

## 2016年度 数理科学III

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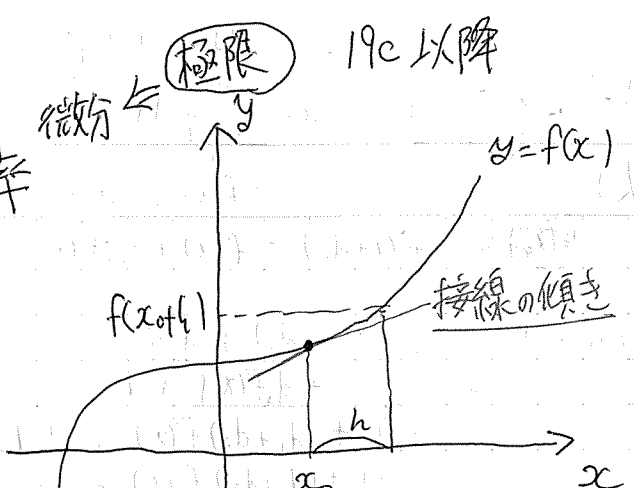
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# 第1回

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$$

平均の変化率

ε-δ



17C Newton, Leibniz

18C Euler, Lagrange

中零無限小  
hが十分小さい時  
 $h^2 = 0 \Rightarrow h = 0$

$$f(x_0+h) - f(x_0) = f'(x_0)h$$

$$f(x_0+h) = f(x_0) + f'(x_0)h$$

$$D = \{d \in \mathbb{R} \mid d^2 = 0\} \neq \{0\}$$

$$f(x+d) = f(x) + a d \quad (\forall d \in D)$$

↑  
∃! a / linear  
f(x)に書く

$$(f+g)' = f' + g'$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) + g(x+h) - \{f(x) + g(x)\}}{h} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$$

$$\begin{aligned} f(x+d) + g(x+h) &= f(x) + f'(x)d + g(x) + g'(x)d \\ &= f(x) + g(x) + d \{f'(x) + g'(x)\} \end{aligned}$$

$$(fg)' = f'g + fg' \quad (\text{Leibniz})$$

$$\begin{aligned} &\lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h} \end{aligned}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} g(x+h) + \lim_{h \rightarrow 0} \frac{f(x) (g(x+h) - g(x))}{h} \\ &= \lim_{h \rightarrow 0} g(x+h) \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + f(x) \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\ &= g(x) f'(x) + f(x) g'(x) \end{aligned}$$

展開

$$\begin{aligned} f(x+d)g(x+d) &= \{f(x) + f'(x)d\} \{g(x) + g'(x)d\} \\ &= f(x)g(x) + d \{f'(x)g(x) + f(x)g'(x)\} \\ &\quad + \underbrace{(d^2)}_0 f'(x)g'(x) \end{aligned}$$

定値

$$f(x) = C$$

$$f'(x) = 0$$

$$f'(x) = 1$$

$$f(x+d) - f(x) = C - C = 0 = 0d$$

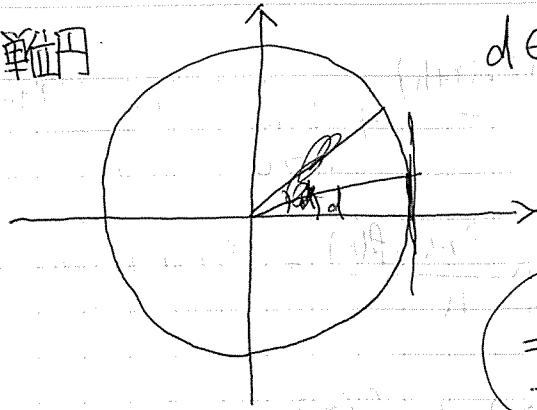
$$f(x) = x \quad f(x+d) - f(x) = x+d - x = d = 1d$$

$$\begin{aligned} f(x) = x^n \quad f(x+d) - f(x) &= (x+d)^n - x^n \\ &= \underbrace{n x^{n-1}}_{2\text{項定理}} d + \dots \\ &= n x^{n-1} \end{aligned}$$

三角関数

単位円

ラジアン

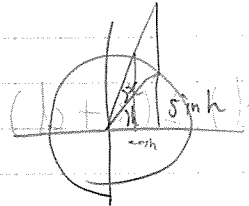


$d \in D$

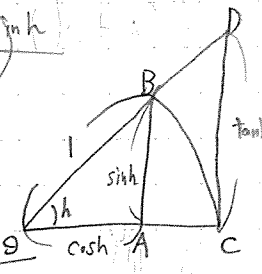
$d \rightarrow 0$  のとき  $\sin d \approx d$

