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Weberian Approach on Economic Development in a Schumpeterian Growth Model

by

Shiro Kuwahara

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UNIVERSITY OF TSUKUBA
Tsukuba, Ibaraki 305-8573
JAPAN
Weberian Approach on Economic Development in a Schumpeterian Growth Model *

Shiro Kuwahara †

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Abstract

This paper combines two important factors of capitalism: the "spirit of capitalism," proposed by Weber (1958), and "creative destruction," emphasized by Schumpeter (1934). The former is captured as a preference for wealth accumulation and the latter is constructed as a production structure with endogenous technological progress. This paper examines how these two factors affect the long-run growth of an economy, and reveals that the spirit of capitalism positively affects the long-run growth.

Key words: innovation; no growth trap; R&D-based growth; spirit of capitalism; wealth effect

JEL classification: O10; O40; B10; B20; P10

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†E-mail: kuwahara@sk.tsukuba.ac.jp TEL/FAX: +81-29-853-6206 Postal Address: Grad. Sch. of Systems & Information Engineering, University of Tsukuba, 1-1-1 Tennodai Tsukuba, Ibaraki, 305-8573 Japan
1 Introduction

The mechanism of economic development has been an important theme since the fast stage of the political economies. This paper takes the first step to incorporate two early but important perspectives of economic development - those of Weber (1905) and Schumpeter (1934) - to analyze economic development (and no/under development). One of the two factors of analysis is Weber (1905)’s the “spirit of capitalism,” and the other is the innovation created by R&D activities, which Schumpeter (1934) denoted as ”creative destruction.”

Modern economists have studied the determinants of economic growth (e.g. Barro 1991). Moreover, certain cultural factors have also intrigued economists. This literature isolates a number of variables that predict the subsequent rates of economic growth. One general conclusion that can be drawn is that successful explanations of economic growth must surpass narrow measures of economic variables and encompass political and social forces (e.g., Hall and Jones, 1996; and Sala-i-Martin, Doppelhofer, and Miller, 2004). In particular, the results reveal important influences on growth from government policies and public institutions. Some researchers (e.g., Inglehart and Baker, 2000) argue that a nation’s culture should be included in a rationalization of its economic growth. Religion is an important dimension in explanations of such literature, including geography (Sachs, 2003), institution (Acemoglu, Johnson, and Robinson, 2001, 2002), ethnic heterogeneity (Easterly and Levine, 1997), and climate (Easterly, and Levine, 2003). There are some positive studies along this lines, such as Dudley and Blum (2001), Barro and McCleary (2003), and Noland (2004).

Following this literature, we analyze the economic growth model using both technological and cultural factors, which will yield the result that long-run economic growth is affected by the spirit of capitalism in addition to the technological conditions of R&D. Weber (1905) emphasized religious ascetic values, termed as the ”Protestant Ethic,”¹ which is an ascetic endeavor for self-help in practicing and executing one’s Beruf (calling) with frugality, and states that the accumulation of wealth is the result of religiousness. This implies that accumulation itself becomes the objective. As early as 1960’s, Kurz (1968) merged this ”spirit” into the Ramsey-type optimal growth model. He

¹The counterpart of this may be Japanese confucianism, and the frontier spirit in the USA.
assumed that the utility of an agent is not derived merely from consumption; rather, it also results from asset holding. This ”wealth effects” can be regarded as one modelization of the spirit of capitalism. Kurz (1968) investigates the results of the existence of these wealth effects, for example, the occurrence of multiple equilibria. In the literature on endogenous growth models with wealth effects, we encounter the pioneering trial of Zou (1994), which demonstrates the relationship between the spirit of capitalism and disappearance of decreasing returns of capital.

On the other hand, Schumpeter (1934) emphasized the important role of innovation on economic growth in a capitalist economy, which has been supported by studies on growth accounting (e.g., Solow, 1957), which clarify that economic growth is mainly attributable to technological progress. This fact was incorporated into an economic growth model as late as the 1990’s by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), among others. They endogenized technological progress derived from R&D activities and demonstrated a mechanism for the long-run growth of capitalist production.

To capture the above properties, the present paper unites Weber’s (1958) spirit of capitalism, which is formulated as an á la Kurz type of wealth effects, and Schumpeter’s (1934) notion of innovation, which is constructed as an á la Aghion and Howitt type of endogenous technological change through the improvement of the quality of intermediate goods.

The obtained results are as follows. The model of the present study demonstrates that the relationship between innovation and the spirit of capitalism determines the long-run growth phase. A more intense spirit capitalism enhances the rate of economic growth and enables the economy’s escape from no growth traps, even if the economy has inefficient R&D structure. It is also demonstrated that these results are maintained under the introduction of capital accumulation.

This paper is organized as follows. Section 2 develops the model; Section 2.1 treats the optimization of consumption and saving with wealth effects, which describe the mechanism of the supply of asset; and section 2.2 discusses the endogenous technological change, which describes the demand of asset. Section 3 derives the steady state of the model and proffers the properties obtained from the model. In Section 4, capital accumulation is introduced and its effects are investigated. Section 5 concludes the paper.
2 The Model

The model of the present study consists of a household with the spirit of capitalism and a production sector with endogenous technological progress. The economy is endowed by the population normalized to 1, the time is continuous, and the final goods are used as a numeraire.

