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monoidal category

Theorem

words of fixed length n

$\Rightarrow G_n = G_{n,b}$ basic graph

$\rho \circ \beta = 0 \quad \rho(-) = 0$

rank

$\rho(w) = \rho(V) + \rho(W) + \text{length}(w) - 1$

$\rho w = 0 \Rightarrow$ all pairs of parentheses in w start at the front

G_n : commute

$V \rightarrow V_1 \leftarrow V_2 \rightarrow V_3 \leftarrow W$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $W^{(n)} = (W^*) = W^{(n)} = W^{(n)} = W^{(n)}$

any two directed paths (all n canonical path) from V_i to $W^{(n)}$

by induction the rank of $V_A = V$

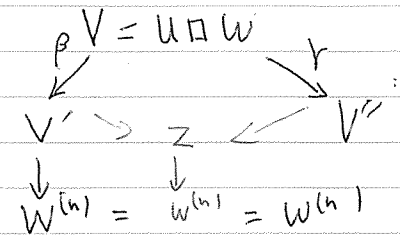
Suppose it true for all V of smaller rank,

consider two different directed paths

starting at V with directed

basic arrows ρ and γ

Both ρ and γ decrease rank



by a case subdivision

(1) $\rho = \gamma \Rightarrow Z = V' = V''$

(2) $\rho \neq \gamma \quad V = U \square W$

$\rho = \beta' \square W$: ρ acts inside the first factor

$\rho = U \square \beta''$: ρ acts inside the second factor

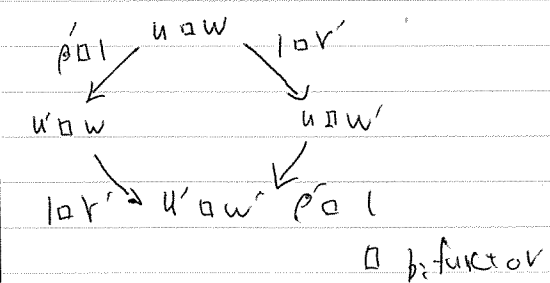
$\rho = d_{u,s,t}$ where $V = U \square W = U \square (s \square t)$

compare ρ and γ

both act inside the same factor U

\Rightarrow we can use induction on the length h

ρ acts inside u and γ



\square is functor

残っている場合

one of β or γ , say β , is $\beta = d = d_{u,s,t}$

since $\gamma \neq \beta$, γ must act inside U or inside W $\lambda \in \rho$ $\lambda \in \rho$ $\lambda \in \rho$ $\lambda \in \rho$ 場合

If u acts inside U , we use a diamond from $U \square (s \square t)$ to $(u' \square s) \square t$, which commutes because d is natural

If γ is inside $w = s \square t$ and actually

inside s or inside t , naturality of d gives a similar diamond

gives a similar diamond

There remains only the case where γ is inside $s \square t$ but not inside s or t .

Then γ must be a n instance of d , after some application of d ,

γ must be a product $t = \rho \square \gamma$

and our diamond must then start with

$v = U \square W = U \square (s \square (\rho \square \gamma))$

$\downarrow \quad \downarrow \quad \downarrow$
 $(U \square s) \square (\rho \square \gamma) \quad \square \quad U \square (s \square \rho \square \gamma)$

pentagon

G_n : commutative in β as far as

associativity in concerned

$\lambda a : e \square a \cong a \quad \rho a : a \square e \cong a$

G_n' : vertices all words of length n including words in $v \square w$ edges all basic arrows.

G_n : infinite

contains G_n

For any word $W \quad W \xrightarrow{\text{path}} W^{(n)}$

composite arrow is equal to that for a different path which first removes

all e 's, then applies d

If e is removed by $\lambda : e \square b \cong b$

then that e can be removed before

either by naturality of d or directly

$a \square (e \square c) \xrightarrow{d} (a \square e) \square c$

$\downarrow \quad \downarrow$
 $a \square c = a \square c$

$e \square (b \square c) \xrightarrow{d} (e \square b) \square c$

$\downarrow \quad \downarrow$
 $b \square c = b \square c$

$$a \square (b \square e) \xrightarrow{\alpha} (a \square b) \square e$$

$$\downarrow \text{id} \quad \downarrow \rho$$

$$a \square b = a \square b$$

$$\lambda e = \rho e = e \square \rho \Rightarrow e$$

This reduced path composite equal to that for a canonical path in which all e 's are removed in some specified order (say starting with the left-most occurrence of e)

$$G^h \Rightarrow G_n$$

monoidal category

(191) $\langle \text{Set}, X, I \rangle$

$\langle \text{Top}, X, * \rangle$

$\langle \text{Ab}, \otimes, \mathbb{Z} \rangle$

$\langle \text{Cat}, X, II \rangle$

Category \mathcal{C}

objects a, b class

$\mathcal{C}(a, b)$ $\cong \mathcal{C}(b, a)$

$\mathcal{C}(b, c) \times \mathcal{C}(a, b) \rightarrow \mathcal{C}(a, c)$

$* \xrightarrow{\text{id}} \mathcal{C}(a, a)$

$\mathcal{C}(e, d) \times \mathcal{C}(b, c) \times \mathcal{C}(a, b) \rightarrow \mathcal{C}(c, d) \times \mathcal{C}(a, c)$

$\mathcal{C}(b, d) \times \mathcal{C}(a, b) \rightarrow \mathcal{C}(a, d)$

$\mathcal{C}(a, b) \xrightarrow{\text{id}} \mathcal{C}(b, b) \times \mathcal{C}(a, b) \rightarrow \mathcal{C}(a, b)$

$\langle \text{Set}, X, * \rangle$

$\langle V, X, * \rangle$ monoidal

enriched category theor

enriched over $\langle V, X, * \rangle$

monoidal category