$\sin d = d$   
 $\cos d = 1$

$\sin^2 d + \cos^2 d = d^2 + 1 = 1$



$\lim_{h \rightarrow 0} \frac{\sinh}{h} = 1$

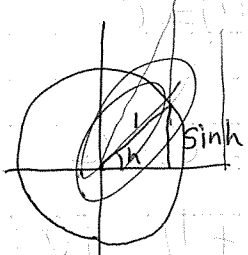


$\sin(x+d) = \sin x \cos d + \cos x \sin d = \sin x + d \cos x$

$\cos(x+d) = \cos x \cos d - \sin x \sin d = \cos x - d \sin x$

指数関数  $(e^x) = e^x$

$e = \lim_{h \rightarrow 0} (1 + \frac{1}{h})^h$



$|OA| = \cosh$   
 $|AB| = \sinh$   
 $|CD| = \tanh$   
 $|\Delta OAB| = \frac{1}{2} \cosh \sinh$   
 $|\Delta OCD| = \frac{1}{2} \tanh$

$\frac{1}{2} \cosh \sinh < |\Delta OCB| < \frac{1}{2} \tanh$

$f(x) = 10^x$

$g(x) = e^x$   
 $e = 10^{\log_{10} e}$

$f(d) = 1 + a d$

$g(x) = (10^{\log_{10} e})^x = 10^{x \log_{10} e}$

$\cosh \sinh < h < \frac{\sinh}{\cosh}$   
 $\Rightarrow \cosh < \frac{\sinh}{h} < \frac{1}{\cosh}$   
 $\lim_{h \rightarrow 0} \frac{\sinh}{h} = 1$

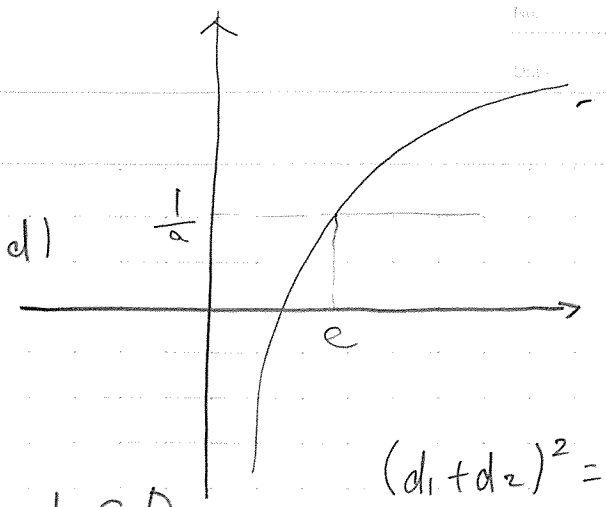
$g(d) = 10^d = 10^{d(\log_{10} e)}$

$= 1 + a d (\log_{10} e)$

$= 1 + d a (\log_{10} e)$   
 $\frac{1}{a}$

$g'(0) = 1$

$e^{x+d} = e^x e^d = e^x (1+d) = e^x + d e^x$



$(d_1 + d_2)^2 = d_1^2 + d_2^2 + 2d_1 d_2 = 2d_1 d_2$

$d_1, d_2 \in D \Rightarrow d_1 + d_2 \in D$

$f(x+d_1) = f(x) + f'(x) d_1$

$f(x+d_1+d_2) = f(x+d_1) + f'(x+d_1) d_2$

$= f(x) + f'(x) d_1 + \{ f'(x) + f''(x) d_1 \} d_2$

$= f(x) + f'(x) (d_1 + d_2) + f''(x) d_1 d_2$

$= f(x) + f'(x) (d_1 + d_2) + f''(x) \frac{(d_1 + d_2)^2}{2}$

$f(x+d_1+d_2+d_3) = f(x+d_1+d_2) + f'(x+d_1+d_2) d_3$

$= \{ f(x) + f'(x) (d_1 + d_2) + f''(x) d_1 d_2 \} + \{ f'(x) + f''(x) (d_1 + d_2) + f'''(x) d_1 d_2 \} d_3$

$= f(x) + f'(x) (d_1 + d_2 + d_3) + f''(x) (d_1 d_2 + d_1 d_3 + d_2 d_3)$

$= f(x) + f'(x) (d_1 + d_2 + d_3) + f''(x) \frac{(d_1 + d_2 + d_3)^2}{2} + f'''(x) \frac{(d_1 + d_2 + d_3)^3}{3!}$

宿

と5の場合を解け!