

2016年度 数理科学III

著者	西村 泰一
著者別名	Nishimura Hirokazu
内容記述	数理科学IIIA (春学期) 数理科学IIIB (秋学期)
発行年	2016
その他のタイトル	Mathematical Science III
URL	http://hdl.handle.net/2241/00140875

第4回 数理科学ⅢB

10/25(火)

Weil 代数

R 代数

$$\mathbb{R}[x_1, \dots, x_n] / (x_1^{m_1}, \dots, x_n^{m_n}, f_1(x_1, \dots, x_n), \dots, f_r(x_1, \dots, x_n))$$

finitely presented \mathbb{R} -algebra $\mathbb{R}[x] / (x^4) \rightarrow \mathbb{R}$

$\text{Spec}_{\mathbb{R}} W$ infinitesimal space 無限小空間 $W = \text{Weil 代数}$

= W から \mathbb{R} への \mathbb{R} 代数としての準同型の全体

$$\text{Spec}_{\mathbb{R}} \mathbb{R}[x] / (x^2) = D$$

$$\text{Spec}_{\mathbb{R}} \mathbb{R}[x, y] / (x^2, y^2) = D^2$$

$$\text{Spec}_{\mathbb{R}} \mathbb{R}[x, y] / (x^2, y^2, xy) = D(2) := \{(d_1, d_2) \in D^2 \mid d_1 d_2 = 0\}$$

$W \rightarrow \mathbb{R}$ $\text{Spec}_{\mathbb{R}} W$ \mathbb{R} 代数 $\text{Spec}_{\mathbb{R}} W$ \mathbb{R} 代数

canonical homomorphism of \mathbb{R} -algebras

$\lambda_{x \in W} \lambda_{f \in \text{Spec}_{\mathbb{R}} W}$ 同型

-一般化された-
Kock - Lawvere の公理

Weil 代数

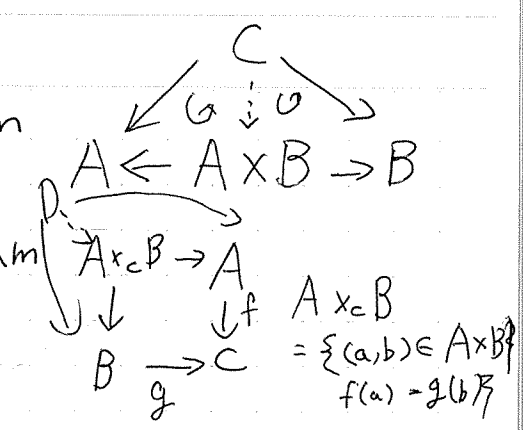
finite limit diagram

$\text{Spec}_{\mathbb{R}}$

quasi-colimit diagram

\mathbb{R}^0

反変
contravariant



$$\{0\} = 1 \rightarrow D$$

$$\downarrow \quad \downarrow i_2$$

$$D \xrightarrow{i_1} D(2)$$

$$i_1: d \in D \mapsto (d, 0)$$

$$i_2: d \in D \mapsto (0, d)$$

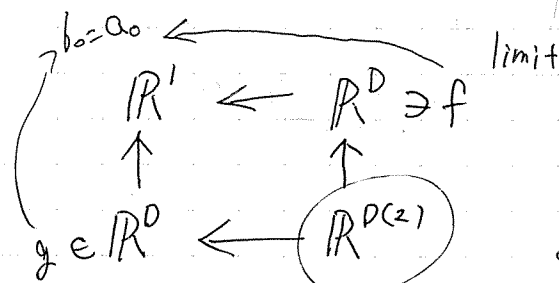
$$W = \mathbb{R} \cong \mathbb{R}[x] / (x)$$

