2017年度 数理科学III

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No. 11/14(火) 第5回

Date

Theorem

$\forall$ monoidal category B $\forall b \in B$

$\exists!$ morphism $W \to B$ of monoidal categories with $(-) \mapsto b$

proof desired morphism $\boxed{w \mapsto w_b}$

(substitute b in all the blanks of the word w)

$(e_0)_b = e$

$(\_)_b = b$

$(v \square w)_b = v_b \square w_b$

For words of fixed length n

we construct a certain "basic" graph

$G_n = G_{n,b}$

vertices : all words w of length n which do not involve e_0

edges : certain arrows $v_b \to w_b$ in B called basic arrows

$\alpha : u_b \square (v_b \square w_b) \to (u_b \square v_b) \square w_b$ of associativity } basic

α^{-1}

$(1 \square \alpha) \square (1 \square 1)$: one instance of α boxed with identities

{ directed (involving α)
anti directed (involving α^{-1})

G_n path from u to w

composable sequences of basic arrows from u_b to w_b

$u_b \to w_b$ in B

(証明の要点)

Any two paths from u to w yield, by composition, the same arrow $u_b \to w_b$ in B

(the ~~great~~ graph G_n is a commutative diagram in B)

$w^{(n)}$: the unique word of length n which has all pairs of parentheses starting in front

((( )))

$\exists$ directed path in G_n from any word w to $w^{(n)}$ in a canonical way, successively moving outer most parentheses to the front by instances of α

$\forall v, w$: words of length n.

$v \to w^{(n)} \to w$

associativity
⇓
general associative law
$a(bc) = (ab)c$

rank ρ of a word w by recursion

$\rho e_0 = 0 \quad \rho(\_) = 0$

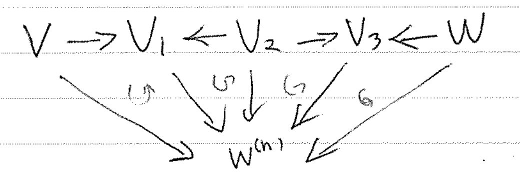
$\rho(v \square w) = \rho(v) + \rho(w) + \mathrm{length}(w) - 1$

$\rho w = 0 \Rightarrow$ all pairs of parentheses in w start at the front

G_n が commute することを示す.

any path $v \to w$

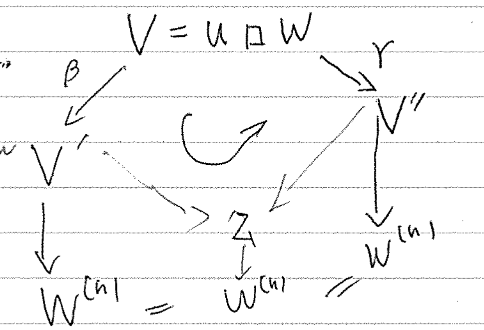
join each vertex to the "bottom" vertex $w^{(n)}$ by the canonical path



by Induction

suppose true for all v of smaller rank consider

two different directed paths starting at v with directed basic arrows β and γ



β, γ decrease the rank

by a case subdivision

$\beta = \gamma \quad z = v' = v'$

$\beta \neq \gamma \quad v = u \square w$

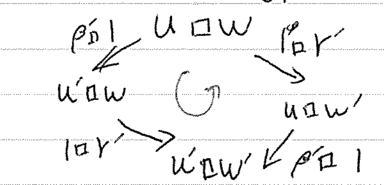
β is　　　γについても同様

$\beta = \beta' \square 1_w$: β acts inside the first factor u

$\beta = 1_u \square \beta''$: β 〃 w

$\beta = \alpha_{u,s,t}$ where $v = u \square w = u \square (s \square t)$

If both act inside the same factor u we can use inductor on the length n



☺ $\square$: bifunctor

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Date

the case when one of β or γ,

Say β is $\beta = \alpha = \alpha_{u,s,t}$. as in the third

case above $\gamma \neq \beta$ γ must act inside u or inside w

If γ acts inside u, we use a diamond

from $u \square (s \square t)$ to $(u' \square s) \square t$

which commute (何故なら α' is natural))