2.1 The Weber Economy

The Weber economy should consist of a representative household with the spirit of capitalism, which, following Kurz (1968), is captured by the assumption that the household derives utility both from consumption \((c)\) and per capita wealth, namely asset holding, \((A)\). Thus, the representative agent in this economy has the following utility:

\[
U = \int_0^\infty u(c, A) e^{-\rho t} dt
\]  

where \(\rho\), and \(u(c, A)\) are the subjective discount rate, and an instantaneous utility function of the representative household, respectively. The representative household has the following budget constraint:

\[
\dot{A} = rA + w - c - nA.
\]  

\(r\) and \(w\) are the interest rate and wage rate, respectively. \(n\) is a population growth rate that is assumed to be non-negative throughout the present study.

We specify the instantaneous utility function as follows

\[
u(c, A) = \left(\frac{c^{1-\beta} A^{\beta}}{1-\sigma} - 1\right),
\]

where \(\beta(\in [0, 1])\) is the parameter that captures the intensity of wealth effects against consumption. When \(\beta = 0\), there are no wealth effects, and the model corresponds to the usual Ramsey type utility structure. For greater \(\beta\), the agent has larger preference on wealth accumulation. When \(\beta = 0.5\), the wealth accumulation has the same weight for the consumption of goods, and when \(\beta > 0.5\), the wealth accumulation has more weight than the consumption of goods. Under this specification, the optimizing condition for the consumption growth rate is derived as

\[
\left\{\sigma - \beta(1 - \sigma)\right\} \dot{c} - \beta(1 - \sigma) \frac{\dot{A}}{A} + \rho = r - n + \frac{\beta}{1 - \beta} \frac{c}{A}.
\]  

4
This equation corresponds to the Euler rule in usual Ramsey models. The additional terms are \( \beta(1 - \sigma)\dot{\chi}/\chi \) and \( (\beta/(1 - \beta))\dot{\chi} \), where \( \chi \equiv c/A \) denotes the consumption-asset ratio. (4) implies \( \dot{\chi} = 0 \) in a steady state; therefore, the former term is canceled out in the steady state and the latter term produces effects on the long-run growth condition.

2.2 The Schumpeter Economy

The Schumpeter economy is one in which innovation is undertaken. This study adopts an Aghion and Howitt (1992) type of Schumpeterian growth model with an intermediate goods quality improvement structure.

2.2.1 Production

The present analysis includes three sectors: final goods, intermediate goods, and R&D. It also contains three factors: labor which is inelastically supplied by the population, final goods devoted to intermediate input, and knowledge captured as a quality of intermediate goods. The final goods are produced by labor and intermediate goods, and one unit of intermediate goods is made from \( \eta \) units of final goods. R&D activity is assumed to be undertaken by using labor. Thus, labor \( (L) \) is utilized in final goods production \( (L_Y) \) and R&D sector \( (L_A) \); therefore, \( L = L_Y + L_A \). Furthermore, we assume that one unit of population inelastically supplies one unit of labor force; therefore, we can identify the aggregate labor supply \( L \) as the population. It is assumed that the final goods are used as consumption goods \( C \), or intermediate goods \( (Z) \); therefore, \( Y = C + Z \).

The producers of final goods utilize a variety of intermediate goods and labor. Each type of intermediate good is indexed as \( i \in (0, N) \), where \( N \) is assumed to be a sufficiently large given constant. Each type of intermediate good has a vertical quality level known as a "quality ladder" along which innovations can occur. Each quality level in the \( i \)th sector has an index \( m_i = 1, 2, ..., M_i \), and the quality ladder has quality levels \( \lambda, \lambda^2, ..., \lambda^{M_i} \), respectively, where \( \lambda(> 1) \) is the exogenously given "width" of one innovation. Thus, the intermediate goods ranked by \( m \) are equivalent to \( \lambda \) units of intermediate goods ranked by \( m - 1 \). Thus, the quality of the cluster of intermediate goods is the source of economic growth in this study; therefore, the productivity of

\[ \text{See Appendix A1 for the detailed derivation.} \]
intermediate goods can be regarded as knowledge in this economy. Thus, we denote the incumbent, as the top quality of the $j$th sector as $q(i)$; therefore, $q(i) \equiv \lambda^M_j$. We also denote the $i$th sector’s next monopolist firm, namely the monopolist firm with one rank upper quality in future, as $i^+$. Using this notation, the quality after one more innovation is denoted as $q(i^+) = \lambda^{M_i+} = \lambda^{M_i+1}$.

In this setting, each quality level $m$ has an efficiency level of $q^m$, and thus, the intermediate goods that are one grade higher in terms of quality are $q$ times more efficient than those that are one grade lower in terms of quality. Since quality-adjusted intermediate goods within the same sector are perfect substitutes, there exists a demand for quality goods with the lowest quality-adjusted cost. Thus, the intermediate goods that are demanded are always those with the top quality.

