

2017年度 数理科学III

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Category (圏) 代数

\mathbb{R} 上の線形空間

準同型
線形写像

$x \in A$
 $a \in \mathbb{R}$

$F: A \rightarrow B$

$F(\lambda x) = \lambda F(x)$
 $F(x+y) = F(x) + F(y)$

演算を保つ

群

$F: A \rightarrow B$

$(x \cdot y) \cdot z = x \cdot (y \cdot z)$

$F(x \cdot y) = F(x) \cdot F(y)$

$F(e_A) = e_B$

$F(x^{-1}) = F(x)^{-1}$

\mathcal{C}, \mathcal{D} : categories (圏)

$F: \mathcal{C} \rightarrow \mathcal{D}$ functor (関手)

$A \in \mathcal{C} \mapsto F(A) \in \mathcal{D}$ objects

$f \in \text{Mor } \mathcal{C} \mapsto F(f) \in \text{Mor } \mathcal{D}$

条件 $F(\text{dom}(f)) = \text{dom}(F(f))$

$f: A \rightarrow B$

$F(\text{cod}(f)) = \text{cod}(F(f))$

$F(f) = F(A) \rightarrow F(B)$

$F(\text{id}_A) = \text{id}_{F(A)}$

$A \xrightarrow{f} B \xrightarrow{g} C \text{ in } \mathcal{C} \quad F(g \circ f) = F(g) \circ F(f)$

\mathcal{C}^{op} opposite category / (dual category)

$F: \mathcal{C} \rightarrow \mathcal{D}$ covariant functor

$F: \mathcal{C}^{\text{op}} \rightarrow \mathcal{D}$ functor

functor

Contravariant functor from \mathcal{C} to \mathcal{D}

$A \xrightarrow{f} B \text{ in } \mathcal{C}$

$F(A) \xleftarrow{F(f)} F(B) \text{ in } \mathcal{D}$

$A \xrightarrow{f} B \xrightarrow{g} C$

$F(g \circ f) = F(f) \circ F(g)$

$\mathcal{C} \xrightarrow[F]{G} \mathcal{D}$

\mathcal{C}, \mathcal{D} : categories

F, G = functors from \mathcal{C} to \mathcal{D}

natural transformation (自然変換)

$A \in \mathcal{C} \mapsto \alpha_A : F(A) \rightarrow G(A) \text{ in } \mathcal{D}$

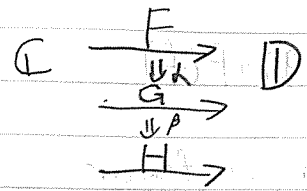
条件 $A \xrightarrow{f} B \text{ in } \mathcal{C}$

$F(A) \xrightarrow{\alpha_A} G(A)$

$F(f) \downarrow \quad \cup \quad \downarrow G(f)$

$F(B) \xrightarrow{\alpha_B} G(B)$

vertical composition

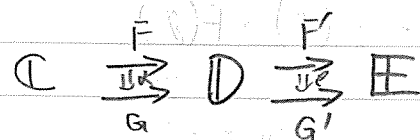
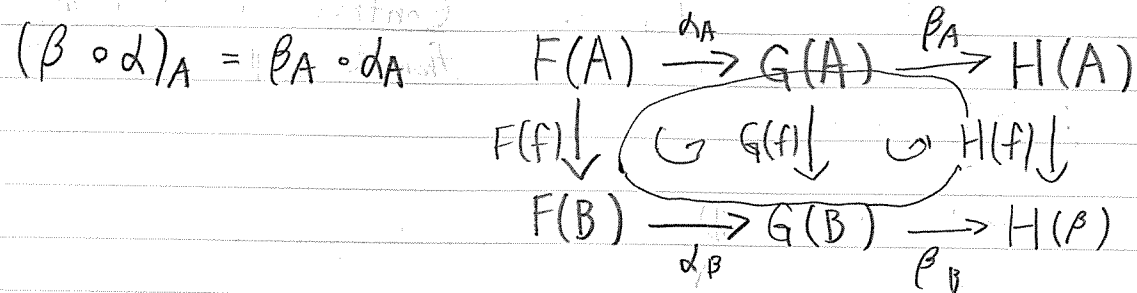


