2017年度 数理科学III

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第2回 数理科学ⅢA

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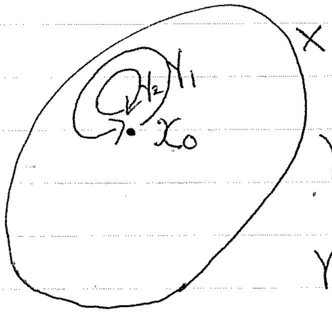
基本群 (fundamental groups)

位相幾何学 (topology)

位相空間 (topological space) ⇒ 代数
取り扱いやすい



$X \ni x_0$ を固定

loop (ループ)

$\gamma : I = [0,1] \longrightarrow X$ 連続

$\gamma(0) = \gamma(1) = x_0$

同値関係 (homotopy)

homotope

$\gamma_1 \sim \gamma_2 \iff \exists F : I \times I \to X$

such that $F(0,t) = \gamma_1(t)$ ($^\forall t \in I$)

$F(1,t) = \gamma_2(t)$ ($^\forall t \in I$)

$F(s,0) = F(s,1) = x_0$ ($^\forall s \in I$)

命題 $\sim$ は 同値関係 (equivalence relation)

proof) (i) $\gamma \sim \gamma$ $\quad F(s,t) = \gamma(t)$ $\quad F : I \times I \to X$

$F(0,t) = F(1,t) = \gamma(t)$

$F(s,0) = \gamma(0) = x_0$

$F(s,1) = \gamma(1) = x_0$

(ii) $\underline{\gamma_1 \sim \gamma_2} \Rightarrow \underline{\gamma_2 \sim \gamma_1}$

$G : I \times I \to X$

$G(s,t) = F(1-s,t)$ $\quad G(0,t) = F(1,t) = \gamma_2(t)$

$G(1,t) = F(0,t) = \gamma_1(t)$

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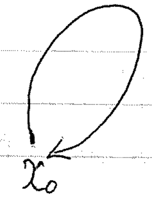
(iii) $\gamma_1 \underset{F}{\sim} \gamma_2 \quad \gamma_2 \underset{G}{\sim} \gamma_3 \Rightarrow \gamma_1 \underset{H}{\sim} \gamma_3$



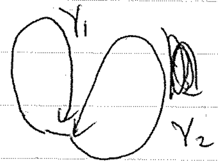
割る

基本群 $[\gamma]$

$\pi_1(X, x_0)$

基点

(base point)



$\gamma_1 \cdot \gamma_2 \qquad I \to X$

$$(\gamma_1 \cdot \gamma_2)(t) = \gamma_1(2t) \quad \left(0 \le t \le \frac{1}{2}\right)$$

$$(\gamma_1 \cdot \gamma_2)(t) = \gamma_2(2t-1) \quad \left(\frac{1}{2} \le t \le 1\right)$$

補題 (Lemma)

演算

$[\gamma_1] \cdot [\gamma_2] = [\gamma_1 \cdot \gamma_2]$

$$\left.\begin{array}{l} \gamma_1 \underset{F}{\sim} \gamma_1' \\ \gamma_2 \underset{G}{\sim} \gamma_2' \end{array}\right\} \Rightarrow \gamma_1 \cdot \gamma_2 \sim \gamma_1' \cdot \gamma_2'$$

証明

$$\gamma_1 \cdot \gamma_2 \underset{H}{\sim} \gamma_1' \cdot \gamma_2'$$

$$H(s,t) = F(s, 2t) \quad \left(0 \le t \le \frac{1}{2}\right)$$

$$H(s,t) = F(s, 2t-1) \quad \left(\frac{1}{2} \le t \le 1\right)$$

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定理 $\pi_1(X, x_0)$ は群 (group) Ⅱ

