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Charm quark system at the physical point of 2 + 1 flavor lattice QCD

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We investigate the charm quark system using the relativistic heavy quark action on 2 + 1 flavor PACS-CS configurations previously generated on $32^3 \times 64$ lattice. The dynamical up, down, and strange quark masses are set to the physical values by using the technique of reweighting to shift the quark-hopping parameters from the values employed in the configuration generation. At the physical point, the lattice spacing equals $a^{-1} = 2.194(10)$ GeV and the spatial extent $L = 2.88(1)$ fm. The charm quark mass is determined by the spin-averaged mass of the 1S charmonium state, from which we obtain $m_{\rm charm}^{\overline{\rm MS}}(\mu = m_{\rm charm}^{\overline{\rm MS}}) = 1.260(1)(6)(35)$ GeV, where the errors are due to our statistics, scale determination and renormalization factor. An additional systematic error from the heavy quark is of order $\alpha_s^2 f(m_Q a)(a\Lambda_{\rm QCD})$, $f(m_Q a)(a\Lambda_{\rm QCD})^2$, which are estimated to be a percent level if the factor $f(m_Q a)$ analytic in $m_Q a$ is of order unity. Our results for the charmed and charmed-strange meson decay constants are $f_D = 226(6)(1)(5)$ MeV, $f_{D_s} = 257(2)(1)(5)$ MeV, again up to the heavy quark errors of order $\alpha_s^2 f(m_Q a)(a\Lambda_{\rm QCD})$, $f(m_Q a)(a\Lambda_{\rm QCD})^2$. Combined with the CLEO values for the leptonic decay widths, these values yield $|V_{cd}| = 0.205(6)(1)(5)(9)$, $|V_{cs}| = 1.00(1)(1)(3)(3)$, where the last error is because of the experimental uncertainty of the decay widths.

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I. INTRODUCTION

Precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix is an indispensable step to establish the validity range of the standard model, and to search for new physics at higher energy scales. Lattice QCD has been making steady progress in this direction. For the matrix elements such as $|V_{ud}|$ and $|V_{us}|$ in the first row which involve only light quarks, dynamical simulations including up, down, and strange quarks have reached the point where the relevant pseudoscalar meson decay constants and form factors are being determined at subpercent precision. On the other hand, for $|V_{cd}|$ in the second row, the precision of the lattice QCD calculation for the decay constants and form factors is still at 5% to 10% level. This is not clearly superior to nonlattice QCD determinations. Indeed, the estimate quoted in Particle Data Group (PDG) 2010, $|V_{cd}| = 0.230(11)$ [1] with an accuracy of 5%, is obtained from neutrino and antineutrino experiments.[1] Much effort is needed on the part of lattice QCD toward a better precision in the charm sector.

One of the difficulties with the charm quark in lattice QCD simulations at a typical cutoff $a^{-1} \approx 2$ GeV resides in significant cutoff errors due to the charm quark mass. The heavy quark mass correction is $m_Q a \sim O(1)$, and hence it is desirable to control $m_Q a$ errors to achieve an accuracy of a few percent. The Fermilab action [2] and the relativistic heavy quark action [3,4] have been proposed to meet this goal. The relativistic heavy quark action removes the leading cutoff errors of $O((m_Q a)^n)$ and the next to leading effects of $O((m_Q a)^n (a\Lambda_{\rm QCD}))$ with arbitrary order n, once all of the parameters in the heavy quark action are determined nonperturbatively. In the present work, we employ the relativistic formalism of Ref. [3] to explore the charm quark system.

Another source that prevents precise evaluations in lattice QCD is the error associated with chiral extrapolations in the light quark masses. This problem has been increasingly alleviated through progress toward simulations with lighter and lighter dynamical quark masses and sophisticated application of chiral perturbation theory techniques. The acceleration of dynamical lattice QCD simulation using multitime steps for infrared and ultraviolet modes [5,6] has made it possible to run simulations with light up, down, and strange quark masses around their physical values [7]. In such simulations, uncertainties due to chiral extrapolations are drastically reduced.

[1]$|V_{cs}|$ is hard to be estimated from neutrino and antineutrino experiments, $|V_{cs}| = 0.94^{+0.32}_{-0.26} \pm 0.13$ [1]. Lattice QCD results are needed for precise determination of $|V_{cs}|$.

In fact, we can proceed one more step and reweight [8] dynamical simulations such that dynamical quark masses take exactly the physical values. A potential difficulty with dynamical lattice QCD is a large fluctuation of quark determinant ratios necessary for reweighting. We have demonstrated the feasibility of this procedure in Ref. [9] by reweighting a set of PACS-CS configurations with $m_\pi = 152(6)$ MeV and $m_K = 509(2)$ MeV to those with $m_\pi = 135(6)$ MeV and $m_K = 498(2)$ MeV. Once the reweighting is successfully made, ambiguities associated with chiral extrapolations are completely removed. In the present work, we employ the reweighting factors and the set of original dynamical configurations employed in Ref. [9]. Hence, our light quark masses sit at the physical point.

In this paper, we present our work for the charm quark system treated with the relativistic heavy quark formalism [3] on the 2 + 1 dynamical flavor PACS-CS configurations of $32^3 \times 64$ lattice generated with the Wilson-clover quark and reweighted to the physical point for up, down, and strange quark masses. The lattice spacing is estimated as $a^{-1} = 2.194(10)$ GeV. We measure the masses and decay constants of charmonia, charmed mesons and charmed-strange mesons. We then calculate the charm quark mass and the CKM matrix elements.

This paper is organized as follows. Section II explains our method and simulation parameters. Section III describes our results for the charmonium spectrum and the charm quark mass. In Sec. IV, we show our charmed meson and charmed-strange meson spectrum. Section V is devoted to present our pseudoscalar decay constants and the CKM matrix elements. Our conclusions are given in Sec. VI.

II. SET UP

Our calculation is based on a set of $N_f = 2 + 1$ flavor dynamical lattice QCD configurations generated by the PACS-CS Collaboration [9] on a $32^3 \times 64$ lattice using the nonperturbatively $O(a)$-improved Wilson quark action with $c_{\rm SW}^{\rm NP} = 1.715$ [10] and the Iwasaki gauge action [11] at $\beta = 1.90$. The aggregate of 2000 MD time units were generated at the hopping parameter given by $(\kappa_{ud}^0, \kappa_s^0) = (0.137\,785\,00, 0.136\,600\,00)$, and 80 configurations at every 25 MD time units were used for measurements. We then reweight those configurations to the physical point given by $(\kappa_{ud}, \kappa_s) = (0.137\,796\,25, 0.136\,633\,75)$. The reweighting shifts the masses of π and K mesons from $m_\pi = 152(6)$ MeV and $m_K = 509(2)$ MeV to $m_\pi = 135(6)$ MeV and $m_K = 498(2)$ MeV, with the cutoff at the physical point estimated to be $a^{-1} = 2.194(10)$ GeV.

Observables at the physical point are evaluated through the formula

$$\langle \mathcal{O}[U](\kappa_{\rm ud}, \kappa_s)\rangle_{(\kappa_{\rm ud}, \kappa_s)} = \frac{\langle \mathcal{O}[U](\kappa_{\rm ud}, \kappa_s) R_{\rm ud}[U] R_s[U]\rangle_{(\kappa_{\rm ud}^0, \kappa_s^0)}}{\langle R_{\rm ud}[U] R_s[U]\rangle_{(\kappa_{\rm ud}^0, \kappa_s^0)}}, \tag{2.1}$$

TABLE I. Simulation parameters. MD time is the number of trajectories multiplied by the trajectory length.

