

2017年度 数理科学III

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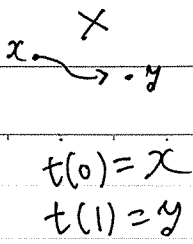
Seifert - Van Kampen の定理 高橋

U, V arcwise connected open subsets

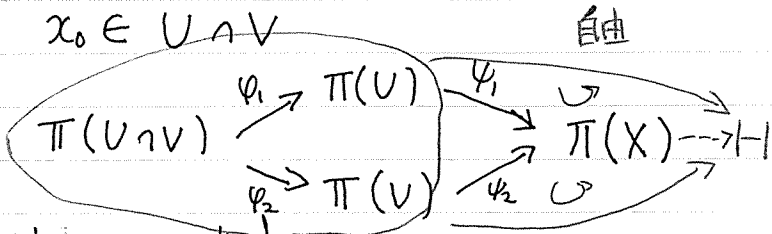
$X = U \cup V$ (和集合)

$U \cap V$ (共通部分) non empty and arcwise connected

base point (基点) $x_0 \in U \cap V$



$t: I \rightarrow X$



前回

Theorem $U \cap V$ simply connected

$\pi(X)$: free product of $\pi(U)$ and $\pi(V)$



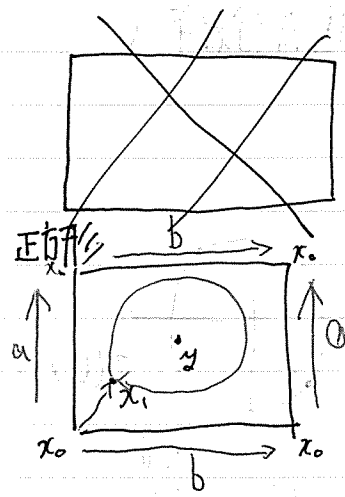
今 V : simply connected
kernel $\psi_1: \pi(U) \rightarrow \pi(X)$ epimorphism (全射)
kernel is the smallest normal subgroup of $\pi(U)$ containing the image $\psi_1(\pi(U \cap V))$

応用 (applications)

torus $T = S^1 \times S^1$ $\pi(S^1)$

$\pi(T) = \pi(S^1) \times \pi(S^1) = \mathbb{Z} \times \mathbb{Z}$

無限巡回群



$U = T - \{y\}$

V : the interior image of



homeomorphic.

deformation retract of the whole square minus a point

U, V open subsets

$U \cap V$ arcwise connected

V : simply connected

$\psi_1: \pi(U, x_1) \rightarrow \pi(T, x_1)$ epimorphism

$\psi_1: \pi(U \cap V, x_1) \rightarrow \pi(U, x_1)$ 正規部分群 α, β

the union of the two circles α and β is a deformation retract of U .

$\pi(U, x_1)$: the free group on two generators.

$\pi(U, x_1)$

$\alpha' = \delta^{-1} \alpha \delta$

$\beta' = \delta^{-1} \beta \delta$

自由群

非可換

UNV circle

γ

$$\varphi_1(\gamma) = \alpha' \beta' \alpha'^{-1} \beta'^{-1}$$

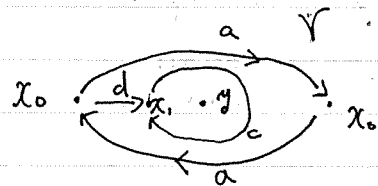
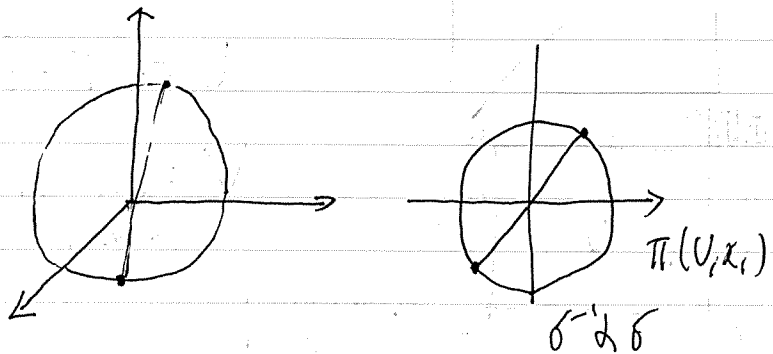
Abel化

結論 $\pi(T)$

α' と β' で生成される自由群を

交換子 $\alpha' \beta' \alpha'^{-1} \beta'^{-1}$ で生成される正規部分群で割ったもの.

射影平面 P_2
直線



$U = P_2 - \{y\}$ $\varphi_1(\gamma) = \alpha'^2$
 V : the image of the interior $\square_{1,2}$ of the polygon under the identification.

the circle a is a deformation retract of U

$\pi(U, x_0)$ α' で生成される無限巡回群 $\alpha' a$