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著者	西村 泰一
著者別名	Nishimura Hirokazu
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Enriched Category Theory (豊穰圏)

例) Mod_R : the category of left modules over a fixed, not necessarily commutative ring R .

$\text{Mod}_R(A, B)$ abel 群
 $(\text{Ab}, \otimes_Z, Z)$ monoidal category

enriched over
admits the structure of a category

The base for enrichment

symmetric monoidal category $(V, X, *)$

V : category

$X : V \times V \rightarrow V$ bifunctor called the monoidal product

$* \in V$ called the unit object

$$V \times W \cong W \times V \quad u \times (v \times w) \cong (u \times v) \times w$$

$$* \times V \cong V \times *$$

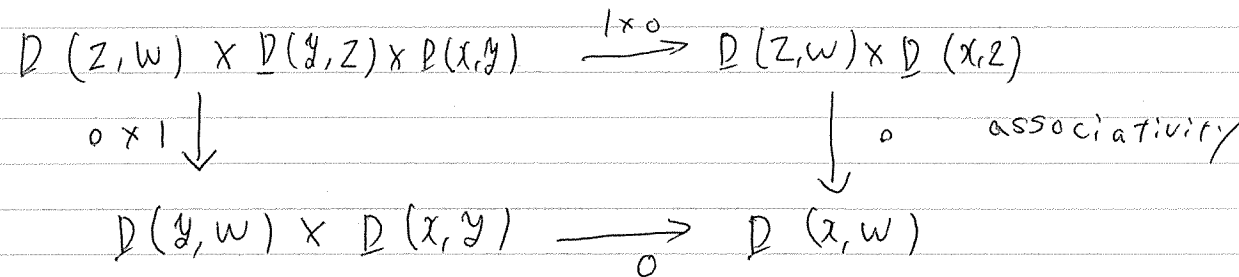
V : complete
co complete

Enriched Categories

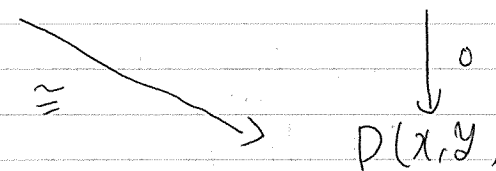
V -category \underline{D}

- a collection of objects $x, y, z \in \underline{D}$
- for each pair $x, y \in \underline{D}$, a hom-object $\underline{D}(x, y) \in V$
- for each $x \in \underline{D}$, a morphism $\text{id}_x : * \rightarrow \underline{D}(x, x)$ in V
- for each triple $x, y, z \in \underline{D}$, a morphism $o : \underline{D}(y, z) \times \underline{D}(x, y) \rightarrow \underline{D}(x, z)$ in V

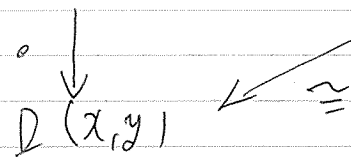
such that the following diagrams commute for all $x, y, z, w \in \underline{D}$



$$\underline{D}(x, y) \times * \xrightarrow{1 \times \text{id}_x} \underline{D}(x, y) \times \underline{D}(x, x)$$



$$\underline{D}(y, y) \times \underline{D}(x, y) \xleftarrow{\text{id}_y \times 1} * \times \underline{D}(x, y) \quad \text{one object monoid}$$



例) $\text{Ab} = (\text{Ab}, \otimes_Z, Z)$ base

one object Ab -category

aring with identity

$$\underline{D} \times \underline{D}(x, x) \otimes_Z \underline{D}(x, x) \rightarrow \underline{D}(x, x)$$

例) Topological spaces are naturally enriched over groupoids.

category $\text{Top}(X, Y)$ object ~~cont~~ continuous maps from X to Y

morphism homotopy classes of homotopies between these maps

例) V has copowers of the unit object that are preserved by the monoidal product in each variable.

\mathcal{C} : 任意の category は 元れに付随する free V -category を持つ.

$$a, b \in \mathcal{C} \quad \coprod_{\mathcal{C}(a,b)} \ast$$

$$\left(\coprod_{\mathcal{C}(b,c)} \ast \right) \times \left(\coprod_{\mathcal{C}(a,b)} \ast \right) \cong \coprod_{\mathcal{C}(b,c)} \left(\ast \times \left(\coprod_{\mathcal{C}(a,b)} \ast \right) \right)$$

$$\cong \coprod_{\mathcal{C}(b,c)} \left(\coprod_{\mathcal{C}(a,b)} \ast \times \ast \right) \cong \coprod_{\mathcal{C}(b,c) \times \mathcal{C}(a,b)} \ast$$

定義 (closed monoidal categories)

$$_ \times V = V \rightarrow V \quad \text{functor}$$

$$\text{right adjoint} \quad \underline{V}(V, _)$$

$$\underline{V}(_, _) = \text{bifunctor}$$

$$V(u \times v, w) \cong V(u, \underline{V}(v, w)) \quad \forall u, v, w \in V$$

命題

V is enriched over itself

$$\underline{V}(v, w) \times \underline{V}(u, v) \rightarrow \underline{V}(u, w) \quad \text{composition law 定義}$$

$$_ \times u \dashv \underline{V}(u, _)$$

$$\text{adjunct} \quad \underline{V}(v, w) \times \underline{V}(u, v) \times u \xrightarrow{1 \times \epsilon} \underline{V}(v, w) \times v \rightarrow w$$

V -category



underlying category

G : fixed discrete group

Top X, Y