β	$\kappa_{\rm ud}$	κ_s	# conf	MD time
1.90	0.137 796 25	0.136 633 75	80	2000

where the reweighting factors are defined as

$$R_{\rm ud}[U] = \left|\det\left[\frac{D_{\kappa_{\rm ud}}[U]}{D_{\kappa_{\rm ud}^0}[U]}\right]\right|^2, \tag{2.2}$$

$$R_s[U] = \det\left[\frac{D_{\kappa_s}[U]}{D_{\kappa_s^0[U]}}\right], \tag{2.3}$$

and $D_{\kappa_q}[U]$ is the Wilson-clover quark operator with the hopping parameter κ_q. We refer to Ref. [9] for details of our evaluation of the determinant ratio. Our parameters and statistics at the physical point are collected in Table I.

The relativistic heavy quark formalism [3] is designed to reduce cutoff errors of $O((m_Q a)^n)$ with arbitrary order n to $O(f(m_Q a)(a\Lambda_{\rm QCD})^2)$, once all of the parameters in the relativistic heavy quark action are determined nonperturbatively, where $f(m_Q a)$ is an analytic function around the massless point $m_Q a = 0$. The action is given by

$$S_Q = \sum_{x,y} \bar{Q}_x D_{x,y} Q_y, \tag{2.4}$$

$$\begin{aligned} D_{x,y} = \delta_{xy} - \kappa_Q & \\ \times \sum_i & [(r_s - \nu\gamma_i) U_{x,i} \delta_{x+\hat{i},y} + (r_s + \nu\gamma_i) U_{x,i}^\dagger \delta_{x,y+\hat{i}}] \\ & - \kappa_Q [(r_t - \gamma_4) U_{x,4} \delta_{x+\hat{4},y} + (r_t + \gamma_4) U_{x,4}^\dagger \delta_{x,y+\hat{4}}] \\ & - \kappa_Q \left[c_B \sum_{i,j} F_{ij}(x) \sigma_{ij} + c_E \sum_i F_{i4}(x) \sigma_{i4} \right] \delta_{xy}, \end{aligned} \tag{2.5}$$

where κ_Q is the hopping parameter for the heavy quark. The parameters r_t, r_s, c_B, c_E, and ν are adjusted as follows. We are allowed to choose $r_t = 1$, and we employ a one-loop perturbative value for r_s [12]. For the clover coefficients c_B and c_E, we include the nonperturbative contribution in the massless limit $c_{\rm SW}^{\rm NP}$ for three-flavor dynamical QCD [10], and calculate the heavy quark mass-dependent contribution to one-loop order in perturbation theory [12] according to

$$c_{B,E} = (c_{B,E}(m_Q a) - c_{B,E}(0))^{\rm PT} + c_{\rm SW}^{\rm NP}. \tag{2.6}$$

The parameter ν is determined nonperturbatively to reproduce the relativistic dispersion relation for the spin-averaged $1S$ states of the charmonium. Writing

$$E(\vec{p})^2 = E(\vec{0})^2 + c_{\rm eff}^2 |\vec{p}|^2, \tag{2.7}$$

for $|\vec{p}| = 0, (2\pi/L), \sqrt{2}(2\pi/L)$, and demanding the effective speed of light $c_{\rm eff}$ to be unity, we find $\nu = 1.1450511$,

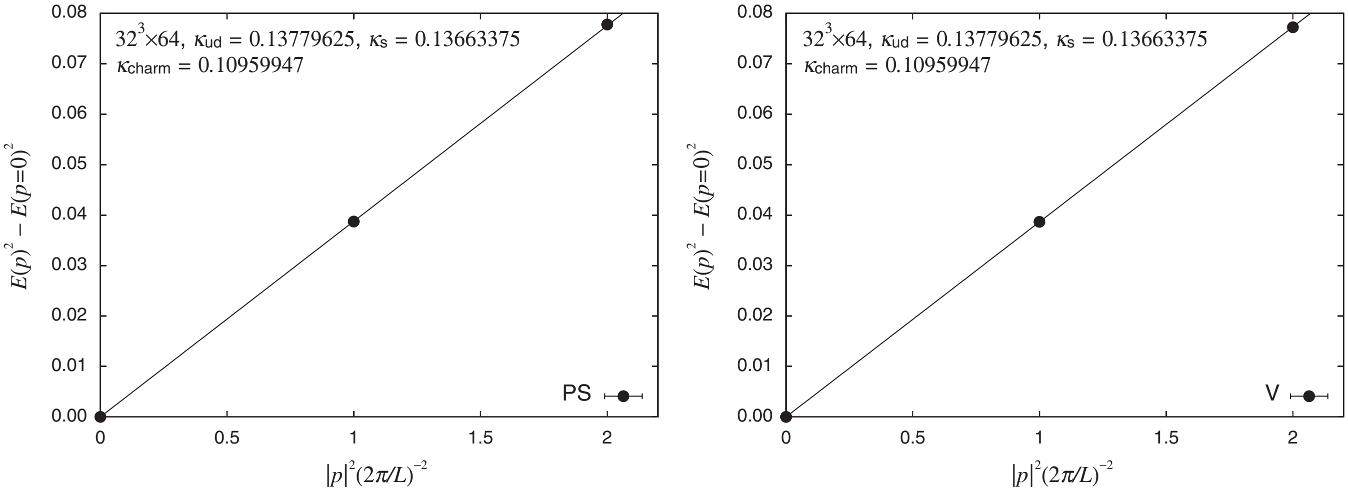

FIG. 1. Dispersion relations for $1S$ states of charmonium in pseudoscalar (left panel) and vector channel (right panel).

with which we have $c_{\rm eff} = 1.002(4)$. Our dispersion relation is shown in Fig. 1 and Table. II. It is noted that the cutoff error of $\alpha_s^2 f(m_Q a)(a\Lambda_{\rm QCD})$ remains, in addition to $f(m_Q a)(a\Lambda_{\rm QCD})^2$, due to the use of one-loop perturbative values in part for the parameters of our heavy quark action.

We tune the heavy quark hopping parameter to reproduce an experimental value of the mass for the spin-averaged $1S$ states of the charmonium, given by

$$M(1S)^{\rm exp} = (M_{\eta_c} + 3M_{J/\psi})/4 = 3.0678(3)\ {\rm GeV}[1]. \tag{2.8}$$

Since the lattice spacing is given by Ω baryon mass, we match $m(1S)^{\rm lat}a = m(1S)^{\rm lat}/m_\Omega^{\rm lat}$ to $M(1S)^{\rm exp}/M_\Omega^{\rm exp}$ for determination of the hopping parameter of the charm quark, $\kappa_{\rm charm}$. $m(1S)^{\rm lat}$ and $m_\Omega^{\rm lat}$ are measured on the same configuration set. This leads to $\kappa_{\rm charm} = 0.109\,599\,47$ for which our lattice QCD measurement yields the value $m(1S)^{\rm lat} = 3.067(1)(14)$ GeV, where the first error is statistical from $m(1S)^{\rm lat}$, and the second is from $m_\Omega^{\rm lat}$. There is no systematic error which requires an additional calculation of observables at different κ shifted from $\kappa_{\rm charm}$ by the error. Our parameters for the relativistic heavy quark action are summarized in Table III.

