Modification of the Baroclinic Instability Associated with Positive and Negative Arctic Oscillation Index: A Theoretical Proof of the Positive Feedback

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Abstract

The modification of the baroclinic instability associated with positive and negative Arctic Oscillation Index (AOI) is theoretically investigated using a linearized 3D spectral primitive equation model. The linear instability analysis shows that the most unstable Charney mode $M_1$ changes its structure to intensify (weaken) the polar jet by the eddy momentum flux associated with the positive (negative) AOI. More importantly, the meridionally dipole Charney mode $M_s$ is modified into the monopole Charney mode $M_1$ to transport eddy momentum flux northward under the positive AOI condition. It is found that this modification is essential to intensify the polar jet during the AOI positive phase. Hence, we have theoretically confirmed that there are positive feedbacks between the baroclinic instability waves and the Arctic Oscillation characterized by the intensity of the polar jet.

1. Introduction

The Arctic Oscillation (AO) is a north-south seesaw pattern of the atmospheric mass between the arctic region pole-ward of 60°N and a surrounding zonal area in mid-latitude (Thompson and Wallace 1998). The AO is defined as a primary mode of empirical orthogonal function (EOF-1) for the sea level pressure (SLP) fields in the Northern Hemisphere (NH). The AO Index (AOI) is then defined as the time series for EOF-1 scores associated with the SLP. The zonal wind pattern of the AO is characterized by the strength of a subtropical jet and a polar jet. When the AOI is positive, the polar jet tends to be strong and the subtropical jet tends to be weak, and vice versa.

Many previous studies indicated that there are positive feedbacks between the AO and the transient baroclinic waves. For example, Yamazaki and Shinya (1999) found that the wave forcing contributes to the transition to the high/low polarity of the AO. This means that the zonal-eddy interaction plays an important role for the transition of the AO. They also showed that the zonal wave components of planetary-scale wavenumber 2 and 3 contribute most to the transition. Lorenz and Hartmann (2002) observationally investigated the variability of the zonal-mean zonal wind in the NH winter using EOF analysis and momentum budget diagnostics. They concluded that the baroclinic eddies are most important for the positive eddy-zonal flow feedback and the quasi-stationary eddies also reinforce the zonal wind anomalies. Moreover, Limpasuvan and Hartmann (1999) demonstrated the important role of transient and stationary eddy fluxes in the maintenance of annular modes of variability with a realistic numerical simulation. They found that the eddy structures evolve with the jets and the transient eddy momentum flux supports the shifts in jet position as the most important eddy momentum forcing. In their study, the ridge or trough axes tend to tilt from southwest to northeast when the jet shifts northward in the North Atlantic. Then the associated stationary wave transports momentum pole-ward to support the local jet displacement. These studies argue that the positive feedbacks between the zonal-mean wind anomalies and the baroclinic eddies are important for the AO. However, many of these studies are classified into the observational studies rather than theoretical studies.

On the other hand, Tanaka and Tokinaga (2002) theoretically investigated the development of the baroclinic instability with the variable basic states depending on the AOI using a 3D spectral primitive equation model in terms of the 3D normal mode expansion. They found that the baroclinicity of the strong polar jet, which is associated with the positive AOI, excites a monopole Charney-type baroclinic instability as $M_1$ (Charney 1947) but is excited by the baroclinicity of the polar jet. The structure of $M_1$ shows the transport of the westerly momentum to the polar jet. Since $M_1$ was replaced with the other most unstable mode only for the strong polar jet, it was theoretically explained that there is a positive feedback between $M_1$ and the polar vortex. However, they investigated only the development of the baroclinic waves under the context of extreme events for the past AOI.

In this study, we investigate the modification of the baroclinic instability associated with the positive or negative AOI. The basic method is following Tanaka and Tokinaga (2002). The baroclinic instability problem is solved for the AO-related basic states. Finally we theoretically explain the positive feedback between the baroclinic instability and the AO. In Section 2 the data and governing equations are described for the linear baroclinic instability problem. The results of the stability analysis are presented for the most baroclinically unstable mode in Section 3. Ultimately the conclusion is summarized with discussion in Section 4.

