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Abstract

This paper considers a bicriteria model to locate a semi-obnoxious facility within a convex polygon, while employing Euclidean push and pull covering criteria. The partial covering context is introduced into an ordinary bicriteria location framework. Although both objectives are neither concave nor convex, low complexity polynomial algorithms to find the efficient solutions and the tradeoffs involved are developed with the help of higher-order Voronoi diagrams. Comparing the tradeoff for the full covering with the others enable decision makers to understand what to extent the maximin and minimax criteria are improved at the expense of uncovering. This is illustrated via numerical examples.

Keywords: partial covering; efficient location; tradeoff curve; higher-order Voronoi diagram; semi-obnoxious facility

1 Introduction

For the last decade, the need for the location of *semi-obnoxious* facilities such as sewage treatment plants, landfills, waste incineration plants, airports and cemeteries, i.e., incorporating both attractive and noxious aspects has been acknowledged in the literature. This need is particularly acute today because emphasis shifts from seeking for the social welfare to evaluating the individuals' environment. Indeed, one of the most difficult tasks faced by governments today is to impose semi-obnoxious facilities on unwilling inhabitants. These facilities are by consensus necessary for development and growth due to their positive external effects. However, they also burden the local communities where they are located with negative external effects: see Lesbirel (1998), Quah and Tan (2002). These facilities are also called *NIMBY* (not-in-my-backyard) facilities. Such positive and negative external effects can be regarded as pulling and pushing forces. These push and pull forces conflict of course, no single location can be optimal for both measures using each force. Accordingly, a bicriteria approach has to be adopted in decision-making for the location of such facilities, as advocated by Carrizosa and Plastria (1999).

In general maximin and minimax criteria have often been used to formulate such pushing and pulling forces. In such models, only extreme distances determine the objective functions' values. This means that a small minority may affect the objective functions in similar way as a large majority. Hence, in siting semi-obnoxious facilities, many types of negotiation and compensation through the transfer of benefits from host populations to a minority have been done in the form of monetary or non-monetary means. For example, the governments have either: 1) often resettled some inhabitants; 2) bought off intense opposition; 3) set up more facilities such as parks, swimming pools, landscaping and clinics in the neighborhood of the semi-obnoxious facilities; 4) improved roads, schools, and public facilities; or 5) protected their property values for the neighbors. On the other hand, many governments today face severe financial problems.

It often seems to be difficult to provide the same service to all inhabitants equally. So some inhabitants far from the facility have been served in different manners such as alternative facilities, or they have obtained subsidies and tax rebates in compensation for the local unavailability of the facility. Thus, real-world location problems may require to take account of only part of the inhabitants. Therefore, rather than full covering formulations which have extensively been introduced in past works, their partial-covering version may be more appropriate for semi-obnoxious facility location. The aim of this paper is to present a polynomial-time algorithm for analytically tracing out the efficient solutions and the tradeoffs within push-pull partial covering context.

Our model is related to several works pertaining to a partial-covering point of view. Brimberg and ReVelle (1999), Carrizosa and Plastria (1998), Daskin and Owen (1999) allowed unserved inhabitants to locate purely *desirable* facilities. In Daskin and Owen (1999), such location models are called *partial location problems.* Inversely, Drezner and Wesolowsky (1994), Muñoz-Pérez and Saameño-Rodríguez (1999), Plastria and Carrizosa (1999) allowed some inhabitants to stay in the neighborhood of purely *undesirable* facilities to be constructed. To date, these existing works considered the two aspects separately, either inhabitants attracting the facility or inhabitants repelling the facility. Independent consideration of these two aspects may result in inefficient outcomes.

In contrast with these works, our analysis deals with *semi-obnoxious* facilities which attract and repel the inhabitants at the same time. Hence, we have to consider simultaneously two types of negotiation in order to lessen opposition by affected minority groups. The first negotiation is used to move out some affected inhabitants who oppose locating a facility in their neighborhood. The second negotiation is used to withhold the service of the facility from some inhabitants. Thus, our formulation attaches great importance to the major group, while it may not necessarily take a serious view of some minor groups, so we hope it is more suitable for real semi-obnoxious facility location decisions than the standard framework.

From the algorithmic point of view, our methodology generalizes the one by Ohsawa (2000) and that by Ohsawa and Tamura (2003). Ohsawa (2000) gave a polynomial algorithm to compute the efficient location and the tradeoff involved for a semi-obnoxious facility in which maximin and minimax Euclidean distances are simultaneously used as push and pull objectives. Ohsawa and Tamura (2003) provided a polynomial algorithm for its rectangular distance version. We obtain the efficient locations associated with the push-pull partial covering bicriteria problems by extending these two algorithms via higher-order Voronoi diagrams, which include the standard and the farthest-point Voronoi diagrams as special cases.

The remainder of this paper is organized as follows. Section 2 characterizes geometrically the solutions of partial anti-center and center problems. Section 3 shows the theoretical findings on the bicritria model generated by these two partial problems. Section 4 takes up our sample problem to better recognize the impacts of partial covering. Section 5 contains our conclusions.