We use the following standard operators to obtain meson masses,

$$M_\Gamma^{fg}(x) = \bar{q}_f(x)\Gamma q_g(x), \tag{2.9}$$

TABLE II. Dispersion relations for the $1S$ states of charmonium.

	$p = (2\pi/L)$	$p = \sqrt{2}(2\pi/L)$
$E(p)^2 - E(p=0)^2$(PS)	0.0387(3)	0.0778(7)
$E(p)^2 - E(p=0)^2(V)$	0.0387(3)	0.0772(8)

where f, g are quark flavors and $\Gamma = I$, γ_5, γ_μ, $i\gamma_\mu\gamma_5$, $i[\gamma_\mu, \gamma_\nu]/2$. The meson correlators are calculated with a point and exponentially smeared sources and a local sink. The smearing function is given by $\Psi(r) = A\exp(-Br)$ at $r \neq 0$ and $\Psi(0) = 1$. We set $A = 1.2$, $B = 0.07$ for the ud quark, $A = 1.2$, $B = 0.18$ for the strange quark, and $A = 1.2$, $B = 0.55$ for the charm quark. The number of source points is quadrupled and polarization states are averaged to reduce statistical fluctuations. Statistical errors are analyzed by the jackknife method with a bin size of 100 MD time units (4 configurations), as in the light quark sector [9].

We extract meson masses by fitting correlators with a hyperbolic cosine function. For charmonium, Fig. 2 shows effective masses, from which we choose the fitting range to be $[t_{\rm min}, t_{\rm max}] = [10, 32]$. Similarly, Figs. 3 and 4 represent effective masses for charmed mesons and charmed-strange mesons. We employ the fitting range $[t_{\rm min}, t_{\rm max}] = [14, 20]$ for pseudoscalar mesons, and $[t_{\rm min}, t_{\rm max}] = [10, 20]$ for the other channels.

We calculate the decay constant $f_{\rm PS}$ of the heavy-light pseudoscalar meson using the improved axial vector current $A_4^{\rm imp}$.

$$i f_{\rm PS} p_\mu = \langle 0|A_\mu^{\rm imp}|{\rm PS}(p)\rangle, \tag{2.10}$$

$$A_4^{\rm imp} = \sqrt{2\kappa_q}\sqrt{2\kappa_Q}Z_{A_4}\{\bar{q}(x)\gamma_4\gamma_5 Q(x) + c_{A_4}^+\partial_4^+[\bar{q}(x)\gamma_5 Q(x)] + c_{A_4}^-\partial_4^-[\bar{q}(x)\gamma_5 Q(x)]\}, \tag{2.11}$$

TABLE III. Parameters for the relativistic heavy quark action.

$\kappa_{\rm charm}$	ν	r_s	c_B	c_E
0.10959947	1.145 051 1	1.188 160 7	1.984 913 9	1.781 951 2

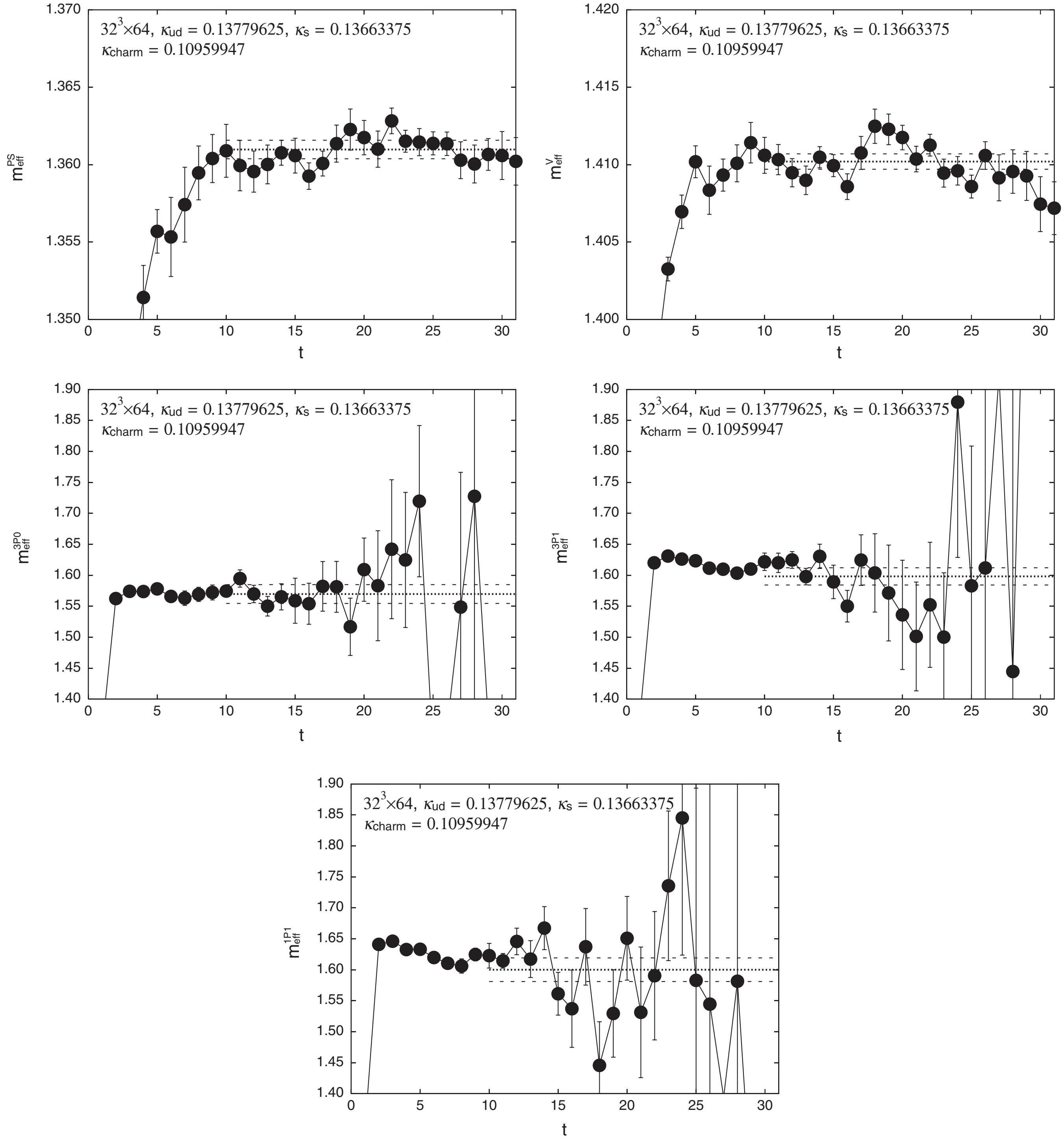

FIG. 2. Effective masses for charmonium.

