

2016年度 数理科学III

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第2回

数理科学ⅢB

10/1 (火)

Synthetic differential geometry

総合的

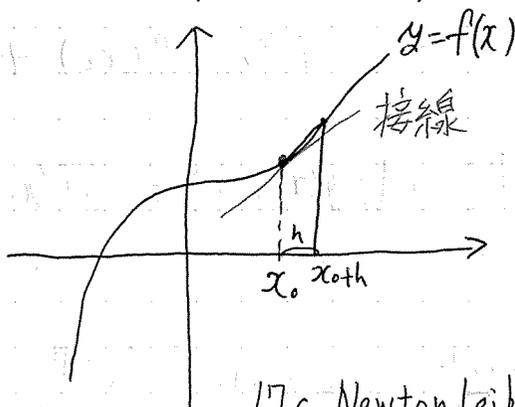
with in homotopy type theory

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$$

平均の
変化率

極限 \Rightarrow 微分

19c ϵ - δ



17c Newton Leibniz

18c Euler, Lagrange

7-77

曲がっている

$$h^2 = 0 \Rightarrow \frac{\text{十分小さい}}{h=0}$$

$$D = \{ d \in \mathbb{R} \mid d^2 = 0 \}$$

$\exists! a \in \mathbb{R}$

$$f(x_0+d) - f(x_0) = \textcircled{a}d \quad (\forall d \in D)$$

$$f(x_0+d) = f(x_0) + \textcircled{a}d$$

$\underbrace{\textcircled{a}}_{f'(x_0)}$

$$(f+g)' = f' + g'$$

$$\lim_{h \rightarrow 0} \frac{f(x_0+h) + g(x_0+h) - \{f(x_0) + g(x_0)\}}{h} = \left(\lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} \right) + \left(\lim_{h \rightarrow 0} \frac{g(x_0+h) - g(x_0)}{h} \right)$$

$$\begin{aligned} f(x_0+d) + g(x_0+d) &= f(x_0) + f'(x_0)d + g(x_0) + g'(x_0)d \\ &= f(x_0) + g(x_0) + \{f'(x_0) + g'(x_0)\}d \end{aligned}$$

$$(fg)' = f'g + fg'$$

$$\lim_{h \rightarrow 0} \frac{f(x_0+h)g(x_0+h) - f(x_0)g(x_0)}{h} = \frac{f(x_0+h)g(x_0+h) - f(x_0)g(x_0+h) + f(x_0)g(x_0+h) - f(x_0)g(x_0)}{h}$$

Leibniz の発見 中零無限小

$$= \lim_{h \rightarrow 0} \frac{\{f(x_0+h) - f(x_0)\}g(x_0+h) + f(x_0)\{g(x_0+h) - g(x_0)\}}{h}$$

$$= f'(x_0)g(x_0) + f(x_0)g'(x_0)$$

$$f(x_0+d)g(x_0+d) = \{f(x_0) + f'(x_0)d\}\{g(x_0) + g'(x_0)d\}$$

$$= f(x_0)g(x_0) + \{f'(x_0)g(x_0) + f(x_0)g'(x_0)\}d + f'(x_0)g'(x_0)d^2$$

Kock - Lawvere の公理 \rightarrow topos
 $\psi: D \rightarrow \mathbb{R}$
 $\exists! \alpha \in \mathbb{R}$

$$\forall d \in D \quad \psi(d) = \psi(0) + \alpha d$$

微分可能な関数

$f: \mathbb{R} \rightarrow \mathbb{R}$ 固定 $x_0 \in \mathbb{R}$

$$\psi(d) = f(x_0+d) \quad \psi(0) = f(x_0)$$

$$f(x_0+d) = \psi(d) = f(x_0) + \alpha d$$

$\mathbb{R}[x]$ \rightarrow \mathbb{R}
 多項式環 \rightarrow \mathbb{R} 代数

\mathbb{R} 代数としての準同型

$$\text{Spec}_{\mathbb{R}}(\mathbb{R}[x]) = \mathbb{R} \rightarrow \mathbb{R} \text{ 同視可能}$$