We assume that the final goods production obeys constant returns to scale and there is no complementarity among intermediate goods. Denoting the demand of the $i$th intermediate goods sector, which is the current top quality of the sector, as $x(i)$, we can specify the production function of final goods $Y$ as

$$Y = L_Y^{1-\alpha} \int_0^N \{q(i)x(i)\}^\alpha di, \quad 0 < \alpha < 1,$$

where $L_Y$ is the labor supply allocated to the final goods production.

The first order condition (FOC) of production is obtained as

$$\frac{\partial Y}{\partial x(i)} = p(i), \quad \text{and} \quad \frac{\partial Y}{\partial L_Y} = w,$$

where $p(i)$ and $w$ are the price of the $i$th intermediate good of the top quality and wage rate, respectively.

R&D firms facilitate technological progress; they create a design that is one grade higher in terms of quality than a design that has the incumbent highest quality level. The R&D activities of the firms are overtaken at the beginning of each period, and the results are immediately evident. A successful research firm retains exclusive rights for the use of the intermediate goods of this new quality level. This exclusive right is referred to as a ”patent.”

In this study, the intermediate goods production are produced by $\eta$ unit of final goods. Hence, the firm that produces the $i$th intermediate good maximizes the profit such that

$$\pi(i) = p(i)x(i) - \eta x(i).$$
The monopolist firm that has the patent term of the current top quality maximizes its profit by considering price as a control variable. Therefore, the FOC of the monopoly firm in the $i$th sector with the $M_i$th quality yields the following:

$$x(i) = \left[ \frac{\alpha^2}{\eta} \right]^{\frac{1}{\alpha-\eta}} L_Y q(i)^{\frac{\alpha}{\alpha-\eta}}, \quad \text{and} \quad p(i) = \frac{\eta}{\alpha}. \quad (8)$$

With regard to monopolistic pricing, we have the following three conditions. First, each unit of the top quality is equivalent to $\lambda > 1$ unit of a good with the next best quality. Second, a good that is one grade lower than the top quality good is supplied at marginal cost $1/\phi$ because the patent for this grade expires. Third, the different quality grades are perfect substitutes if they are weighted by the quality level. Based on the above conditions, it follows that $p(i) < \lambda \eta$ is necessary for the firm to create a top quality good to monopolize the demand of that good. Therefore, a combination of (8) and $p(i) < \lambda \eta$ indicates that the condition of (8) is optimal under the assumption that $1/\alpha < q$. The following discussion is developed such that it satisfies the present assumption. Thus, only top quality goods are supplied.

The aggregate index of quality is defined as

$$Q \equiv \int_0^N q(i)^{\alpha} di. \quad (9)$$

Substituting (8) into (5) and using (9) to rearrange it, we obtain the aggregate output $Y$ as

$$Y = \alpha \frac{2\alpha}{\alpha-\eta} \eta^{-\frac{\alpha}{\alpha-\eta}} L_Y Q \quad (10)$$

We also note the intermediate goods input of final goods denoted by $X$ from the aggregation of (8) across sectors and usage of

$$Z = \eta \int_0^N x(i) di = \alpha \frac{2\alpha}{\alpha-\eta} \eta^{-\frac{\alpha}{\alpha-\eta}} L_Y Q (= \alpha^2 Y). \quad (11)$$

Substituting (11) into (10), we obtain the aggregate consumption as

$$C = (1 - \alpha^2)Y = (1 - \alpha^2) \alpha \frac{2\alpha}{\alpha-\eta} \eta^{-\frac{\alpha}{\alpha-\eta}} L_Y Q \quad (12)$$

\textit{3}This assumption implies that the width of one innovation is sufficiently large. If \(1/\alpha < \lambda\), the optimal pricing is given as $p(i) = \lambda \eta$. This pricing does not alter the main framework of the model; therefore, we assume that $1/\alpha < \lambda$ throughout this study.
By using (7) and (8), the profit of the $i$th sector monopoly firm with a patent of quality $M_i$ is obtained as follows:

$$
\pi(i) = \left( \frac{1}{\alpha} - 1 \right) \eta x(i) \\
= \left( 1 - \alpha \right) \gamma^{\frac{i-\alpha}{1-\alpha}} L_Y q(i)^{\frac{\alpha}{1-\alpha}} \\
= \left( \frac{1}{\alpha} - 1 \right) Y^{\frac{\alpha}{1-\alpha}} q(i)^{\frac{\alpha}{1-\alpha}} \tag{13}
$$

2.2.2 R&D Activities

It is presumed that R&D activities for the purpose of innovating different quality levels are conducted using labor, and the success of R&D stochastically depends on the labor input. When innovation occurs in a sector, the probability that firm $j$ in that sector will be granted a patent is assumed to be proportional to the share of R&D input in the $i$th sector; therefore, $L_A(i)^j/L_A(i)$, where $L_A(i)^j$ and $L_A(i)$ represent the R&D input for the $i$th sector of firm $j$ and the aggregate R&D input for the $i$th sector, respectively.

