No. 5/16 (火)　　第5回 数理科学ⅢA

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基本群

(X, x_0)　　X：位相空間　$x_0 \in X$　　loop　homotopy

$\pi(X, x_0)$　　Xの点x_0を基点とする基本群

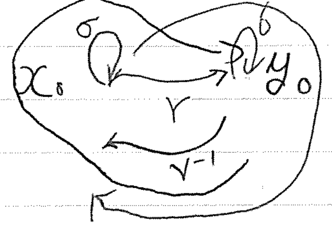
基点をx_0からy_0へ変更するとどうなるか？

X connected

$x \rightsquigarrow y$

$t : I \to X$　　$t(0) = x$
$t(1) = y$

$\pi(X, x_0) \longrightarrow \pi(X, y_0)$



$\sigma \longmapsto \gamma^{-1} \cdot \sigma \cdot \gamma$

$\sigma_1 \sim \sigma_2 \Rightarrow \gamma^{-1}\sigma_1\gamma \sim \gamma^{-1}\sigma_2\gamma$

$\sigma_1\sigma_2$　　$\gamma^{-1}\sigma_1\sigma_2\gamma \sim (\gamma^{-1}\sigma_1\gamma)(\gamma^{-1}\sigma_2\gamma)$

演算を保つ。

群の準同型写像

$\pi(X, y_0) \to \pi(X, x_0)$　　群の同型写像

$\sigma \mapsto \gamma\sigma\gamma^{-1}$

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$\pi(X, x_0) \to \pi(X, y_0) \to \pi(X, x_0)$

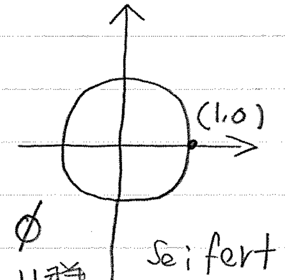
恒等写像

$\sigma \longmapsto \gamma^{-1}\sigma\gamma$

$\gamma(\gamma^{-1}\sigma\gamma)\gamma^{-1} \sim \sigma$

$\pi(X, y_0) \to \pi(X, x_0) \to \pi(X, y_0)$　恒等写像に

$\pi(S^1, (1,0))$



$X = U \cup V$　(U, Vも開集合)

$x_0 \in U \cap V \neq \emptyset$

Seifert and van Kampen の定理
ザイフェルト　ファンカンペン

universal　H群

$$\begin{tikzcd} & \pi(U) \arrow[dr] \arrow[d, phantom, "\circlearrowleft"] & \\ \pi(U \cap V) \arrow[ur] \arrow[dr] & {} & H \\ & \pi(V) \arrow[ur] & \end{tikzcd}$$

$\exists ! \sigma : \pi(X) \to H$

such that

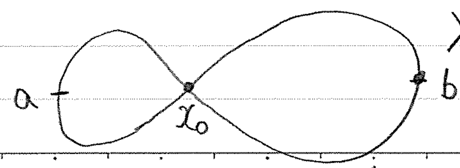
$$\begin{tikzcd} & \pi(X) \arrow[d] \\ \pi(U) \arrow[ur] \arrow[r] \arrow[r, phantom, "\circlearrowleft"] & H \end{tikzcd} \qquad \begin{tikzcd} & \pi(X) \arrow[d] \\ \pi(V) \arrow[ur] \arrow[r] \arrow[r, phantom, "\circlearrowleft"] & H \end{tikzcd}$$

$$\begin{tikzcd} & \pi(X) \arrow[d] \\ \pi(U \cap V) \arrow[ur] \arrow[r] \arrow[r, phantom, "\circlearrowleft"] & H \end{tikzcd}$$

$U \cap V$ simply connected　$\{e\}$

$\pi(U \cap V) =$ 単位元のみ

deformation retract



X

$U = X - \{b\}$　S^1

$V = X - \{a\}$　S^1

$\pi(U_p) = \pi(V_q) = \mathbb{Z}$　　$p^{m_1} q^{n_1} p^{m_2} q^{n_2} \cdots$

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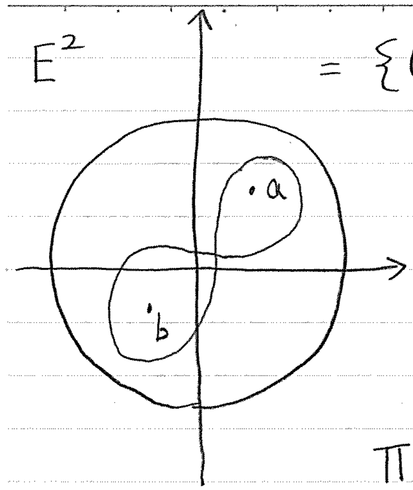
$E^2 = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leqq 1\}$



$Y = E^2 - \{a, b\}$

$X \subset Y$ X ; deformation retract of Y

$\pi(Y) = \pi(X)$

closed disk

Y: open disc minus two points.

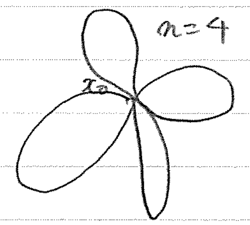
Y = entire plane minus two points.

例 n circles with a single point in common

$n > 2$

$X = A_1 \cup A_2 \cup \cdots \cup A_n$

A_i : homeomorphic to S^1 $A_i \cap A_j = \{x_0\}$



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命題

$\pi(X)$ is a free group on n generators $\alpha_1, \alpha_2, \cdots, \alpha_n$

証明 by induction on n

$a_i \in A_i$ such that $a_i \neq x_0$

$U = X - \{a_n\}$ $A_1 \cup \cdots \cup A_{n-1}$: deformation retract of U

$V = X - \{a_1, a_2, \cdots, a_{n-1}\}$

A_n : deformation retract of V

$\pi(U)$ と $\pi(V)$ の free product

$U \cap V$: contractible