ to $q = 6$, only NCs are selected to be fortified. The most critical supermarket is SM 10, which is standalone. Other supermarkets are located in groups with other supermarkets or near stations. As shown in Figure 8, the decrease in the total sum of the distances is largest when $q = 2$.

The result differs for $r = 2$ (Table 3) in that even stores that are supposed to be fortified in the previous fortification step may not necessarily be included in the next fortification pattern (for example, see $q = 7$). Similar to the case of $r = 1$, only NCs are protected, up to $q = 6$. The decrement is largest when $q = 3$, which is extreme.

Meanwhile, when $r = 1$, the results of RICF indicate that stores that are supposed to be interdicted are necessarily included in the next fortification patterns, similar to the case with RIMF. NC 21 has the highest priority and NC 12 and SM 10 take the next highest. The priority of SM 10 is higher in the covering method than in the median method.

For $r = 2$, among the supermarkets, SM 10, SM 1, and SM 5 are included in the fortification patterns as q increases but not in sequence. That is, although SM 1 and SM 10 are supposed to be protected when $q = 7$, only SM 10 remains when $q = 8$, and SM 5 and SM 10 are protected when $q = 12$. Moreover, for both $r = 1$ and $r = 2$, the maximum increase in coverage occurs when $q = 3$ (Table 4). Nevertheless, the impact of stores with the highest priority on an increase in accessibility is relatively lower when compared with the results of RIMF.

Considering the results of both RIMF and RICF, the importance of supermarkets is higher when the covering method is used compared with the median method, particularly for SM 10.

The results in this chapter show different features compared to those in Chapter 4. Although the protection priority lies with the fringes on a uniform space when r is relatively small, this result does not generalize to a practical situation. For example, comparing the results of $r = 2$ and $q = 1$, the most-desired facility to protect is NC 12 in RIMF and NC 21 in RICF. Comparing locations, NC 12 is located near the center, whereas NC 21 is on the peripheral area. This result can also be seen between supermarkets: SMs located outside of the boundary are not prioritized when the demand is not uniform. On the other hand, it is observed that the agglomeration of facilities—for example, SM 1, 2 and 3—makes the facilities in the group more robust towards damage, which results in little improvement in system robustness from the protection of individual facilities within a group. As a consequence, it is seen in both RIMF and RICF that NCs, which are planned to be located evenly over the area, have higher priority than supermarkets, which occur in groups. Therefore, these results indicate that the distribution pattern of facilities, or the spatial relationship between facilities, strongly influences the protection pattern, even compared to the absolute location of a facility in the area. These characteristics were not observed in the uniform space analyzed in the previous chapter.

As previously noted, the majority of supermarkets form groups, whereas neighborhood centers are dispersed. In other words, supermarkets are regarded as having a complementary relationship with one another but neighborhood centers do not. In addition, supermarkets are located in such places as stations, which provide transit convenience and, hence, have a low elderly population nearby; however, neighborhood centers have more elderly people living around them. This situation implies that neighborhood centers should be protected prior to supermarkets.

6. Concluding Remarks

In this study, we proposed interpretations of what RIMF and RICF mean which can be generalized to urban areas. Furthermore, a specific geographical dataset was utilized to emphasize the necessity of considering the locational patterns of existing facilities for planning protection strategies. For the urban spaces in general, higher damage to accessibility is expected when the facilities in the center are closed compared to those on the peripheral area when the expected amount of total damage is large, while the peripheral area is prioritized for protection when the total amount of damage is relatively small. It is also found that the protection of evenly divided sections is desired when multiple sites can be protected to avoid overlapping of the protection area of each protected facility. However, this generalization can be violated for some distributions of demand and facilities as seen in Chapter 5 when facilities in the center could also be prioritized for protection.

Moreover, using the median and covering methods, the actual patterns can differ even with the same r and q, although the evaluations are similar. Therefore, these methods may provide a more detailed and persuasive basis for decision-making processes with respect to related policies. At the same time, offering an optimal strategy that accommodates the expected disruption/damage and potentially constrained planned budgets is crucial.

Generalizing the closure and protection problems of facilities on a linear urban space with a regular pattern made clear that the protection patterns for critical facilities do not depend only on the importance level of the facilities, especially with potentially large closures. Suppose that the amount of damage expected is $r = 3$, which was not possible to calculate at this time because of the limited calculation performance. For three supermarkets concentrated near the station such as SM 1, 2, and 3, the protection priority for these supermarkets can be raised. The risk of diminution of accessibility attributable to the damage can be sharply reduced if only one among the group is protected. The results of the linear urban spaces in Chapter 4 correspond to this phenomenon, and showed that a greater effect results from setting up the strong points of fortification in the midsections as r increases.

Future research may consider the application of closure in the risk minimizing approach, which was not considered in this study. Modeling the actual behavioral pattern of users and employing this pattern into the calculation of accessibility is also important for future research.

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