2017年度 数理科学III

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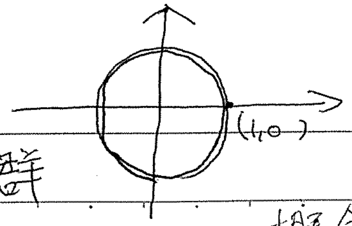
第4回 数理科学IIIA



$\pi(S^1,(1,0))$ の決定　無限巡回群

概念

前回　cyclic group（巡回群）

位数　$\mathbb{Z}_2$　$x+x=0$

degree　closed path

$S^1 \subseteq \mathbb{C}$（複素平面）

z: 複素数

$a(z)$　angle　偏角　$\theta+2k\pi$

$$\left.\begin{array}{ll}\theta_1 & a(z_1)\\ \theta_2 & a(z_2)\end{array}\right\} \Rightarrow \begin{array}{ll}\theta_1+\theta_2 & a(z_1 z_2)\\ \theta_1-\theta_2 & a(z_1/z_2)\end{array}$$

$h: I \to S^1$　closed path　$h(0)=h(1)=1$

$0=t_0<t_1<\cdots<t_n=1$

such that　$t, t' \in [t_{i-1}, t_i] \Rightarrow |h(t')-h(t)|<1$

Lebesgue 数

θ_i　偏角　$h(t_i)/h(t_{i-1})$

such that　$-\frac{\pi}{2}<\theta_i<\frac{\pi}{2}$

$|h(t_i)-h(t_{i-1})|<1$

degree of $h = \frac{1}{2\pi}\sum_{i=1}^{n}\theta_i$　整数　for　$\prod_{i=1}^{n}\frac{h(t_i)}{h(t_{i-1})} = \frac{h(t_n)}{h(t_0)} = \frac{1}{1} = 1$

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independent of the choice of the subdivision of I

two subdivisions have a common refinement

$[t_{i-1}, t_i]$　$t_{i-1}<s<t_i$　細分（refine）

θ_i

θ_i'　θ_i''

$\theta_i' = a\left(\frac{h(s)}{h(t_{i-1})}\right)$　$\theta_i'' = a\left(\frac{h(t_i)}{h(s)}\right)$　$|\theta_i'|<\frac{\pi}{2}$　$|\theta_i''|<\frac{\pi}{2}$

$$\left.\begin{array}{l}\theta_i\\ \theta_i'+\theta_i''\end{array}\right\} h(t_i)/h(t_{i-1})$$

$\theta_i-(\theta_i'+\theta_i'') = 2\pi m$　$|\ |<\frac{\pi}{2}$

$h \sim g$ (homotope) $\Rightarrow$ $\boxed{\text{degree of } h = \text{degree of } g}$　$m=0$

$F: I\times I \to S^1$ ~~st~~ such that

$F(t,0)=h(t)$

$F(t,1)=g(t)$

$F(0,s)=F(1,s)=1$

$t_0=0<t_1<\cdots<t_n=1$

$s_0=0<s_1<\cdots<s_m=1$

F　$[t_{i-1},t_i]\times[s_{j-1},s_j]$　S^1　diameter <1

$(t,s),(t',s') \in [t_{i-1}, t_i] \times [s_{j-1}, s_j]$

$\Rightarrow |F(t,s) - F(t',s')| < 1$

$\theta_i' = a\left(\dfrac{F(t_i, s_{j-1})}{F(t_{i-1}, s_{j-1})}\right) \qquad |\theta_i'| \leq \dfrac{\pi}{2}$

$\theta_i'' = a\left(\dfrac{F(t_i, s_j)}{F(t_{i-1}, s_j)}\right) \qquad |\theta_i''| < \dfrac{\pi}{2}$

$j = 1, 2, \cdots, m$

以下を証明したい。

$$\boxed{\sum_{i=1}^{n} \theta_i' = \sum_{i=1}^{n} \theta_i''}$$

$\varphi_i = a\left(\dfrac{F(t_i, s_j)}{F(t_i, s_{j-1})}\right) \qquad |\varphi_i| < \dfrac{\pi}{2}$ for $i = 0, 1, \cdots, n$

$\theta_i'' - \theta_i' \qquad \varphi_i - \varphi_{i-1}$

偏角 $2\pi m$

$\dfrac{F(t_i, s_j) F(t_{i-1}, s_{j-1})}{F(t_{i-1}, s_j) F(t_i, s_{j-1})} \qquad \theta_i'' - \theta_i' = \varphi_i - \varphi_{i-1}$

$\Sigma \theta_i'' - \Sigma \theta_i' = \Sigma(\varphi_i - \varphi_{i-1}) = \varphi_n - \varphi_0$

$\varphi_0 - \varphi_n = 0 \qquad \beta \in \pi(S^1)$

$E^n = \{x \in \mathbb{R}^n \mid |x| \leq 1\}$

$n \leq 2$

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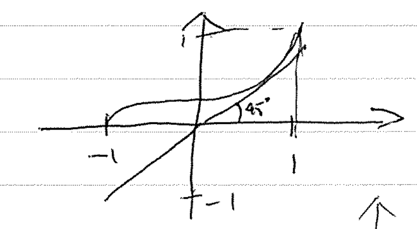
Theorem Any continuous map f of E^n into itself has at least one fixed point

$n = 1$

$E^1 = [-1, 1]$



$\pi_1(S^1) \to \pi_1(E^2) \to \pi_1(S^1)$ 基本群

$\mathbb{Z}$ 0 球

$S^1 \to E^2 \to S^1$

S^1 は E^2 の retract