where $|\mathrm{PS}\rangle$ is the pseudoscalar meson state and $\partial^{\pm}$ is the lattice forward and backward derivative. For the renormalization factor Z_{A_4} and the improvement coefficients of the axial current $c_{A_4}^{+}$ and $c_{A_4}^{-}$, we employ one-loop perturbation theory to evaluate the mass-dependent contributions [13], adding the nonperturbative contributions in the chiral limit by

$$c_{A_4}^{+} = (c_{A_4}^{+}(m_Q a) - c_{A_4}^{+}(0))^{\mathrm{PT}} + c_A^{\mathrm{NP}}, \tag{2.12}$$

$$Z_{A_4} = (Z_{A_4}(m_Q a) - Z_{A_4}(0))^{\mathrm{PT}} + Z_A^{\mathrm{NP}}, \tag{2.13}$$

with $c_A^{\mathrm{NP}} = -0.038\,761\,06$ [14] and $Z_A^{\mathrm{NP}} = 0.781(20)$ [15]. The one-loop mass-dependent contributions are

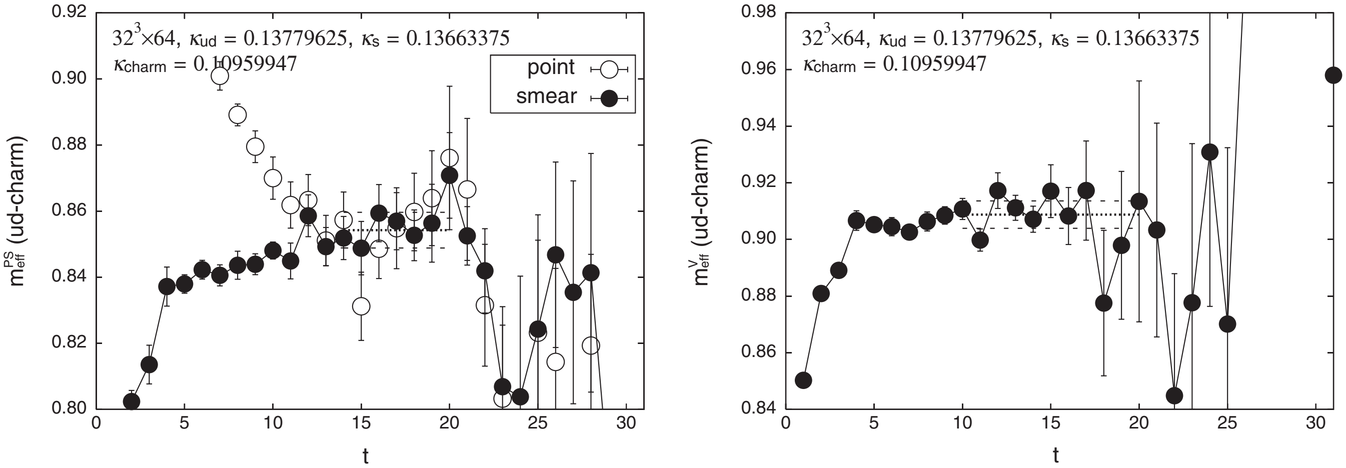

FIG. 3. Effective masses for charmed mesons.

$(c^+_{A_4}(m_Q a) - c^+_{A_4}(0))^{\rm PT} = 0.009\,643\,83$ and $(Z_{A_4}(m_Q a) - Z_{A_4}(0))^{\rm PT} = 0.289$ for $m_Q a = m_{\rm charm} a$. We note that $Z_A^{\rm NP}$ has been updated from that in Ref. [9]. It gives $f_\pi = 113.2(7.8)(2.9)$ MeV, $f_K = 150.9(3.1)(3.9)$ MeV.

The bare quark mass is determined through the axial vector Ward-Takahashi identity,

$$m_f^{AWI} + m_g^{AWI} = m_{\rm PS} \frac{\langle 0|A_4^{\rm imp}|{\rm PS}\rangle}{\langle 0|P|{\rm PS}\rangle}, \qquad (2.14)$$

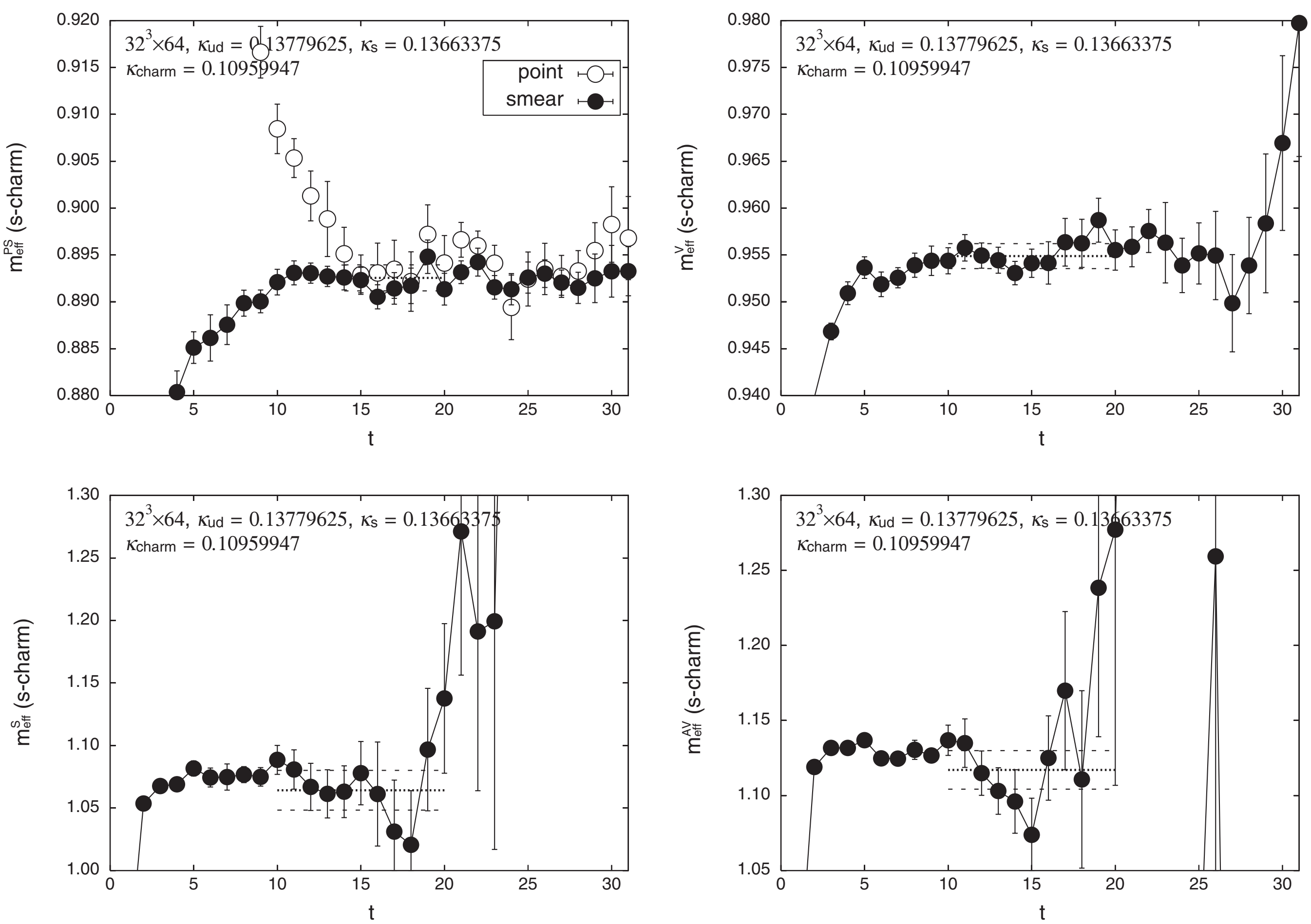

FIG. 4. Effective masses for charmed-strange mesons.