Therefore, from the above assumptions, it is shown that the profit of sector $i$ of R&D firm $j$ is

$$
\max_{L_A(i)^j} \frac{\mu(i)L_A(i)^j}{L_A(i)} v(i) - wL_A(i)^j.
$$

The presence or absence of investment in R&D activities is determined as follows. If $\frac{\mu L_A(i)^j}{L_A(i)} v(i) < wL_A(i)^j$ holds, the R&D activity is not profitable. Consequently, the R&D input stops and equilibrium is attained without R&D; therefore the probability of R&D success is 0, that is, $\mu = 0$. If $\mu = 0$ is realized, the quality of the intermediate goods would remain constant over time. If $\frac{\mu L_A(i)^j}{L_A(i)} v(i) = wL_A(i)^j$, a positive amount of labor would be devoted to R&D activities and the market would be in equilibrium. The above points can be summarized as follows:

$$
\nu(i) \leq \frac{wL_A(i)}{\mu(i)} \quad \text{with equality whenever } L_A(i) > 0. \tag{14}
$$

Thus, if the profitability of R&D is positive, (14) holds with equality, and if not, it holds with inequality and $L_A(i) = 0$. The former case presents endogenous growth with positive economic growth rate, and the latter with no growth or poverty traps.
First, we assume that the former case depicts the steady state with positive long-run growth. Under this assumption, the time differentiation of (14) is calculated as

$$r + \mu(i^+) = \frac{\pi(i)}{v(i)} + g_v(i). \quad (15)$$

Substituting (6), (13) and (14) into (15), we obtain

$$r + \mu(i^+) = \frac{\mu(i) \left( \frac{1}{\alpha} - 1 \right) \sum q(i)^{1-\alpha}}{(1 - \alpha) \sum L_A(i)} + v(i). \quad (16)$$

We assume that the probability of innovation success follows

$$\mu(i) = \xi \frac{Q L_A(i)}{q(i)^{1-\alpha} L}. \quad (17)$$

where it should be noted that the aggregate labor supply $L$ corresponds to the population. This arrangement implies that the whole quality of intermediate goods $Q$ has positive effects, the sector’s quality has negative effects and the scale of economy, which is assumed to be captured by population $L$, has negative effects on creation of the creation of the newest quality. This function is assumed to be linear and positively related to the R&D input rate for labor, $L_A(i)/L$, and negatively related to the product relative quality, $q(i)^{1-\alpha}/Q$.

We assume symmetric equilibrium for intermediate goods sector. Using $g_v(i) = n$ in a steady state, which can be obtained from (13), (16) in a steady state is produced as

$$\mu = \xi l + n - r, \quad (18)$$

where $l \equiv L_Y/L$ is the rate of labor division on final goods production.

(17) and (18) yields

$$L_A(i) = \frac{q(i)^{1-\alpha} L}{\xi Q} \left[ \frac{\xi l + n - r}{\alpha} \right]. \quad (19)$$

Aggregating (19) about $i$ yields the aggregate R&D spending, denoted by $L_A$, as

$$L_A = (1 - l)L = \int_0^N L_A(i) di = L \left[ \frac{1}{\alpha} l - \frac{r - n}{\xi} \right]. \quad (20)$$
Hence, $L_A$ is proportional to $L$ for a given variable $r$. (20) immediately yields the relationship between $r$ and $l$ as follows:

$$r - n = \left\{ \frac{1}{\alpha} + 1 \right\} l - 1 \xi, \quad \text{or} \quad l = \frac{1}{1 + \alpha} \left( \frac{r - n}{\xi} + \alpha \right). \quad (21)$$

Uniting (18) and (21), we obtain the following equilibrium $\mu$ as a function of $l$

$$\mu = \xi(1 - l). \quad (22)$$

Thus, the innovation probability of the economy demonstrates a linearity relationship against the rate of labor input on R&D activity.

### 3 Dynamics and Steady State

From the dynamics of $Q$ defined in (9), the increment of the $i$th sector’s innovation $R(i) \equiv q(i^+)^{\frac{\alpha}{\alpha - 1}} - q(i)^{\frac{\alpha}{\alpha - 1}}$ is calculated as $R(i) = q(i)^{\frac{\alpha}{\alpha - 1}}(\lambda^{\frac{\alpha}{\alpha - 1}} - 1)$. Therefore, the aggregate dynamics of $Q$ are

$$E(\dot{Q}) = \int_0^N R(i) di = \mu(\lambda^{\frac{\alpha}{\alpha - 1}} - 1)Q \quad (23)$$

From (22) and (23), the dynamics of $Q$ as a function of $l$ are derived as follows:

$$g_Q = \mu(\lambda^{\frac{\alpha}{\alpha - 1}} - 1) = \xi(1 - l)\Lambda, \quad (24)$$

where $\Lambda \equiv \lambda^{\frac{\alpha}{\alpha - 1}} - 1 > 0$ and $g_Z \equiv \dot{Z}/Z$. Since $\partial \Lambda/\partial \lambda > 0$, $\Lambda$ is the parameter that immediately captures the scale of one innovation.