$\mathbb{C}, \mathbb{D} = \text{categories}$

$H, F, G = \text{functors from } \mathbb{C} \text{ to } \mathbb{D}$

$\alpha = F \Rightarrow G$

$\beta = G \Rightarrow H$



$\mathbb{C}, \mathbb{D}, \mathbb{E} = \text{categories}$

$F, G : \mathbb{C} \rightarrow \mathbb{D}$ functors

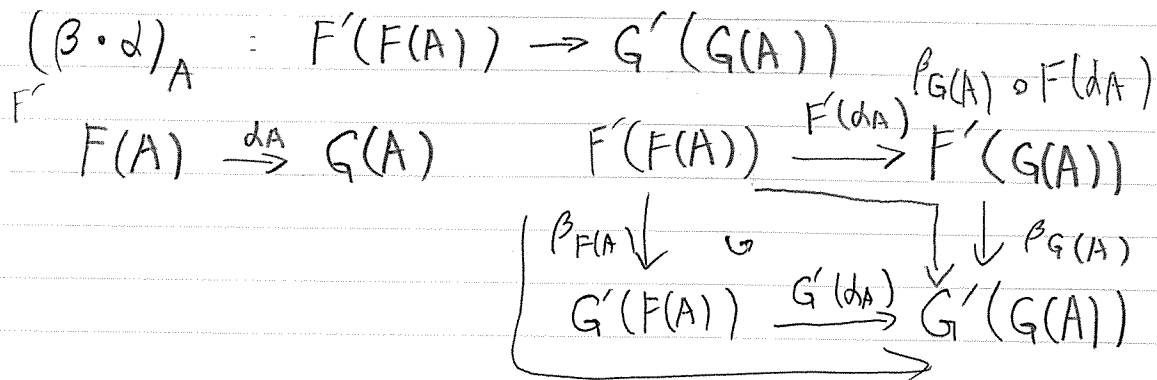
$F', G' : \mathbb{D} \rightarrow \mathbb{E}$ functors

$\alpha = F \Rightarrow G$ natural trans

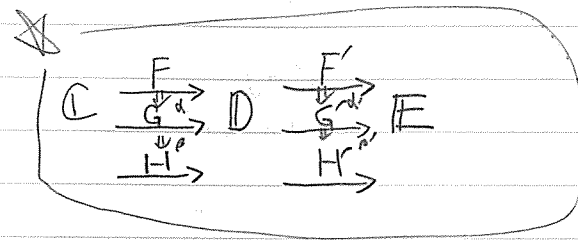
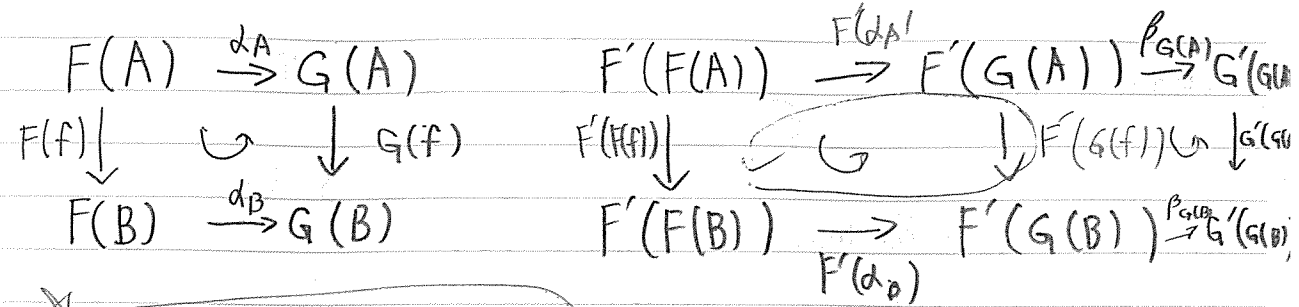
$\beta = F' \Rightarrow G'$

$\beta \circ \alpha : F' \circ F \Rightarrow G' \circ G$

horizontal composition



$A \xrightarrow{f} B$ in \mathbb{C} . F' を掛ける.



$(\beta \circ \alpha') \circ (\beta \circ \alpha) = \dots$

$(\beta' \circ \beta) \circ (\alpha' \circ \alpha)$

証明せよ.