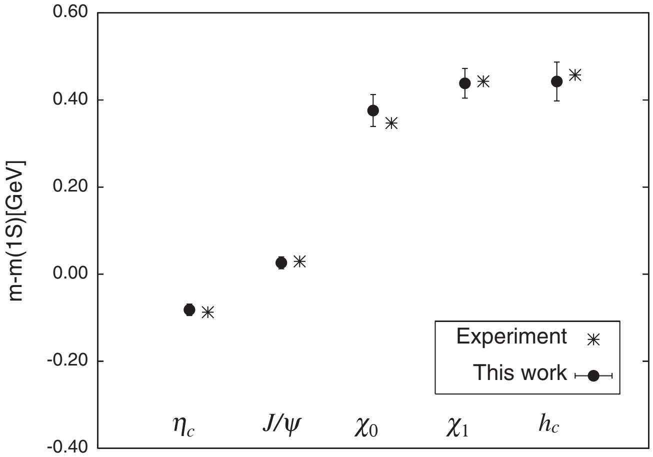

FIG. 5. Our results for the charmonium mass spectrum.

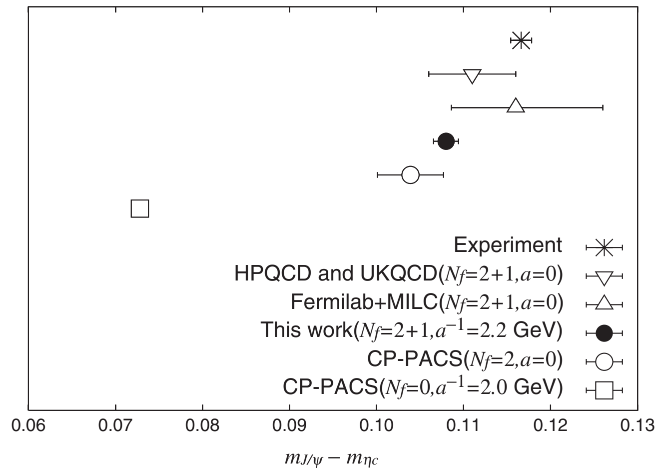

FIG. 6. Hyperfine splitting of the charmonium with different number of flavors.

where P is the pseudoscalar meson operator. The renormalized quark mass in the $\overline{MS}$ scheme is given by

$$m_f^{\overline{\rm MS}}(\mu) = Z_m(\mu) m_f^{AWI}. \tag{2.15}$$

Similar to the case of Z_{A_4}, the renormalization factor for the quark mass at the renormalization scale μ, $Z_m(\mu)$, is nonperturbatively determined at the massless point,

$$Z_m(\mu) = (Z_m(m_Q a) - Z_m(0))^{\rm PT}(\mu) + Z_m^{\rm NP}(\mu), \tag{2.16}$$

with $Z_m^{\rm NP}(\mu = 1/a) = 1.308(35)$ [15]. The one-loop mass-dependent contribution is $(Z_m(m_Q a) - Z_m(0))^{\rm PT}(\mu) = -0.048$ for $m_Q a = m_{\rm charm} a$. The charm quark mass is then evolved to $\mu = m_{\rm charm}^{\overline{\rm MS}}$ using $N_f = 3$ or $N_f = 4$ four-loop beta function [16].

III. CHARMONIUM SPECTRUM AND CHARM QUARK MASS

Our results for the charmonium spectrum on the physical point are summarized in Fig. 5 and Table IV. Within the error of 0.5% to 1%, the predicted spectrum is in reasonable agreement with experiment.

Let us consider the 1S states more closely. Since these states are employed to tune the charm quark mass, the central issue here is the magnitude of the hyperfine splitting. Our result $m_{J/\psi} - m_{\eta_c} = 0.108(1)(0)$ GeV, where the first error is statistical and the second error is from the scale determination, is 7% smaller than the experimental value of 0.117 GeV. In Fig. 6, we compare the present result on $N_f = 2 + 1$ flavor dynamical configurations with previous attempts on $N_f = 2$ dynamical and quenched configurations using the same heavy quark formalism and the Iwasaki gluon action [17]. Other results by recent lattice QCD simulations by Fermilab lattice and MILC Collaborations [18], HPQCD and UKQCD Collaborations [19] are also plotted. We observe a clear trend that incorporation of dynamical light quark effects improves the agreement.

We should note that we have not evaluated our systematic errors for the hyperfine splitting, yet. The continuum extrapolation needs to be performed. A naive order counting implies that the cutoff effects of $O(\alpha_s^2 f(m_Q a) \times (a\Lambda_{\rm QCD}), f(m_Q a)(a\Lambda_{\rm QCD})^2)$ from the relativistic heavy

TABLE IV. Charmonium spectrum in GeV units. The first error is statistical, and the second is from the scale determination. Experimental data are also listed [1].

	$J^{\rm PC}$	Γ operator	Lattice	Experiment
m_{η_c}[GeV]	0^{-+}	γ_5	2.986(1)(13)	2.980(1)
$m_{J/\psi}$[GeV]	1^{--}	γ_i	3.094(1)(14)	3.097(0)
$m_{\chi_{c0}}$[GeV]	0^{++}	I	3.444(33)(15)	3.415(0)
$m_{\chi_{c1}}$[GeV]	1^{++}	$\gamma_i\gamma_5$	3.506(30)(15)	3.511(0)
m_{h_c}[GeV]	1^{+-}	$\gamma_i\gamma_j$	3.510(42)(15)	3.525(0)

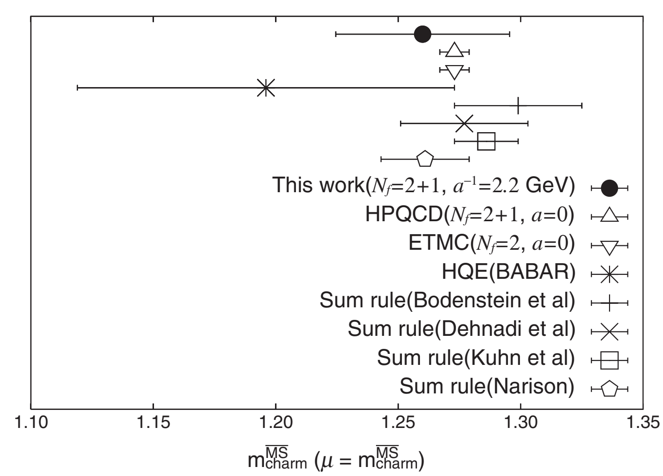

FIG. 7. Comparison of the charm quark mass. The charm quark mass is obtained at $\mu = a^{-1}$, and evolved to $\mu = m_{\rm charm}^{\overline{\rm MS}}$ using four-loop beta function [16]. We employ $N_f = 4$ running in this plot.