From (14) and (17), the aggregate value of R&D firms $V$ is calculated as

$$V = \int_0^N v(i) di = \int_0^N \frac{wL_A(i)}{\mu(i)} di = \frac{(1 - \alpha)Y}{\xi l}, \quad (25)$$

where we use $w = (1 - \alpha)Y/(IL)$ for this derivation. Since we assume symmetric equilibrium about household, and only the asset of this economy is the equity of R&D firms, the per capita asset holding $A$ is denoted as

$$A = \frac{V}{L} = \frac{(1 - \alpha)Y}{\xi l}. \quad (26)$$
A substitution of $y$ from (10) into (26) yields

$$A = \frac{1 - \alpha}{\xi} - \alpha \frac{2\alpha + \eta}{1 - \alpha} Q. \quad (27)$$

This equation implies that asset holding in the economy is proportionally related to the technological level, and it grows at the same rate as the quality index. (12) implies that the per capita consumption $c \equiv C/L$ grows at the same rate as the quality index. Thus, the steady state, wherein all variables grow at constant rates and the Euler equation (4) is satisfied, can exists.

Taking $C = cL$ into account, the time differential on (12) and (26) provides the following steady state growth rate:

$$g_{c} = g_{y} - n = g_{y} = g_{A} = g_{Q}. \quad (28)$$

Substituting (12), (26), and (28) into (4), we obtain

$$\sigma g_{Q} = r - n - \rho + B(1 + \alpha) \xi l, \quad (29)$$

where $B \equiv \beta/(1 - \beta)$. Since $\partial B/\partial \beta > 0$, this parameter can be regarded as capturing the intensity of wealth effects.

Substituting (21), (22) and (24) into (29), we can analytically obtain the equilibrium division of labor to production,

$$l^{*} = \frac{\rho \xi + 1 + \sigma \Lambda}{(\frac{1}{\alpha} + 1) + B(1 + \alpha) + \sigma \Lambda}, \quad (30)$$

and the growth rate of the economy,

$$g_{y}^{*} = g_{Q}^{*} = \frac{\frac{1}{\alpha} + B(1 + \alpha) - \frac{\rho}{\xi}}{(\frac{1}{\alpha} + 1) + B(1 + \alpha) + \sigma \Lambda} \Lambda. \quad (31)$$

(31) implies that a higher efficiency of R&D, $\xi$ and $\Lambda$, and a lower subjective discount rate, $\rho$, accelerate the growth rate. These properties are shared with the usual R&D-based growth model. Regarding the wealth effects, $B$, we have the following proposition.

**Proposition I** The wealth effects increase the economic growth rate.

Proof: Differentiating (31) by $B$, we immediately obtain $\frac{\partial g_{y}^{*}}{\partial B} > 0$. (Q.E.D.)
**Condition for long-run positive endogenous growth**  We have assumed the positive profitability of R&D, namely holding equality with (14). \( l \in (0, 1) \) is necessary for the steady state obtained above to be a feasible equilibrium that is consist with the positive R&D investment. Since (30) shows that \( l^* > 0 \) constantly holds, the restriction is eventually determined to be \( l^* < 1 \), which yields

\[
B > \frac{1}{1 + \alpha} \left( \frac{\rho}{\xi} - \frac{1}{\alpha} \right). \tag{32}
\]

Namely, the economy with sufficiently high \( \xi \) has a balance growth path with a positive growth rate for all \( B \); however, a sufficiently high \( B \) is necessary for the economy with low \( \xi \) to achieve positive long-run growth. Since \( B \) represents the intensity of the wealth effects, we can sum up the following proposition about the relationship between wealth effects and no growth traps.

**Proposition II**  If an economy has low R&D efficiency, sufficiently high wealth effects captured by \( B \) are necessary for positive endogenous growth.

If this condition is not met, the economy would be caught in poverty traps.

### 4 An Extension: Economy with Capital

In this section, we add an extension of the basic model developed in the previous part of the study. We introduce capital accumulation into the economy, and then, illustrate the robustness of the main results derived in the basic model. For this purpose, we restrict our concern to the steady state analysis.

#### 4.1 Production and R&D Activities

Following in an *à la* Romer (1990) manner, we introduce capital into the basic model. Namely, the new arrangement is that one unit of intermediate goods is made by \( \eta \) units of durable goods, instead of final goods, and we call the durable goods capital. Then, final goods are used as consumption goods \( C \), and are accumulated as the capital goods \( (K) \); therefore, \( Y = C + K \).

Thus, the production function of final goods \( Y \) and the FOCs of production are the same as (5) and (6), respectively. In this section, the intermediate
goods of production are generated by \( \eta \) unit of final goods. Hence, the firm producing the \( i \)th intermediate good maximizes the profit such that

\[
\pi(i) = p(i)x(i) - \eta r x(i). \tag{33}
\]

The FOC of the monopoly firm in the \( i \)th sector with the \( M_i \)th quality yields the following:

\[
x(i) = \left[ \frac{\alpha^2}{\eta r} \right]^{\frac{1}{1-\alpha}} L_Y q(i)^{\frac{\alpha}{1-\alpha}}, \quad \text{and} \quad p(i) = \frac{\eta r}{\alpha}. \tag{34}
\]