2. Data and method

2.1 Data for the zonal-mean basic state

The data used in this study are National Centers for Environmental Prediction (NCEP)-National Center for Atmospheric Research (NCAR) reanalysis of four-times-daily zonal wind at constant pressure levels for December to February (DJF) during 1958–1997 (Kalnay et al. 1996). Figure 1a shows the climatology of zonal-mean zonal wind in January during this period. In order to construct the basic states representing the AOI positive and negative, we added (or subtracted) the zonal wind anomaly regressed with the normalized AOI (Fig. 1b) to the climatology of the zonal-mean zonal wind (Fig. 1a). For example, adding the three-fold zonal wind anomaly to the climatology, we constructed the basic state of zonal-mean zonal wind for the AOI $+3.0\sigma$ (standard deviation) in Fig. 1c. Although Fig. 1c is an artificial basic state for the AOI $+3.0\sigma$, it is almost identical to the observation for DJF in 1988/89 (see Tanaka and Tokinaga 2002). Similarly, the artificial zonal-mean zonal wind for the AOI $-3.0\sigma$ (Fig. 1d) represents the realistic one for the large negative AOI. In this study, the basic states of the AOI ranging from $-3.0\sigma$ to $+3.0\sigma$ are prepared for the linear instability analysis.
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2.2 Model

In order to amplify the baroclinically unstable wave for the basic states associated with the positive and negative AOI and to solve the linear instability analysis, we adopted a 3D spectral primitive equation model in terms of the 3D normal mode expansion. The detail of the model description was provided by Tanaka and Tokinaga (2002). We present a brief description here.

Transforming the primitive equations on a sphere into the wavenumber space with 3D normal mode functions, and linearizing them with respect to a given zonal basic state, we obtain a prognostic equation for the anomaly of the normal mode expansion coefficients \( w_i \) as:

\[
\frac{d w_i}{d \tau} = -i \omega w_i - i \sum_{j=1}^{K} \sum_{k=1}^{6} \left( r_{ik} + r_{kj} \right) w_j, \\
i = 1, 2, 3, \ldots, K,
\]

(1)

where \( \tau \) is a dimensionless time, symbol \( i \) is an imaginary unit, \( \omega \) is the eigenfrequency of Laplace's tidal equation, \( r_{ik} \) are the interaction coefficients for nonlinear wave-wave interactions, and subscripts \( i \) \((or \ j, k)\) denote the 3 dimensional wavenumbers. The external forcing for transient components is disregarded in this study. Although the model is based on the primitive equations on a sphere, we can filter out the redundant unstable gravity modes due to the normal mode expansion technique.

The AO-modified basic states are substituted into the zonal basic state \( \overline{w}_n \) as a prescribed parameter. Since \( \omega \) and \( r_{ik} \) are determined accurately from the primitive equations, we can solve the linear equation as an eigenvalue problem by assuming a wave-type solution for the prognostic equation:

\[
w_i(\tau) = \psi \exp(-i \nu \tau),
\]

(2)

where \( \psi \) is a complex-valued eigen vector describing the structure of the unstable mode, and \( \nu \) is an eigenvalue whose real and imaginary parts represent the phase speed and the growth rate for the unstable modes, respectively. The matrix to be solved becomes a block diagonal for each zonal wavenumber so that we can solve the eigenvalue problem for each zonal wavenumber. We used vertical modes of \( n = 0 \) to 6 and meridional modes \( l = 0 \) to 20 but for the symmetric Rossby modes. The problem of the baroclinic instability can thus be solved for arbitrary zonal basic states on a sphere by this method in the framework of the primitive equation.

The fastest growing mode represents the most unstable mode.

The structures of the unstable modes may be analyzed by the inverse Fourier transform of \( \psi \) from the wavenumber space to the physical space using the 3D normal mode functions. Selecting the most unstable modes, we can analyze the relationship between the positive and negative AOI and the baroclinically unstable waves.

