

2017年度 数理科学III

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Seifert - van Kampen の定理

knot theory (結び目理論) (筑波大学石井 丹下)
 $I = [0, 1]$ a simple closed curve in Euclidean 3-space

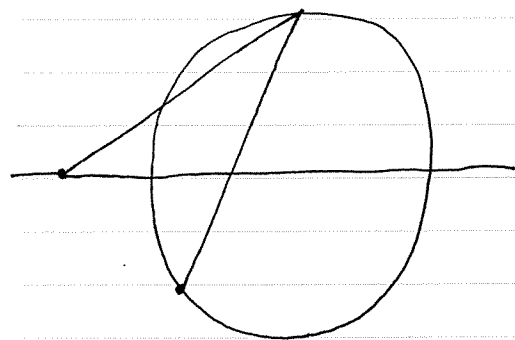
$t: I \rightarrow \mathbb{R}^3$ $t(0) = t(1)$ 分類

ex) 線形代数 $P^{-1}AP = B$
Jordan 標準形, 有限生成のAbel群 \cong 同群

k_1, k_2 contained in \mathbb{R}^3 are equivalent +
 if \exists orientation-preserving homeomorphism $h: \mathbb{R}^3 \rightarrow \mathbb{R}^3$
 with $h(k_1) = k_2$

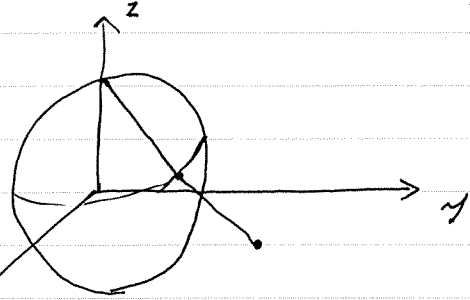
$\mathbb{R}^3 - k_1$ $\mathbb{R}^3 - k_2$ homeomorphic
 同相

$\pi(\mathbb{R}^3 - k_1) = \pi(\mathbb{R}^3 - k_2)$



$\mathbb{R} \subset S^1$ 1点 compact.

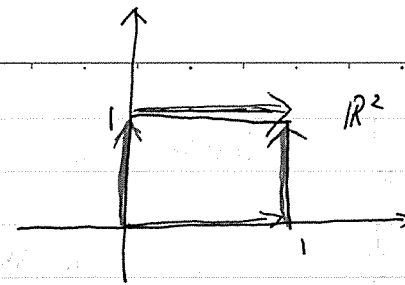
$\mathbb{R}^2 \subset S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$



$\mathbb{R}^3 \subset S^3 = \{(x, y, z, u) \in \mathbb{R}^4 \mid x^2 + y^2 + z^2 + u^2 = 1\}$

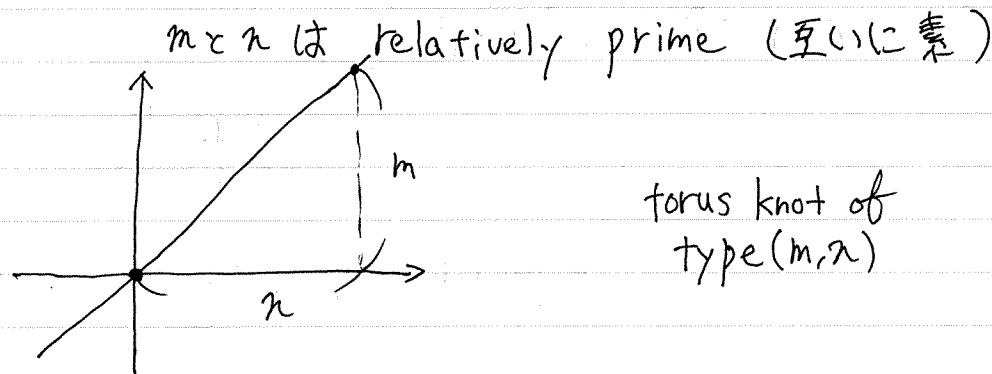
torus knots

torus



$(x, y) \sim (x', y')$
 $x - x' \in \mathbb{Z}$
 $y - y' \in \mathbb{Z}$

L line through the origin
 with slope m/n ($1 < m < n$)



torus knot of type (m, n)

目標 \rightarrow

$(2, 5)$
 $(7, 9)$ は同じではない。

torus knot of type (m, n) S^3

$A = \{(x_1, x_2, x_3, x_4) \in S^3 \mid x_1^2 + x_2^2 \leq x_3^2 + x_4^2\}$

$B = \{(x_1, x_2, x_3, x_4) \in S^3 \mid x_1^2 + x_2^2 \geq x_3^2 + x_4^2\}$

A, B = closed subsets of S^3 with $A \cup B = S^3$

$A \cap B = \{(x_1, x_2, x_3, x_4) \in S^3 \mid x_1^2 + x_2^2 = x_3^2 + x_4^2 = \frac{1}{2}\}$

$x_1^2 + x_2^2 = \frac{1}{2}$ circle in the (x_1, x_2) plane

$x_3^2 + x_4^2 = \frac{1}{2}$ circle in the (x_3, x_4) plane

$A \cup B$ solid torus (homeomorphic to the product of a disc and a circle)

$D = \{ (x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq \frac{1}{2} \}$ closed disc

$S^1 = \{ (x_3, x_4) \in \mathbb{R}^2 : x_3^2 + x_4^2 = \frac{1}{2} \}$ circle

radius $\frac{\sqrt{2}}{2}$
(半径)

$f: D \times S^1 \rightarrow A$

report I 全単射であることを示せ。

$f(x_1, x_2, x_3, x_4) = (x_1, x_2, \sqrt{2}x_3, (1 - (x_1^2 + x_2^2))^{\frac{1}{2}}, \sqrt{2}x_4(1 - (x_1^2 + x_2^2))^{\frac{1}{2}})$
continuous

$A \cap B$
 k torus knot of type (m, n)

$S^3 - k = (A - k) \cup (B - k)$
 $A - k, B - k, (A - k) \cap (B - k) =$ arcwise connected.

Seifert - van Kampen の定理.

小さな $\epsilon > 0$

$A \cup B$ は deformation retract of $U \cup V$

N : tubular neighborhood of k of radius ϵ

厚-円柱の近傍

$S^3 - N$ deformation retract of $S^3 - k$

U, V $\frac{1}{2}\epsilon$ neighborhoods of A and B respectively

U, V homeomorphic to the product of an open disc and circle

$U \cap V$ thickened torus

the product of $A \cap B$ and the open interval $(-\frac{1}{2}\epsilon, \frac{1}{2}\epsilon)$

$S^3 - N = (U - N) \cup (V - N)$
 $\pi(S^3 - N) = \pi(S^3 - k)$

$U - N$
 $V - N$ } homotopy type of a circle 示せ (II)

無限巡回群

$(U - N) \cap (V - N) = (U \cap V) - N$
 $(A - k) \cap (B - k) = (A \cap B) - k$ } same homotopy type

$(A - N) \cap (B - N) = (A \cap B) - N$ deformation retract of each of these spaces.

$(A \cap B) - k$ subset of $A \cap B$ torus homeomorphic to the product of a circle and an open interval

$\varphi_1 = \pi(U \cap V - N) \rightarrow \pi(U - N)$
 $\varphi_2 = \pi(U \cap V - N) \rightarrow \pi(V - N)$ } 二本を決定せよ (III)

命題 torus knot of type (m, n) $\{ \alpha, \beta \}$ $\alpha^m \beta^n$
本質的