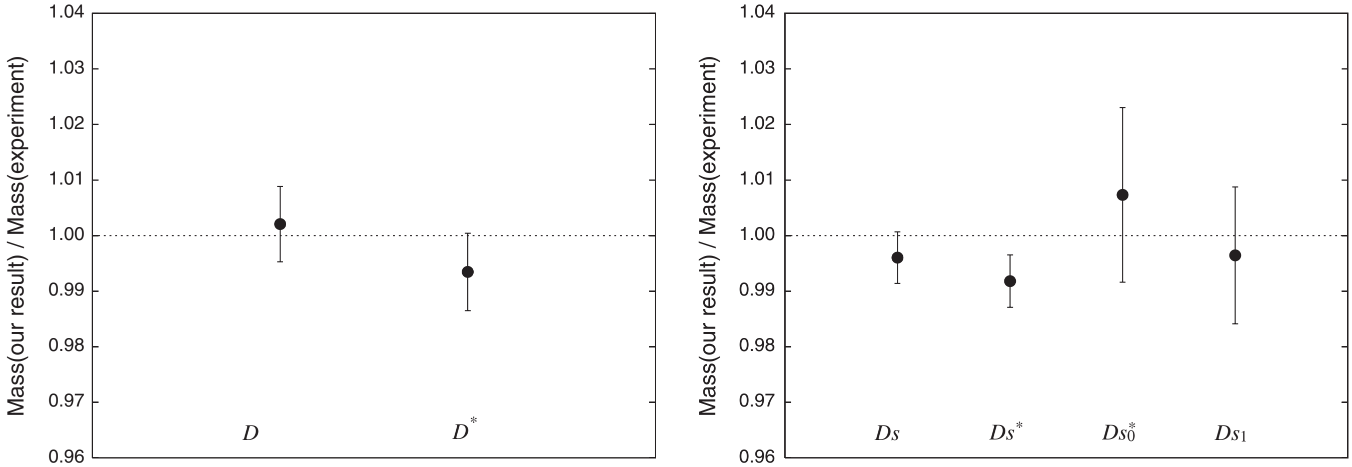

FIG. 8. Our results for charmed meson masses (left panel) and charmed-strange meson masses (right panel) normalized by the experimental values.

quark action are at a percent level. Another aspect is that dynamical charm quark effects and disconnected loop contributions, albeit reported to give a shift of only a few MeV [20], are not included in the present work. Additional calculations are needed to draw a definite conclusion for the hyperfine splitting of the charmonium spectrum. We leave it for a future work.

Using Eq. (2.15), the charm quark mass is obtained as

$$m^{\overline{\rm MS}}_{\rm charm}(\mu = m^{\overline{\rm MS}}_{\rm charm}, N_f = 3\,\text{running}) = 1.260(1)(6)(35)\ \text{GeV}, \quad (3.1)$$

$$m^{\overline{\rm MS}}_{\rm charm}(\mu = m^{\overline{\rm MS}}_{\rm charm}, N_f = 4\,\text{running}) = 1.249(1)(6)(35)\ \text{GeV}, \quad (3.2)$$

where the first error is statistical, the second is from the scale determination, and the third from uncertainty in the renormalization factor. The systematic error due to the heavy quark of $O(\alpha_s^2 f(m_Q a)(a\Lambda_{\rm QCD}), f(m_Q a)(a\Lambda_{\rm QCD})^2)$ will be estimated by using data on finer lattices in the future. Figure 7 compares our result with a recent $N_f = 2 + 1$ lattice QCD estimation by the HPQCD Collaboration [21] in the continuum limit, which uses the HISQ form of the staggered quark action for the heavy quark on the MILC dynamical configurations. Another result by ETM Collaboration with the twisted mass quarks is also plotted [22]. In addition to lattice QCD determinations, recent continuum results using the Heavy Quark Expansions(HQE) [23], as well as sum rules [24], are shown. All these results are consistent.

TABLE V. Charmed meson mass spectrum in GeV units. The first error is statistical, and the second is from the scale determination. Experimental data are also listed [1].

	J^P	Γ operator	Lattice	Experiment
m_D[GeV]	0^-	γ_5	1.871(10)(8)	1.865(0)
m_{D^*}[GeV]	1^-	γ_i	1.994(11)(9)	2.007(0)

TABLE VI. Charmed-strange meson mass spectrum in GeV units. The first error is statistical, and the second is from the scale determination. Experimental data are also listed [1].

	J^P	Γ operator	Lattice	Experiment
m_{D_s}[GeV]	0^-	γ_5	1.958(2)(9)	1.968(0)
$m_{D_s^*}$[GeV]	1^-	γ_i	2.095(3)(10)	2.112(1)
$m_{D_{s0}^*}$[GeV]	0^+	I	2.335(35)(10)	2.318(1)
$m_{D_{s1}}$[GeV]	1^+	$\gamma_i\gamma_5$	2.451(28)(11)	2.460(1)

IV. CHARMED MESON AND CHARMED-STRANGE MESON SPECTRUM

We calculate the charmed meson and charmed-strange meson masses which are stable on our lattice with the spatial size of $L = 2.88(1)$ fm and a lattice cutoff of $a^{-1} = 2.194(10)$ GeV. The D^* and D_s^* meson decay channels are not open in our lattice setup. D_{s0}^* and D_{s1} meson masses are below the DK threshold [1] but above the $D_s\pi$ threshold. Their decays, however, are prohibited by the isospin symmetry. On the other hand, D_0^* and D_1 meson masses are not computed since their decay channels are open, and therefore a calculation involving $D\pi$ contributions is needed.

Our results are summarized in Fig. 8 and in Tables V and VI. All our values for the heavy-light meson quantities are predictions, because the physical charm quark mass has already been fixed with the charmonium spectrum. The experimental spectrum are reproduced in 2σ level. The potential model predicts the D_{s0}^* meson mass is above the DK threshold [25], which deviates from the experiment significantly.[2] But, our result does not indicate such a large

[2]For a recent review, see Ref. [26].

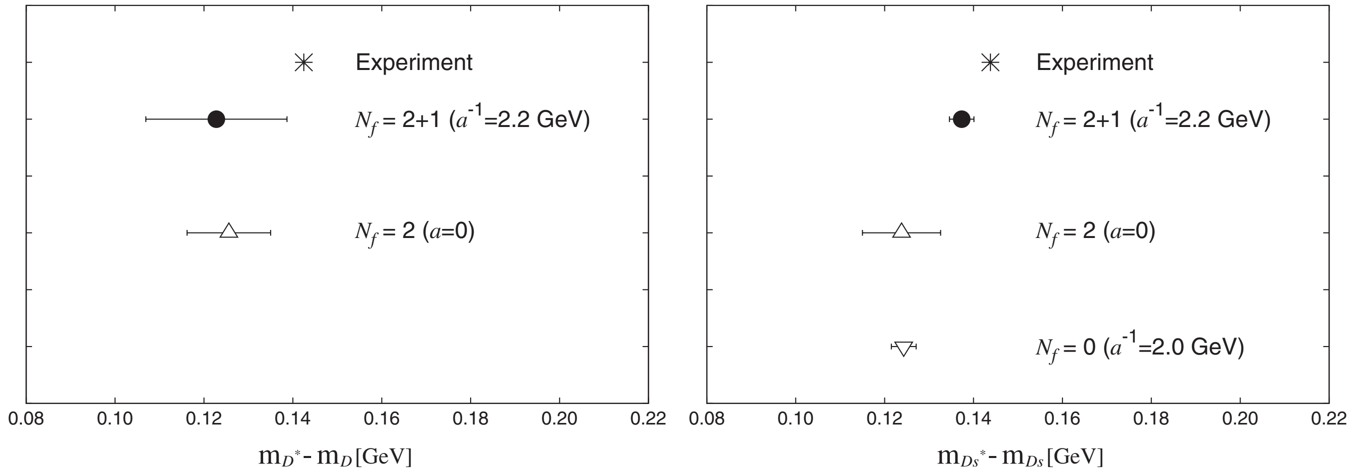

FIG. 9. Our results for the hyperfine splittings of charmed meson (left panel) and charmed-strange meson (right panel).

difference from the experimental value. A similar result is obtained in other lattice QCD calculations [27,28]. The D_{s0}^* meson mass is below the DK threshold. It should be noticed that our calculation does not cover DK scattering states yet. DK contamination for D_{s0}^* and D_{s1} meson masses can be considerably large. Further analysis is required to validate our results for D_{s0}^* and D_{s1} meson spectrum.