3. Results

First, we demonstrate the baroclinic instability for each artificial basic state. Figure 2 illustrates the growth rates (day \(^{-1}\)) and phase speeds (\(^{\circ} \) day \(^{-1}\)) of unstable modes computed for the zonal-mean basic state of the AOI +3.0\( \sigma \) (Fig. 1c). In the synoptic scale of zonal wavenumber \( n = 6 \) to 12, the fastest growing unstable mode is Charney type monopole mode \( M_C \) (see Charney 1947; Tanaka and Kung 1989). However the most unstable mode in the planetary scale of \( n = 1 \) to 5 is replaced by the other monopole mode \( M_I \) which is located at higher latitude than \( M_C \) (see Tanaka and Tokinaga 2002). Meridional dipole Charney mode \( M_M \) also appears in the planetary scale (see Tanaka and Kung 1989). We find tripole Charney mode \( M_T \) in other basic states (not shown). \( M_C \) and \( M_I \) are identified as the high-order meridional modes of \( M_C \) that have nodes in the meridional direction. These characteristics described in Fig. 2 agree with Fig. 9 in Tanaka and Tokinaga (2002) showing the growth rates and phase speeds of each unstable mode related to the observed basic state for the AOI +3.0\( \sigma \). This agreement is also seen in the cases of other basic states.

In order to find the relationships between the AOI and the baroclinic instability waves, we analyzed the most unstable modes at each scale. In this paper, \( M_I \) at \( n = 6 \) and \( M_T \) at \( n = 2 \) are discussed in detail since the characteristics of their modal structure are represented by these two examples.

3.1 Charney mode \( M_C \)

Figure 3a illustrates the 500 hPa geopotential deviations from the zonal mean in the NH for \( M_C \) at zonal wavenumber 6 with the climatological basic state (Fig. 1a). The maxima of geopotential amplitudes are located at the mid-latitudes. In the northern part of the maxima their ridge (or trough) axes tilt from northwest to southeast and in the southern part their axes tilt from northeast to southwest. This means that westerly eddy momentum is transported and converged at the mid-latitudes. The momentum flux thus intensifies the subtropical jet.

On the other hand, Fig. 3b shows the same structure but for the basic state of AOI +3.0\( \sigma \) (Fig. 1c). The growth rate is approximately 0.27 day \(^{-1}\) (Fig. 2a). It moves eastward with its phase speed of approximately 8.6 \(^{\circ} \) day \(^{-1}\) (Fig. 2b). The ridge axes predominantly tilt from northeast to southwest. Although the amplitude maxima stay at mid-latitudes even if the AOI becomes large positively (Fig. 3b), the tilted axes tend to transport westerly momentum northward and reinforce the polar jet. Hence, the structure of \( M_I \) is modified to intensify the polar jet by the enhanced northward eddy momentum flux so that the AOI becomes positive. \( M_I \) in other zonal wavenumbers also indicate the same modification.

To investigate whether the modification in the modal structure is continuous or not, the growth rates for each mode are examined as a function of the AOI (Fig. 3c). According to the result, the growth rate of \( M_T \) continuously changes with varying in the AOI.

The same characteristics are found in the phase speeds of \( M_I \) (not shown). \( M_T \) continuously changes its structure from a typical Charney mode to a different shaped Charney mode transporting eddy momentum poleward.

Therefore, as the AOI becomes positive, \( M_T \) transports more...
This implies that the unstable planetary mode \( M_1 \) positive AOI. jet, but is identified as not a distinct mode associated with the baroclinicity of the polar structure is identical to the monopole Charney mode higher latitudes as the AOI increases. The modified monopole for Fig. 4d. It is found that the eddy momentum flux extends to figure, we can infer the upward and southward wave-activity flux wave activity flux is perpendicular to the phase contour in the latitudes as the AOI becomes positive (Figs. 4c, 4d). Since the wave tends to tilt westward with respect to height at the high-latitudes as the AOI becomes positive (Figs. 4c, 4d). Since the wave activity flux is perpendicular to the phase contour in the figure, we can infer the upward and southward wave-activity flux for Fig. 4d. It is found that the eddy momentum flux extends to higher latitudes as the AOI increases. The modified monopole structure is identical to the monopole Charney mode \( M_1 \) advocated by Tanaka and Tokinaga (2002). It is found in this study that \( M_1 \) is not a distinct mode associated with the baroclinicity of the polar jet, but is identified as \( M_2 \) modified to a monopole structure by the positive AOI.

Consequently, \( M_2 \) is modified into a monopole structure or \( M_1 \) to intensify the polar jet as the AOI becomes large positive. This implies that the unstable planetary mode \( M_2 \) shows a positive feedback with the AOI.

