

2016年度 数理科学III

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type $A \rightarrow B$ A から B への関数の全体 $x:A$
 A, B

universe すべての集合が作る集合 \Rightarrow paradox

category theory small

$$A = U$$

$$U_0 = U_1 = U_2 = U_3$$

$$A = U_0 \Rightarrow A = U_1$$

階層的

U

dependent function type

$$A = \text{type}$$

$$B : A \rightarrow U$$

$$\prod_{x:A} B(x)$$

$$\prod_{x:A} B(x)$$

$$\prod (x:A), B(x)$$

$$B(x) = B$$

$$\Phi = B(x)$$

$$A \rightarrow B$$

$$f(x) \equiv \Phi \text{ for } x:A$$

$$\lambda x. \Phi = \prod_{x:A} B(x)$$

$$\text{Fin} = \mathbb{N} \rightarrow U$$

Values are the standard finite sets with elements

$$0_n, 1_n, \dots, (n-1)_n = \text{Fin}(n)$$

$$\prod_{n:\mathbb{N}} \text{Fin}(n+1)$$

$$f_{\max}(n) \equiv \kappa_{n+1}$$

f_{\max}

largest element

A, B

$A \times B$ 直積

dependent pair type

$$\sum_{x:A} B(x)$$

$$\sum_{x:A} B(x)$$

$$\sum (x:A), B(x)$$

$$\sum_{x:A} B \equiv A \times B$$

\mathbb{Z} the type of booleans

$0_{\mathbb{Z}}, 1_{\mathbb{Z}}$

natural number \mathbb{N} $0 = \mathbb{N}$ $\text{succ} : \mathbb{N} \rightarrow \mathbb{N}$

Formation $\frac{\Gamma \text{ context}}{\Gamma \vdash \mathbb{N} = U}$ \mathbb{N} formation

introduction
elimination
computation

$\frac{\Gamma \text{ context}}{\Gamma \vdash 0 = \mathbb{N}}$ \mathbb{N} - introduction₁

$\frac{\Gamma \vdash n = \mathbb{N}}{\Gamma \vdash \text{succ}(n) = \mathbb{N}}$ \mathbb{N} - introduction₂

$\Gamma, x = \mathbb{N} \vdash C = U$ $\Gamma \vdash C_0 = C[0/x]$ $\Gamma, x = \mathbb{N}, y = C \vdash C_s = C[\text{succ}(x)/x]$
 $\Gamma \vdash n = \mathbb{N}$

$\Gamma \vdash \text{ind}_{\mathbb{N}}(x = C, C_0, x, y, C_s, n) = C[n/x]$

$\Gamma, x = \mathbb{N} \vdash C = U$ $\Gamma \vdash C_0 = C[0/x]$ $\Gamma, x = \mathbb{N}, y = C \vdash C_s = C[\text{succ}(x)/x]$

$\Gamma \vdash \text{ind}_{\mathbb{N}}(x = C, C_0, x, y, C_s, 0) \equiv C_0 = C[0/x]$ \mathbb{N} - computation
 $\leftarrow C \text{ (if } x \text{) depend } C_0, C_s.$

$\Gamma, x = \mathbb{N} \vdash C = U$ $\Gamma \vdash C_0 = C[0/x]$ $\Gamma, x = \mathbb{N}, y = C \vdash C_s = C[\text{succ}(x)/x]$
 $\Gamma \vdash n = \mathbb{N}$

$\Gamma \vdash \text{ind}_{\mathbb{N}}(x = C, C_0, x, y, C_s, \text{succ}(n))$

$\equiv C_s[n, \text{ind}_{\mathbb{N}}(x = C, C_0, x, y, C_s, n) / x, y] = C[\text{succ}(n)/x]$

computation_s

dependent function types (Π -types)

$$\frac{\Gamma \vdash A = U \quad \Gamma, x:A \vdash B = U}{\Gamma \vdash \Pi_{x:A} B = U} \quad (\Pi\text{-formation})$$

$$\frac{\Gamma, x:A \vdash b = B}{\Gamma \vdash \lambda(x:A). b = \Pi_{x:A} B} \quad (\Pi\text{-introduction})$$

$$\frac{\Gamma \vdash f = \Pi_{x:A} B \quad \Gamma \vdash a = A}{\Gamma \vdash f(a) = B[a/x]} \quad (\Pi\text{-elimination})$$

$$\frac{\Gamma, x:A \vdash b = B, \quad \Gamma \vdash a = A}{\Gamma \vdash (\lambda(x:A). b)(a) \equiv b[a/x] = B[a/x]} \quad \Pi\text{ computation}$$

~~the~~ dependent pair types (Σ -types)

$$\frac{\Gamma \vdash A = U \quad \Gamma, x:A \vdash B = U}{\Gamma \vdash \Sigma_{x:A} B = U} \quad (\Sigma\text{-formation})$$

$$\frac{\Gamma, x:A \vdash B = U \quad \Gamma \vdash a = A \quad \Gamma \vdash b = B[a/x]}{\Gamma \vdash (a, b) = \Sigma_{x:A} B} \quad (\Sigma\text{-introduction})$$

$$\frac{\Gamma, z = \Sigma_{x:A} B \vdash C = U \quad \Gamma, x:A, y = B \vdash g = C[(x, y)/z]}{\Gamma \vdash p = \Sigma_{x:A} B} \quad (\Sigma\text{-elimination})$$

$$\Gamma \vdash \text{ind}_{\Sigma_{x:A} B}(z, c, x, \lambda y. g, y, p) = C[p/z] \quad \Sigma\text{-elimination}$$

\star double = $\mathbb{N} \rightarrow \mathbb{N}$ recursion

$$\text{double } (0) \equiv 0$$

$$\text{double } (\text{succ } (u)) \equiv \text{succ } (\text{succ } (\text{double } (u)))$$

$$\text{double}(2) \equiv \text{double}(\text{succ}(\text{succ}(0)))$$

$$\equiv C_s(\text{succ}(0), \text{double}(\text{succ}(0)))$$

$$\equiv \text{succ}(\text{succ}(\text{double}(\text{succ}(0))))$$

$$\equiv \text{succ}(\text{succ}(C_s(0, \text{double}(0))))$$

$$\equiv \text{succ}(\text{succ}(\text{succ}(\text{succ}(\text{double}(0)))))$$

$$\equiv \text{succ}(\text{succ}(\text{succ}(\text{succ}(0)))) = 4$$

addition

$$C_0: \mathbb{N} \rightarrow \mathbb{N}$$

$$C_0(n) \equiv n$$

$$C_s: \mathbb{N} \rightarrow (\mathbb{N} \rightarrow \mathbb{N}) \rightarrow (\mathbb{N} \rightarrow \mathbb{N})$$

$$C_s(m, g)(n) \equiv \text{succ}(g(n))$$

$$\mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$$

$$\text{add}: \mathbb{N} \rightarrow (\mathbb{N} \rightarrow \mathbb{N})$$

$$\text{add}(0, n) \equiv n$$

$$\text{add}(\text{succ}(m), n) \equiv \text{succ}(\text{add}(m, n))$$