We compare our results for the hyperfine splittings $m_{D^*} - m_D$ and $m_{D_s} - m_D$ with experiments in Fig. 9, where we also plot our previous results for $N_f = 2$ and quenched QCD [17]. The deviation from the experimental value is 1.2σ for charmed mesons, and 2.3σ for charmed-strange mesons.

Figure 10 represents mass differences between a charmed meson and charmonium, as well as those between a charmed-strange meson and charmonium, where $m(\bar{D})$ is the spin-averaged mass of D and D^* mesons, and $m(\bar{D}_s)$ is

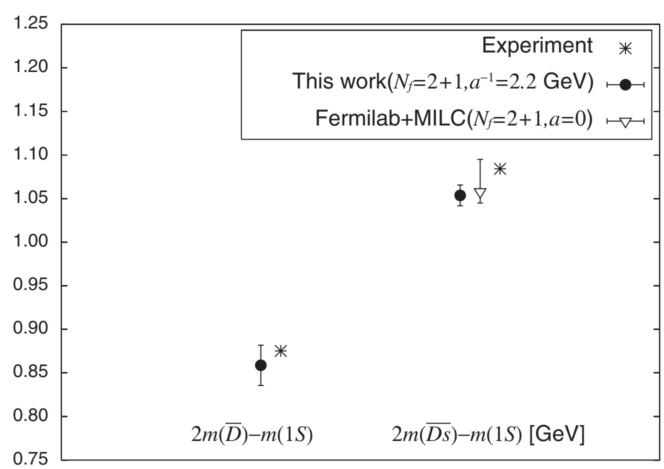

FIG. 10. Mass difference between a charmed meson and charmonium, as well as that between a charmed-strange meson and charmonium. $m(\bar{D})$ is the spin-averaged mass of D and D^* mesons, and $m(\bar{D}_s)$ is that of D_s and D_s^* mesons.

that of D_s and D_s^* mesons. The result by Fermilab lattice and MILC Collaborations [18] is also plotted for comparison. While the mass difference for the charmed meson agrees with the experimental value well, that for the charmed-strange meson is 3% smaller, 2.6σ deviation. This result suggests necessity of continuum extrapolations.

V. CHARMED MESON AND CHARMED-STRANGE MESON DECAY CONSTANTS AND CKM MATRIX ELEMENTS

Table VII presents our estimate of the pseudoscalar decay constants for D and D_s mesons. The error budgets are compiled in Table VIII. Figure 11 shows the

TABLE VII. Our results for decay constants of D meson and D_s meson. The first error is statistical, the second is from the scale determination, and the third is from the renormalization factor. Experimental data are also listed [1].

	Lattice	Experiment
f_D[MeV]	226(6)(1)(5)	206.7(8.9)
f_{D_s}[MeV]	257(2)(1)(5)	257.5(6.1)
f_{D_s}/f_D	1.14(3)(0)(0)	1.25(6)

TABLE VIII. Error budgets for f_D and f_{D_s}. Finite volume effects are evaluated using the heavy meson chiral perturbation theory [29]. Discretization errors are estimated by the naive order counting.

Error	f_D	f_{D_s}
Statistics	3%	1%
Scale uncertainties	0	0
Renormalization	2	2
Finite volume	0	0
Discretization	6	5
Total	7%	5%

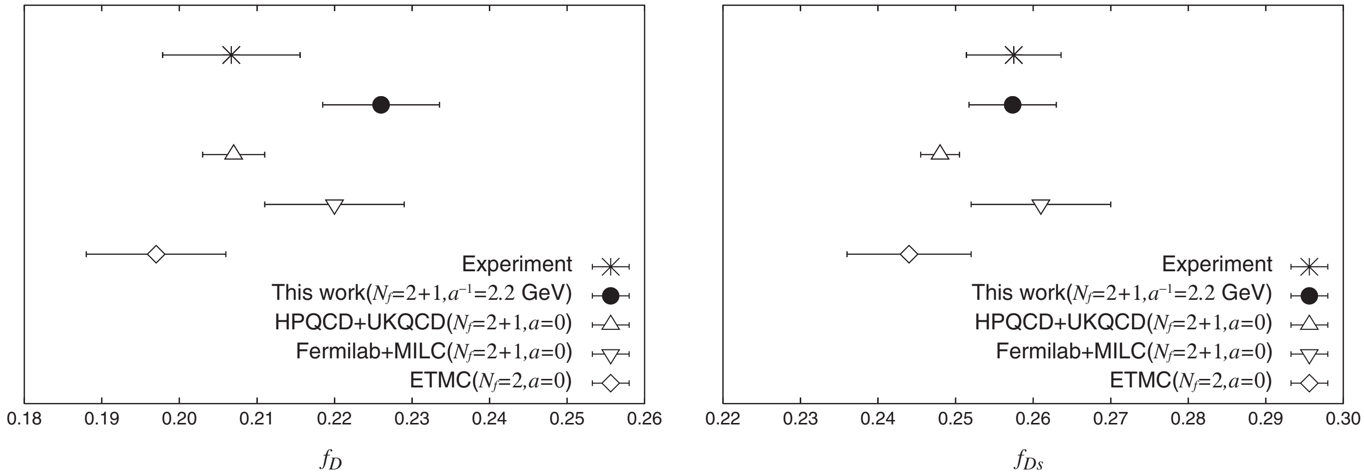

FIG. 11. Comparison of pseudoscalar decay constants for the charmed meson (left panel) and charmed-strange meson (right panel).

experimental values [1] and our decay constants, as well as three recent lattice QCD results: HPQCD and UKQCD Collaboration [21] using HISQ heavy quark on the MILC staggered dynamical configurations, Fermilab lattice and MILC group [30] using the Fermilab heavy quark on the MILC configurations, and ETM Collaboration [31] who uses the twisted mass formalism. Our value for f_{D_s} is in accordance with experiment, while that for f_D is somewhat larger. Comparing four sets of lattice determinations, we observe, both for f_D and f_{D_s}, an agreement between our values and those of the Fermilab group, while there seems to be a discrepancy between our values and those by the HPQCD and UKQCD Collaboration and ETM Collaboration, though continuum extrapolation is needed on our part.