Figure 5 illustrates the growth rates for zonal wavenumber 2 as a function of the AOI for various types of unstable modes. The growth rates show that \( M_1 \) is modified to \( M_1 \) continuously as the AOI becomes large positive. In addition, \( M_1 \) is modified to \( M_2 \) as the AOI becomes large positive. Of course, these features also appear in the variation of the phase speed (not shown). The same modification in the structure described for \( M_2 \) at the zonal wavenumber 2 is also seen for other zonal wavenumbers.

4. Conclusion and discussion

The AO is characterized by the intensity of the polar jet and polar vortex. Observational studies revealed that the westerly jet is built up by the meridional flux convergence of the eddy momentum provided by the baroclinic instability waves (Limpasuvan and Hartmann 1999; Yamazaki and Shinya 1999; Lorentz and Hartmann 2002). In this study, the relationship between the AO and baroclinic instability waves is examined theoretically using a 3D spectral primitive equations linearized with respect to the zonal basic states for various positive and negative AOI.

As a result, the basic state with a positive AOI modifies the ridge axes of \( M_1 \) and \( M_2 \) to tilt from northeast to southwest. Thus the eddy momentum is transported further to the north to enhance the polar jet. Moreover, the southern amplitude maximum of \( M_1 \) disappears as the AOI becomes positive. The single amplitude maximum is located at very high latitudes (Fig. 4d). Its ridge axes also tilt in the horizontal plane to feed the westerly momentum northward to intensify the polar jet inducing the positive AOI. This theoretically shows that there is a positive feedback between the AO and baroclinically unstable Charney mode \( M_1 \). In addition, \( M_2 \) is also modified into another monopole mode as the AOI becomes positive in Fig. 3c. We will refer to this in the next section.

3.2 Dipole Charney mode \( M_1 \)

Figure 4 illustrates latitude-height structures of geopotential amplitudes and their phase (longitudes of ridges) for the dipole mode \( M_1 \) at zonal wavenumber 2. When the AO is negative with a weak polar vortex, it is obvious that \( M_1 \) has the meridional dipole structure with two amplitude maxima in the north and south (Fig. 4a). But the amplitude of the southern maximum gets weaker as the AOI increases to positive (Figs. 4b, 4c). And eventually \( M_1 \) is modified to the monopole structure with only northern pole (Fig. 4d). Compared with \( M_1 \) for the high AOI (Fig. 3b), the northern amplitude maximum of \( M_1 \) is located at very high latitudes (Fig. 4d). Its ridge axes also tilt in the horizontal plane to feed the eddy momentum flux northward as in the case of \( M_1 \). Similarly, the phase tends to tilt westward with respect to height at the high-latitudes as the AOI becomes positive (Figs. 4c, 4d). Since the wave activity flux is perpendicular to the phase contour in the figure, we can infer the upward and southward wave-activity flux for Fig. 4d. It is found that the eddy momentum flux extends to higher latitudes as the AOI increases. The modified monopole structure is identical to the monopole Charney mode \( M_1 \) advocated by Tanaka and Tokinaga (2002). It is found in this study that \( M_1 \) is not a distinct mode associated with the baroclinicity of the polar jet, but is identified as \( M_2 \) modified to a monopole structure by the positive AOI.

Consequently, \( M_2 \) is modified into a monopole structure or \( M_1 \) to intensify the polar jet as the AOI becomes large positive. This implies that the unstable planetary mode \( M_2 \) shows a positive
momentum flux for the positive and negative AOI are presented observationally by Ikeda and Tanaka (2008), which support quantitatively with the present theoretical explanation.

The positive feedbacks accelerate the polarity of the AO. However, the extremely large value of the AOI must be terminated by the nonlinearity of the atmosphere, which is beyond the scope of the present linear theory with the 3D spectral primitive equation model. Also the wave-wave interactions for the non-zonal basic state may be important to modify the structure and behavior of the baroclinic waves. As a future subject, we need to expand the problem for the general non-zonal basic state to construct a general linear baroclinic model.

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References


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SOLA: http://www.jstage.jst.go.jp/browse/sola

Fig. 4. Latitude-height structures of geopotential amplitudes (solid line in arbitrary unit) and phases (dotted line denoting longitudes of ridges) for $M_2$ at zonal wavenumber 2 for (a) AOI $-1.0\sigma$, (b) climatological basic state, (c) AOI $+1.0\sigma$, and (d) AOI $+2.0\sigma$ in NH.

Fig. 5. Growth rates for the various types of the unstable modes at zonal wavenumber 2 as a function of the AOI.