証明 $\underline{(\gamma_1 \cdot \gamma_2) \cdot \gamma_3} \sim \gamma_1 \cdot (\gamma_2 \cdot \gamma_3)$

$$
((\gamma_1 \cdot \gamma_2) \cdot \gamma_3)(t) = \begin{cases} \gamma_1(4t) & (0 \leq t \leq \frac{1}{4}) \\ \gamma_2(4t-1) & (\frac{1}{4} \leq t \leq \frac{1}{2}) \\ \gamma_3(2t-1) & (\frac{1}{2} \leq t \leq 1) \end{cases}
$$

結合律 (associativity) $\gamma \cdot \varepsilon_{x_0} \sim \gamma$

$\varepsilon_x \cdot \gamma \sim \gamma$

単位元 $\varepsilon_{x_0} : I \to X$ $\varepsilon_{x_0}(t) = x_0$

$(\gamma \cdot \varepsilon_{x_0})(t) = \gamma(2t)$ $(0 \leq t \leq \frac{1}{2})$

$(\gamma \cdot \varepsilon_{x_0})(t) = x_0$ $(\frac{1}{2} \leq t \leq 1)$

γ 逆元 $\gamma^{-1}(t) = \gamma(1-t)$

$\gamma \cdot \gamma^{-1} \sim \varepsilon_{x_0}$

$\gamma^{-1} \cdot \gamma \sim \varepsilon_{x_0}$

$f : (X, x_0) \longrightarrow (Y, y_0)$ $f : X \to Y$

$f(x_0) = y_0$

$\gamma \sim \gamma' \in \pi_1(X, x_0)$

$\longrightarrow f \circ \gamma$

$f_* : \pi_1(X, x_0) \to \pi_1(Y, y_0)$ 群の準同型

$(g \circ f)_* = g_* \circ f_*$

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$X = \{x_0\}$ $\quad \pi_1(X, x_0)$ つぶれる.

$A \subseteq X \qquad i : \underset{\in}{\overset{x_0}{}} A \to X$

$f : X \to A \qquad f \circ i = id_A$

AはXの retract(レトラクト)

$i_* : \pi_1(A, x_0) \to \pi_1(X, x_0)$

$f_* : \pi_1(X, x_0) \to \pi_1(A, x_0)$

$$\underset{全射}{f_*} \circ \underset{単射}{i_*} = (\underline{f \circ i})_* = id$$

$A \subseteq X \qquad i : A \to X$

$f, g : (X, x_0) \to (Y, y_0)$

homotope $\quad f \sim g$

$\exists F : I \times I \to Y$ homotopy

such that $F(0, x) = f(x)$
$F(1, x) = g(x)$

$F(s, x_0) = y_0$

$\Rightarrow f_* = g_*$

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$x_0 \in A \subseteq X$ $\qquad i : A \to X$

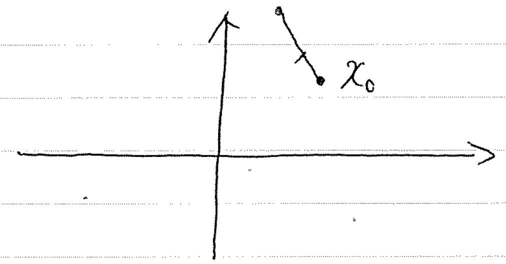
$f : X \to A$

$F \quad i \circ f \sim id_X \quad F(s,a) = a \quad ({}^\forall a \in A)$

Aは X の deformation retract

$\Rightarrow \pi_1(A, x_0) = \pi_1(X, x_0)$ $\qquad$ III

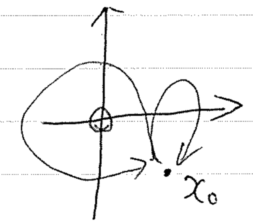
$x_0 \in \mathbb{R}^n$ $\qquad \{x_0\}$は $\mathbb{R}^n$ の deformation retract



$\pi_1(\mathbb{R}^n, x_0) =$ つぶれる.

$\mathbb{R}^n - \bigcirc$

$\mathbb{R}^2$ を



$\pi_1(\mathbb{R}^2 - \{(0,0)\}, x_0)$

$\mathbb{Z}$