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A Comment on Section 4 of D.P.1002

by

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A comment on Section 4 of D. P. 1002.

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Abstract. In this paper the author considers the same problem as that in Section 4 of D. P. 1002 (Nogami(2002)) and shows an improved procedure of testing the hypothesis \( H_0 : \theta = \theta_0 \) and the alternative hypothesis \( H_1 : \theta \neq \theta_0 \) with some constant \( \theta_0 \).

Comment. Let us consider the same problem as that in Section 4 of Nogami(2002). Let \( X_1, \ldots, X_n \) be a sample of size \( n \) taken from (1) of Nogami(2002). Let \( t_0 = t_1 + t_2 \) and \( c = t_1 - t_2 \) (\( c > 0 \)) as in p. 2 of Nogami(2002). Let \( X_{(1)} \) be the 1-th smallest observation of \( X_1, \ldots, X_n \) and define \( Y = 2^{-1} (X_{(1)} + X_{(n)} - t_0) \).

Instead of \( T \) in (6) of Nogami(2002) we use

\[
(1) \quad S = \frac{(Y - \theta)}{\{(n+1)(n-1)^{-1}Z/2(n+1)(n+2)\}}
\]

where \( (n+1)(n-1)^{-1}Z \) is an unbiased estimate for \( c \). Making a variable transformation \( S = \sqrt{(n+1)(n+2)} T \) for \( h_r(t) \) in p. 7 of Nogami(2002) we obtain

\[
h_r(s) = \sqrt{(n+1)/(2(n+2))} \{(n-1)^{-1}/2(n+1)/(n+2)|s|+1\}^{-\delta}, \quad \text{for } 0 < |s| < \infty.
\]

Let \( \delta \) be a real number such that \( 0 < \delta < 1 \). We call \((U_1, U_2)\) a \((1-\delta)\) interval estimate for the parameter \( \gamma \) if \( P_r [ U_1 < \gamma < U_2 ] = 1-\delta \). To get the conditional (or restricted) minimum-length \((1-\delta)\) interval estimate for \( \delta \) we shall find real numbers \( r_1 \) and \( r_2 \) (\( r_1 < r_2 \)) which minimize \( r_2-r_1 \) subject to

\[
(2) \quad P_r [ r_1 < S < r_2 ] = \gamma. \quad h_r(s) \ ds = 1-\delta.
\]

Letting \( \epsilon \) be a real number we define
\[ r_1 \]

\[ L = r_2 - r_1 - \int \{ h_2(s) \ ds \mid s < 1 \}. \]

By Lagrange's method, \( \text{d}L/\text{d}r_1 = 0 = \text{d}L/\text{d}r_2 \), which leads to

(3) \[ h_3(r_1) = h_3(r_2)(= e^{-1}). \]

Since \( \text{d}L/\text{d}r_1 = 0 \) is equivalent to (2), (2) and (3) leads to \( r_2 = r_1 (= r) \). Taking

\[ \int h_3(s) \ ds = e/2 \]

we have

\[ r = (n-1)/(n+2)/(2(n+1)) \left( a^{-1/(n-1)} - 1 \right). \]

Thus, in view of (1) and (2) the conditional minimum-length \((1-\alpha)\) interval estimate for \( \theta \) is as follows:

(4) \[ (Y_r - X_r)/\left\{ \sqrt{\frac{n+1}{2(n+2)(n-1)}} \right\}, Y_r + X_r/\left\{ \sqrt{\frac{n+1}{2(n+2)(n-1)}} \right\} \]

Therefore, to test the hypothesis \( H_0 : \theta = \theta_0 \) versus the alternative hypothesis \( H_1 : \theta \neq \theta_0 \) we invert (4) with respect to \( \theta = \theta_0 \) and get the following acceptance region of our two-sided test.

(5) \[ -r < (Y_r - X_r)/\left\{ \sqrt{\frac{n+1}{2(n+2)(n-1)}} \right\} < r. \]

Here, since \( \lim_{x \to -\infty} (x^3)/(2(x+2)) = 2^{-1/2} \) and \( \lim_{x \to -\infty} x(e^{-1/x} - 1) = -\log_e e \), it follows that as \( n \to \infty \), \( r \to -2^{-1/2}\log_e e \). Hence, the acceptance region (5) is more natural than that with \( t_0 \) appeared on the 5-th line from the bottom in p. 7 of Nogami(2002).
REFERENCES.
Nogami, Y. (2002). Hypothesis testing based on Lagrange's method: Application to the uniform distribution (II)., Discussion Paper Series No. 1002, Institute of Policy and Planning Sciences, University of Tsukuba, August.