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Enriched Category Theory (豊穣圏)

例) Mod_R : the category of left modules over a fixed, not necessarily commutative ring R

$\mathrm{Mod}_R(A,B)$ abel群
$(\mathrm{Ab}, \otimes_{\mathbb{Z}}, \mathbb{Z})$ monoidal category

admits the structure of a category enriched over

The base for enrichment

symmetric monoidal category $(V, \times, *)$

V : category

$\_\times\_ : V \times V \to V$ bifunctor called the monoidal product

$* \in V$ called the unit object

$v \times w \cong w \times v$ $u \times (v \times w) \cong (u \times v) \times w$

$* \times v \cong v \times *$

V : complete
co complete

Enriched Categories

V-category D
- a collection of objects $x, y, z \in D$
- for each pair $x, y \in D$, a hom-object $D(x,y) \in V$
- for each $x \in D$, a morphism $id_x : * \to D(x,x)$ in V
- for each triple $x, y, z \in D$, a morphism $\circ : D(y,z) \times D(x,y) \to D(x,z)$ in V

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such that the following diagrams commute for all $x, y, z, w \in D$

$$\begin{tikzcd}
D(z,w) \times D(y,z) \times D(x,y) \arrow[r, "1\times\circ"] \arrow[d, "\circ\times 1"'] & D(z,w) \times D(x,z) \arrow[d, "\circ"] \\
D(y,w) \times D(x,y) \arrow[r, "\circ"'] & D(x,w)
\end{tikzcd}$$
associativity

$$\begin{tikzcd}
D(x,y) \times * \arrow[r, "1\times id_x"] \arrow[dr, "\cong"'] & D(x,y) \times D(x,x) \arrow[d, "\circ"] \\
& D(x,y)
\end{tikzcd}$$

$$\begin{tikzcd}
D(y,y) \times D(x,y) \arrow[d, "\circ"'] & * \times D(x,y) \arrow[l, "id_y\times 1"'] \arrow[dl, "\cong"] \\
D(x,y) &
\end{tikzcd}$$
one object
monoid

例) $\mathrm{Ab} = (\mathrm{Ab}, \otimes_{\mathbb{Z}}, \mathbb{Z})$ base
one object Ab-category a ring with identity

D x $D(x,x) \otimes_{\mathbb{Z}} D(x,x) \to D(x,x)$

例) Topological spaces are naturally enriched over gropoids.

category

$\underline{Top}(X, Y)$ — $\underline{\text{object}}$ ~~con~~ continuous maps from X to Y

$\underline{\text{morphism}}$ homotopy classes of homotopies between these maps

例) V has copowers of the unit object that are preserved by the monoidal product in each variable.

$\mathbb{C}$: 任意の category は それに付随する free V-category を持つ.

$a, b \in \mathbb{C}$ $\quad \coprod_{\mathbb{C}(a,b)} \ast$

$$\left(\coprod_{\mathbb{C}(b,c)} \ast\right) \times \left(\coprod_{\mathbb{C}(a,b)} \ast\right) \cong \coprod_{\mathbb{C}(b,c)} \left(\ast \times \left(\coprod_{\mathbb{C}(a,b)} \ast\right)\right)$$

$$\cong \coprod_{\mathbb{C}(b,c)} \left(\coprod_{\mathbb{C}(a,b)} \ast \times \ast\right) \cong \coprod_{\mathbb{C}(b,c) \times \mathbb{C}(a,b)} \ast$$

定義 (closed monoidal categories)

$- \times v : V \to V$ functor

right adjoint ~~V(u~~ $\underline{V}(v, -)$

$\underline{V}(-,-)$: bifunctor

$V(u \times v, w) \cong V(u, \underline{V}(v, w)) \quad {}^\forall u, v, w \in V$

命題

V is enriched over itself

$\underline{V}(v, w) \times \underline{V}(u, v) \to \underline{V}(u, w)$ composition law 定義

$- \times u \dashv \underline{V}(u, -)$

adjunct

$\underline{V}(v, w) \times \underline{V}(u, v) \times u \xrightarrow{1 \times \epsilon} \underline{V}(v, w) \times v \to w$

V-category

$\Downarrow$

underlying category

G : fixed discrete group

$\underline{Top\, X, Y}$