The structure of intermediate goods sectors is essentially same as the model without capital. We can also use the same quality index defined in (9). Here, we introduce a new variable \( K \), which denotes aggregate capital accumulation. Aggregating (34) across sectors yields

\[
K = \eta \int_0^N x(i)di = \eta \int_0^N x(i)di = \eta \left[ \frac{\alpha^2}{\eta r} \right]^{\frac{1}{1-\alpha}} L_Y Q. \tag{35}
\]

Therefore, we obtain the interest rate \( r \) as

\[
r = \alpha^2 \eta^{-\alpha} K^{\alpha-1} L_Y^{1-\alpha} Q^{1-\alpha} \tag{36}
\]

Eliminating \( r \) and \( x(i) \) from (34) by using (35) and (36) yields

\[
Y = \eta^{-\alpha} K^\alpha L_Y^{1-\alpha} Q^{1-\alpha} \quad \text{or} \quad y = \eta^{-\alpha} k^\alpha (Q l)^{1-\alpha}, \tag{37}
\]

where \( k \equiv K/L \) denotes per capita capital stock.

By using (5), (33) and (34), the profit of the \( i \)th sector monopoly firm with a patent of quality \( M_i \) is obtained as follows:

\[
\pi(i) = \left( \frac{1}{\alpha} - 1 \right) \eta r x(i)
\]

\[
= (1 - \alpha) \alpha \frac{Y}{Q} q(i)^{\frac{\alpha}{1-\alpha}}. \tag{38}
\]

Because R&D structure is assumed to be same as the basic model, obtained equilibrium conditions are shared by those of the basic model except for the determination of interest rate \( r \) given in (36) and profit of R&D \( \pi(i) \) derived as (38).
The counterparts of (18) - (21) in this version are respectively given as

\[ \mu = \alpha \xi l + n - r, \]  
(39)

\[ L_A(i) = \frac{q(i)^{\alpha} L}{\xi Q} (\alpha \xi l + n - r), \]  
(40)

\[ L_A = (1 - l) L = \int_0^N L_A(i) di = L \left[ \alpha l - \frac{r - n}{\xi} \right], \]  
(41)

\[ r - n = \{(1 + \alpha)l - 1\} \xi, \quad \text{or} \quad l = \frac{1}{1 + \alpha} \left( \frac{r - n}{\xi} + 1 \right). \]  
(42)

The introducing capital doesn’t change (22) - (25) on the other hand.

### 4.2 Steady State

Because the model studied here contains capital accumulation, the system has transition path. However, we concentrate our analysis on long-run steady state.

We assume symmetric equilibrium about household, and assets of this economy consist of equity of R&D firms \( V \) derived in (25) and capital stock \( K \), therefore the per capita asset holding \( A \) in this case is denoted as

\[ A^* = \frac{V^* + K^*}{L^*} = \left[ \frac{(1 - \alpha) r^*}{\xi l^* \alpha^2} + 1 \right] k^*, \]  
(43)

where we use \( r = \alpha^2 y/k \), which can be derived from (36).

The resource constraint of the final goods \( Y = C + \dot{K} \) gives \( c = y - \dot{k} - nk \). From this, (36), and (37), we obtained the followings:

\[ c^* = \frac{r}{\alpha^2} k^* - g_k^* k^* - n k^* = \left( \frac{r^*}{\alpha^2} - g^*_q - n \right) k^*, \]  
(44)

\[ g_c^* = g_k^* = g_y^* = g_Y^* = g^*_Q \]  
(45)

From (22) and (39), we can give the steady state interest rate \( r^* \) as a function of labor allocation rate \( l^* \) as

\[ r^* = (1 + \alpha) \xi l^* + n - \xi. \]  
(46)

Substituting (24), (42), (43), (44), (45) and (46) into (4), and using the notation \( \chi \), we can obtain the equilibrium condition about \( l \) as

\[ \chi = \Gamma(l) = \frac{1}{B} \left[ -\{\alpha \Lambda + (1 + \alpha)\} \xi l + \alpha \xi \Lambda + \xi + \rho \right]. \]  
(47)
The definition of $\chi$ yields
\[ \chi = \chi(l) \frac{(1 + \alpha + \alpha^2 \Lambda)\xi l + (1 - \alpha^2)n - (1 + \alpha^2 \Lambda)\xi}{1 - \alpha (n - \xi) + l}. \] (48)

These two equations determine the $l$ in the steady state. (47) and (48) are depicted in Figure 1. (See Appendix A2 for detail derivations.) From Figure I (and the discussion in Appendix A2), increase of $B$ gins up the growth rate in the steady state through decrease of $l^*$. Thus, we can obtain the equivalent of Proposition I as follows:

**Proposition I’** The wealth effects increase economic growth rate.

Proof: Since $\frac{\partial g}{\partial l} < 0$, proving $\frac{\partial g}{\partial B} > 0$ is equivalent to proving $\frac{\partial l}{\partial B} < 0$. $\frac{\partial l}{\partial B} < 0$ is provided in Appendix II. (Q.E.D.)