$$\text{Spec}_{\mathbb{R}} W = 0$$

$$D = \{d \in \mathbb{R} \mid d^2 = 0\}$$

quasi-colimit diagram

$$D = \text{Spec}_{\mathbb{R}} \mathbb{R}[x] / (x^2)$$



$$f(d) = a_0 + a_1 d$$

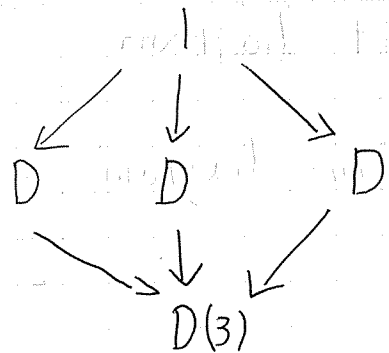
$$g(d) = b_0 + b_1 d$$

$$c_1 = b_1$$

$$c_2 = a_1$$

$$h(d_1, d_2) = \underbrace{c_0 + c_1 d_1 + c_2 d_2}_{c_0 + c_1 d}$$

$a_0 = b_0$ のこと等しい.

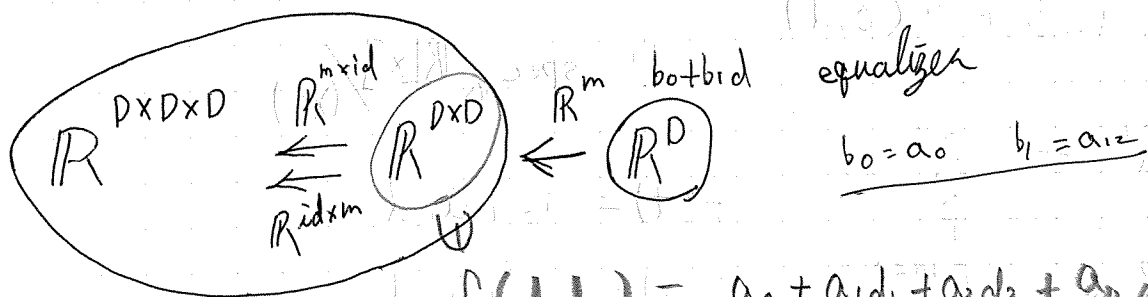


$$D(3) = \{ (d_1, d_2, d_3) \in D^3 \mid d_1 d_2 = d_1 d_3 = d_2 d_3 = 0 \}$$

$$D \times D \times D \xrightarrow[\text{id} \times m]{m \times \text{id}} D \times D \xrightarrow{m} D \quad \text{multiplication}$$

$$m: (d_1, d_2) \in D \times D \mapsto d_1 d_2 \in D$$

$$\begin{array}{l} m \times \text{id} \\ \text{id} \times m \end{array} (d_1, d_2, d_3) \mapsto (d_1 d_2, d_3) \mapsto (d_1, d_2 d_3)$$



$$f(d_1, d_2) = a_0 + a_1 d_1 + a_2 d_2 + a_{12} d_1 d_2$$

$$\leftarrow a_0 + a_1 d_1 d_2 + a_2 d_3 + a_{12} d_1 d_2 d_3$$

$$\leftarrow a_0 + a_1 d_1 + a_2 d_2 d_3 + a_{12} d_1 d_2 d_3$$

$$\Downarrow \\ a_1 = a_2 = 0$$

宿

$$(I) D \xrightarrow[\underline{0}]{\begin{array}{l} \bar{i}_1 \\ \bar{i}_2 \end{array}} D \times D \xrightarrow{m} D$$

$$\bar{i}_1(d) = (d, 0)$$

$$\bar{i}_2(d) = (0, d)$$

$$0(d) = (0, 0)$$

$$(II) D \xrightarrow[\underline{0}]{\bar{i}} D_2 \xrightarrow{q} D \quad \text{where } D_2 \cong \mathbb{R}[x]/(x^3)$$

\bar{i} = zero map

$0 = 0$ 对应

$$q(d) = d^2$$

$$(III) D \times D \xrightarrow[\underline{V}]{\text{id}} D \times D \xrightarrow{+} D_2$$

$$(d_1, d_2) \mapsto d_1 + d_2$$

$$V(d_1, d_2) = (d_2, d_1)$$