We plot the ratio of f_{D_s} to f_D in Fig. 12. Uncertainties coming from the renormalization factors cancel out, and

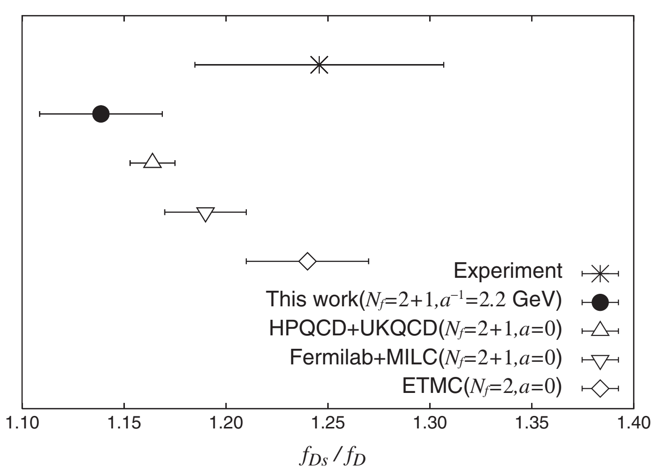

FIG. 12. Ratios of pseudoscalar decay constants for the charmed meson and charmed-strange meson.

that of the lattice cutoff to some extent. Our result is slightly smaller, but still $N_f = 2 + 1$ lattice results are mutually consistent within the errors of a few percent.

Estimating the CKM matrix elements

The standard model relates $|V_{cd}|$ to the leptonic decay width of the D meson $\Gamma(D \to l\nu)$ by

$$\Gamma(D \to l\nu) = \frac{G_F^2}{8\pi} f_D^2 m_l^2 m_D \left(1 - \frac{m_l^2}{m_D^2}\right)^2 |V_{cd}|^2, \qquad (5.1)$$

where G_F is the Fermi coupling constant, and m_l is the lepton mass in the final state. A lattice determination of the D meson decay constant f_D with the experimental value of $\Gamma(D \to l\nu)$ gives $|V_{cd}|$. $|V_{cs}|$ can be obtained in the same way.

We estimate $|V_{cd}|$ from our D meson mass and decay constant with the CLEO value of $\Gamma(D \to l\nu)$ [32]. Up to our heavy quark discretization error of $O(\alpha_s^2 f(m_Q a) \times (a\Lambda_{\rm QCD}), f(m_Q a)(a\Lambda_{\rm QCD})^2)$, we obtain

$$|V_{cd}|(\text{lattice}) = 0.205(6)(1)(5)(9), \qquad (5.2)$$

where the first error is statistical, the second is due to the scale determination, the third is uncertainty of the renormalization factor, and the fourth represents the experimental error of the leptonic decay width. For comparison, the PDG value given by $|V_{cd}| = 0.230(11)$ [1] is about 10% larger (see Fig. 13).

Similarly, using the CLEO value of $\Gamma(D_s \to l\nu)$ [33], we find

$$|V_{cs}|(\text{lattice}) = 1.00(1)(1)(3)(3), \qquad (5.3)$$

as compared to $|V_{cs}| = 1.02(4)$ from PDG [1].

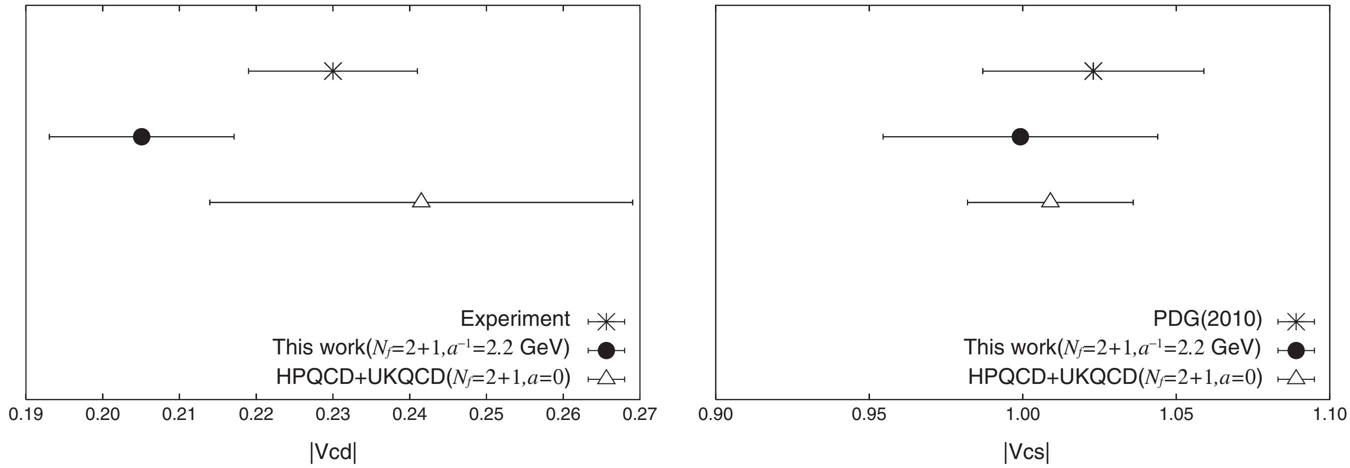

FIG. 13. Comparison of the CKM matrix elements, $|V_{cd}|$ (left panel) and $|V_{cs}|$ (right panel).

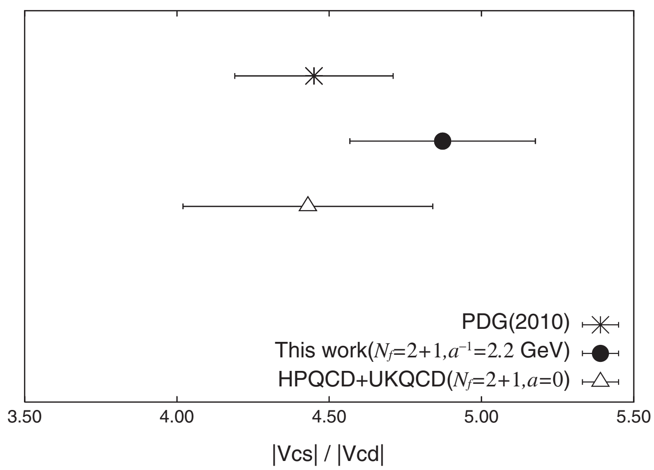

FIG. 14. Ratio of the CKM matrix elements, $|V_{cs}|$ and $|V_{cd}|$.

For completeness, we also record the ratio $|V_{cs}|/|V_{cd}|$ in Fig. 14, for which the systematic errors are partially dropped out.

$$\frac{|V_{cs}|}{|V_{cd}|}(\text{lattice}) = 4.87(14)(0)(0)(27). \qquad (5.4)$$

The PDG value is $|V_{cs}|/|V_{cd}| = 4.45(26)$.

VI. CONCLUSION

We have reported our study of the charm quark system in $N_f = 2 + 1$ dynamical lattice QCD. Although carried out at a finite lattice spacing of $a^{-1} = 2.194(10)$ GeV, our results for the spectra of mesons involving charm quarks are consistent with experiment at a percent level, and so are those for the decay constants within a few percent accuracy. These results indicate that the heavy quark mass correction $m_Q a$ in the charm quark system is under control by the relativistic heavy quark formalism of Ref. [3]. Of course, the continuum extrapolation and further reductions of statistical noises are required to obtain the result competitive with other approaches in the literature.

From methodological point of view, we have shown that the realistic heavy quark simulations with the light dynamical quark masses precisely tuned to the physical values are feasible. With the technique of reweighting, configuration generations are needed to be carried out approximately around the physical point, and a residual fine-tuning to reach the physical point only requires a much less time consuming evaluation of the quark determinant ratios. Combined with the PACS-CS configuration generation at a smaller lattice spacing of $a^{-1} \approx 3$ GeV underway, we hope to return to the issue of continuum extrapolation for the charm quark system in the future.

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