$l \in (0, 1)$ is also necessary for the steady state with positive long-run growth. From the discussion in Appendix 2, the condition is $\chi(1) > \Gamma(1)$, which is transformed into
\[ B > \frac{\alpha \xi + n(1 - \alpha)}{\alpha \xi + n(1 - \alpha^2)} \left( \frac{\rho}{\xi} - \alpha \right). \] (49)

Because $\Delta$ is positive, the determination on positive long-run growth depends on $(\rho/\xi) - \alpha$; therefore, the essentially akin mechanism between $\rho$ and $\xi$ is obtained as follows.

**Proposition II’** If the economy has low R&D efficiency, sufficiently high wealth effects captured by $B$ are necessary for positive endogenous growth.

5 Conclusion

This paper demonstrates the relationship between long-run growth, realized by endogenous technological progress, which is captured by activities intended to improve for quality of intermediate goods, and the "spirit of capitalism," which is captured by the preference for wealth accumulation. If the efficiency of innovation is low, large parameter of the spirit is necessary for long-run positive growth, and the growth rate positively depends on the parameter. Thus, a combination of the spirit of capitalism and innovation...
affects the long-run growth of an economy; thus, this paper can be considered as a trial that incorporates the culture factor into an orthodox R&D-based growth model.

There are some topics left to be examined. The determination mechanism of the intensity of the wealth effects remains because we assume it constantly given. These cultural, as well as technological, properties vary based on regional, racial, and historical differences. Last, but not least, one important factor that the present study ignores is the monetary effect. Since the transmission mechanism of money demand and supply on the economy are so complicated and controversial, we concentrated our analysis on the real effects. Importing of these factors will constitute the future agenda for this study.

Appendix

A1 Optimization on the household with wealth effects

The representative agent in this economy is assumed to have the utility (1). The optimal policy for the representative agent is to maximize (1) under the constraint of (2). The Hamiltonian is given as

\[ H(t) = u(c(t), A(t)) + \lambda(t) \{ r(t)A(t) + w(t) - c(t) - nA(t) \}. \]

and we obtain the two following first order conditions:

\[ \frac{\partial H(t)}{\partial c(t)} = \frac{\partial u(c(t), A(t))}{\partial c(t)} + \lambda(t)(-1) = 0, \]

\[ \rho \lambda(t) - \dot{\lambda}(t) = \frac{\partial H(t)}{\partial A(t)} = \frac{\partial u(c(t), A(t))}{\partial A(t)} - \lambda(t)(r(t) - n). \]

The transversality condition is given as follows:

\[ \lim_{t \to \infty} \lambda(t)A(t) = 0. \]

From these conditions, we obtain the Euler equation as follows:

\[ \rho - \frac{\dot{\lambda}(t)}{\lambda(t)} = r(t) - n + \frac{u_{A}(c(t), A(t))}{u_{c}(c(t), A(t))}. \]
We specify the instantaneous utility function as (3). Using this specification, the above equation is rewritten as

\[
\{\sigma + \beta(1 - \sigma)\} \frac{\dot{c}(t)}{c(t)} - \beta(1 - \sigma) \frac{\dot{A}(t)}{A(t)} + \rho = r(t) - n + \beta \frac{c(t)}{A(t)}.
\]

**A2 Determination of \( l^* \) in the economy with capital**

(47) and (48) determine the steady state labor allocation \( l^* \). We can easily verify that \( \Gamma \) is a decreasing linear function, and has a fixed point \((\bar{l}, 0)\), where \( \bar{l} \equiv \frac{1 + \alpha}{1 + \alpha + \alpha^2} (> 0) \). \( \bar{l} > 1 \) holds for sufficiently small \( \xi \) \((\xi < \rho/\alpha)\), and \( \bar{l} \in (0, 1) \) holds for sufficiently large \( \xi \) \((\xi > \rho/\alpha)\). As \( \xi \) can be the efficiency of R&D, the former case corresponds to the economy with low R&D efficiency, and the latter case corresponds to the one with high R&D efficiency.

\( \chi(l) \) is a non-linear function with \( \chi(0) = 0 \) and \( \chi(1) > 0 \). From the definition of \( \chi \), we have two feasible conditions on \( c^* \) and \( A^* \), which are derived as

\[
c^* > 0 \implies l > \frac{(1 + \alpha^2 \Lambda)\xi - (1 - \alpha^2)n}{(1 + \alpha + \alpha^2 \Lambda)\xi} (\equiv l_c),
\]

\[
A^* > 0 \implies l > \frac{1 - \alpha}{\xi} (\equiv l_A).
\]

Therefore, the steady state conditioned by (47) and (48) must satisfy \( l > \max\{l_c, l_A\} \). Since \( l_c - l_A = \frac{\alpha^2(1 + \alpha)\xi + (1 - \alpha)n}{(1 + \alpha + \alpha^2 \Lambda)\xi} > 0 \), we obtain the condition about the steady state labor allocation \( l^* \) as \( l^* > l_c \equiv \bar{l} \).