$$\mathbb{R}[x]/(x^2) \rightarrow \mathbb{R}$$

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$$

$$\text{Spec}_{\mathbb{R}}(\mathbb{R}[x]/x^2) = D$$

$$D_1 = D = \{d \in \mathbb{R} \mid d^2 = 0\}$$

$$\left. \begin{array}{l} d \in D \\ d \in \mathbb{R} \end{array} \right\} \Rightarrow (dd)^2 = d^2(d^2) = 0$$

$$dd \in D$$

$$d_1, d_2 \in D \Rightarrow (d_1 + d_2)^2 = \overset{=0}{d_1^2} + 2d_1d_2 + \overset{=0}{d_2^2} = 2d_1d_2$$

$$d_1 + d_2 \notin D$$

$$(d_1 + d_2)^3 = \underbrace{d_1^3}_0 + 3 \underbrace{d_1^2 d_2}_0 + 3 \underbrace{d_1 d_2^2}_0 + \underbrace{d_2^3}_0 = 0$$

$$D_n = \{d \in \mathbb{R} \mid d^{n+1} = 0\}$$

$D_1 =$ 1次の無限小

$$d_1, d_2 \in D_1 \Rightarrow d_1 + d_2 \in D_2$$

$D_2 =$ 2次の無限小

Lemma $d_m \in D_m, d_n \in D_n \Rightarrow d_m + d_n \in D_{m+n}$ を示せ

$$\left. \begin{array}{l} d_m \in D_m \\ d_n \in D_n \end{array} \right\} \Rightarrow \begin{array}{l} d_m^{m+1} = 0 \\ d_n^{n+1} = 0 \end{array}$$

$$(d_m + d_n)^{m+n+1} = \sum_{k=0}^{m+n+1} \binom{m+n+1}{k} (d_m)^k (d_n)^{m+n+1-k}$$

$$= d_m^{m+1} + \binom{m+n+1}{1} d_m^m d_n + \dots + d_n^{m+n+1} = 0$$

$\therefore d_m + d_n \in D_{m+n}$
 (i), (ii) の条件を同時に満たすことは出来ない

Corollary

$$\{0\} \cup \mathbb{R} = \mathbb{R} = \mathbb{D}$$

$$d_1, \dots, d_n \in D = \mathbb{D} \Rightarrow d_1 + d_2 + \dots + d_n \in D_n$$

証明 by induction on n

$$d_1 + \dots + d_{n-1} \in D_{n-1}, d_n \in D = \mathbb{D}$$

(復習)

$$f(x_0 + d_1) = f(x_0) + f'(x_0) d_1$$

$$\begin{aligned} f(x_0 + d_1 + d_2) &= f(x_0 + d_1) + f'(x_0 + d_1) d_2 \\ &= (f(x_0) + f'(x_0) d_1) + \{ f'(x_0) + f''(x_0) d_1 \} d_2 \\ &= f(x_0) + f'(x_0) (d_1 + d_2) + f''(x_0) d_1 d_2 \\ &= f(x_0) + f'(x_0) (d_1 + d_2) + \frac{f''(x_0)}{2} (d_1 + d_2)^2 \end{aligned}$$

$$\begin{aligned} f(x_0 + d_1 + d_2 + d_3) &= f(x_0 + d_1 + d_2) + f'(x_0 + d_1 + d_2) d_3 \\ &= f(x_0) + f'(x_0) (d_1 + d_2) + \frac{f''(x_0)}{2} (d_1 + d_2)^2 \\ &\quad + \{ f'(x_0) + f''(x_0) (d_1 + d_2) + \frac{f'''(x_0)}{2} d_1 d_2 \} d_3 \\ &= f(x_0) + f'(x_0) (d_1 + d_2 + d_3) + f''(x_0) (d_1 d_2 + d_1 d_3 + d_2 d_3) \\ &\quad + f'''(x_0) d_1 d_2 d_3 \end{aligned}$$