We have two cases: \( l \in (0, 1) \) and \( l < 0 \). Under the case of \( l < 0 \), \( l_c < 0 \) produces \((1 + \alpha^2 \Lambda)\xi - (1 - \alpha)n < 0 \). Uniting this and \( \Lambda > 1 \), we obtain \((1 + \alpha^2)\xi - (1 - \alpha^2)n < 0 \). Therefore, \( \xi < \frac{1 - \alpha}{1 - \alpha^2}n < n \) holds. Noting \( n - \xi > 0 \), differentiating \( \chi(l) \) produces

\[
\frac{d\chi(l)}{dl} = \chi(l) \left[ \frac{(1 + \alpha + \alpha^2 \Lambda)\xi}{(1 + \alpha + \alpha^2 \Lambda)\xi l + (1 - \alpha^2)n - (1 + \alpha^2 \Lambda)\xi} + \frac{\frac{1 - \alpha}{\xi}(n - \xi)}{\frac{1 - \alpha}{\xi}(n - \xi) + l} \right] > 0.
\]

\(^4\chi(1) > 0 \) is immediately proved as follows:

\[
\chi(1) = \frac{\alpha\xi + (1 - \alpha^2)n}{\frac{1 - \alpha}{\xi}(n - \xi) + 1} = \frac{\alpha\xi + (1 - \alpha^2)n}{(1 - \alpha)n + \alpha\xi} > 0.
\]

\(^5\)It is easily verifiable that \( l > 1 \) is infeasible under the assumption of a non-negative population growth rate because \( l_c > 1 \) is made as \(-\alpha \xi > (1 - \alpha)n \).
Thus, function $\chi(l)$ is an increasing function in $l \in (0, 1)$.

Under the case of $l \in (0, 1)$, $\chi(l)$ is defined on $l \in (l, 1)$ because of $l_A > 0$, which yields $n - \xi > -\frac{\xi}{1 - \alpha} l$. Noting this relationship, differentiating $\chi(l)$ yields

$$
\frac{d\chi(l)}{dl} > \chi(l) \left[ \frac{(1 + \alpha + \alpha^2\Lambda)\xi}{(1 + \alpha + \alpha^2\Lambda)\xi l + (1 - \alpha^2)n - (1 + \alpha^2\Lambda)\xi} - \frac{\xi l}{1 - \alpha} \left\{ \frac{1 - \alpha}{\xi} (n - \xi + l) \right\} \right]
$$

$$
= \chi(l) \frac{\alpha^2 \{(1 - \alpha)\Lambda n + (1 + \alpha\Lambda)\xi\}}{\{(1 + \alpha + \alpha^2\Lambda)\xi l + (1 - \alpha^2)n - (1 + \alpha^2\Lambda)\xi\} \left\{ \frac{1 - \alpha}{\xi} (n - \xi + l) \right\} } > 0.
$$

(51)

(50) and (51) demonstrate that $\chi(l)$ is increasing in the domain of $l$ in both cases; therefore, the two functions in the two cases of $\bar{l} > 1$ and $\bar{l} \in (0, 1)$ are depicted in Figure 1 (a) and (b), respectively.\(^6\)

Noting the increasing property of $\chi(l)$, a totally differentiation of (47) yields

$$
\sigma \xi (-1) \Lambda dl^* = B\chi^* \frac{\partial \chi^*}{\partial l^*} dl^* + \chi^* dB + (1 + \alpha) \xi dl^*,
$$

therefore,

$$
\frac{dl^*}{dB} = -\chi^* \left[ \sigma \xi \Lambda + (1 + \alpha) \xi + B\chi^* \frac{\partial \chi^*}{\partial l^*} \right]^{-1} > 0.
$$

(52)

Thus, we can conclude that $dl^*/dB < 0$ always holds, which can also be confirmed by Figure 1, and is summarized in Proposition I'.

Next, we seek the condition on the relationship between positive long-run growth and the spirit of capitalism captured by $B$. As is depicted in Figure 1, if $\Gamma(1) < \chi(1)$, the economy has inner equilibrium $l^*$ and if not, corner solution $l^* = 1$ is an equilibrium. Therefore, the condition that the economy has a positive long-run growth is $\Gamma(1) < \chi(1)$, it can be transformed into (49), and it is the counterpart of the capital accumulation version of (32). In

\(^6\)In drawing panel (b), we use $\bar{l} > \bar{l}$ for all $\bar{l}$. The case of $\bar{l} < 0$ is trivial because $\bar{l} > 0$. Moreover, the case of $\bar{l} \in (0, 1)$ is also illustrated by the following simple calculation:

$$
\bar{l} - \bar{l} = \frac{\alpha^2 (1 - \alpha)\Lambda}{(1 + \alpha + \alpha^2\Lambda)(1 + \alpha + \alpha\Lambda)} + \frac{\rho}{\xi (1 + \alpha + \alpha^2\Lambda)} + \frac{(1 - \alpha^2)n}{\xi (1 + \alpha + \alpha\Lambda)} > 0.
$$
the case of $l \in (0, 1)$ (represented in Figure 1 (b)), it is trivial that an inner solution $l^*$ always exists because $\lim_{l \to 0} \chi(l) = 0$ for $l < 0$ and $\lim_{l \to 1} \chi(l) = 0$ for $l \in (0, 1)$. Summing up these results, we obtain Proposition II'.

References


Upper bound of $\Gamma(l)$ for endogenous growth

Case of $l > 1$

Case of $l < 1$

Figure 1: The Determination of $l^*$