2 Single-Objective Models

2.1 Formulations

Given a convex polygon Ω on a Euclidean plane where a facility can be built. Let $\partial\Omega$ be its boundary, and $|\partial\Omega|$ be the number of straight-line segments forming it. Let I and $\{\mathbf{p}_1, \cdots, \mathbf{p}_{|I|}\}$ be the index and location sets of the affected inhabitants on the plane, respectively. In what follows, the index set I is used interchangeably with the location set $\{\mathbf{p}_1, \cdots, \mathbf{p}_{|I|}\}$. To avoid the technicalities of degenerate cases, we assume that $|I| \geq 2$, and $\mathbf{p}_i$'s are in general position.

In the *partial anti-center location problem*, which is equivalent to the *maxquantile location problem* in Plastria and Carrizosa (1999), in order to mitigate opposition to locating a facility to be constructed, an exogenously specified number n^- of inhabitants in the neighborhood of the facility will be resettled farther from the facility, and it may therefore be considered that their current location will be neglected. The problem then consists of finding the point within Ω which maximizes the nearest distance from the facility to $|I| - n^-$ inhabitants. Hence, this is defined by

$$\max_{\mathbf{x}\in\Omega}\left(F_{n^-}(\mathbf{x}) \equiv \max_{J^-\subseteq I, |J^-|=|I|-n^-}\left(\min_{u\in J^-}\|\mathbf{x}-\mathbf{p}_u\|\right)\right). \tag{1}$$

Thus, the objective function $F_{n^-}(\mathbf{x})$ stands for the $(n^- + 1)$-th nearest distance from $\mathbf{x}$ to the original set I. This problem seeks for the center of the largest open circular disk enclosing n^- inhabitants. We denote its solution by $\mathbf{a}^*_{n^-}$. It should be noted that $n^- = 0$ means empty cover, or the traditional maximin problem. Our formulation is also a special case of obnoxious facility location in Muñoz-Pérez and Saameño-Rodríguez (1999).

In the *partial center location problem*, which is equivalent to the *minquantile location problem* in Carrizosa and Plastria (1998), in order to reduce the covering distance, an exogenously specified number n^+ of inhabitants far from the facility will remain unserved. The problem is to determine a point within Ω in such a way that the farthest distance from the facility to $|I| - n^+$ inhabitants is minimized. Its mathematical description is

$$\min_{\mathbf{x}\in\Omega}\left(G_{n^+}(\mathbf{x}) \equiv \min_{J^+\subseteq I, |J^+|=|I|-n^+}\left(\max_{v\in J^+}\|\mathbf{x}-\mathbf{p}_v\|\right)\right). \tag{2}$$

So the objective function $G_{n^+}(\mathbf{x})$ stands for the $(n^+ + 1)$-th farthest, i.e., $(|I| - n^+ - 1)$-th nearest distance from $\mathbf{x}$ to the original set I. In other word, in this problem the smallest closed circular disk uncovering n^+ inhabitants is determined. We denote its optimal location by $\mathbf{c}^*_{n^+}$. It should be noted that $n^+ = 0$ means full covering or the traditional minimax problem.

2.2 Geometrical Solutions

Let $I^k_1, \cdots, I^k_{t(k)}$ be all possible subsets out of I whose cardinality is k, where $t(k) \equiv \frac{|I|!}{k!(|I|-k)!}$. Let $V_{I^k_i}$ be the set of the points from which the largest distance to all $\mathbf{p}_i$'s with $i \in I^k_i$ is equal to or less than the shortest distance to the other points. Its formulation is given by

$$V_{I^k_i} \equiv \{\mathbf{x} \mid \max_{u\in I^k_i}\|\mathbf{x}-\mathbf{p}_u\| \leq \min_{v\notin I^k_i}\|\mathbf{x}-\mathbf{p}_v\|\}. \tag{3}$$

This set, when non-empty is called *order-k Voronoi polygon* associated with I^k_i. The union of all $V_{I^k_i}$'s is called the *order-k Voronoi diagram.* As special cases, the diagram satisfying $k = 1$ and $k = |I| - 1$ reduce to the ordinary Voronoi diagram and the farthest-point Voronoi diagram, respectively. It follows from (3) that the $V_{I^k_i}$'s are defined by the intersection of half-planes, so the feasible region Ω can be partitioned into bounded convex regions delimited by ∂V^k, where ∂V^k is the union of the boundaries of $V_{I^k_i}$'s within Ω. The geometrical properties of the higher-order Voronoi diagram can be found in detail in Lee (1982).

Since $F_{n^-}(\mathbf{x}) = \max_{u\in I^{n^-+1}_i}\|\mathbf{x}-\mathbf{p}_u\|$ for $\mathbf{x} \in V_{I^{n^-+1}_i}$, the level set of $F_{n^-}(\mathbf{x})$ for a fixed α within $V_{I^{n^-+1}_i}$, i.e., $\{\mathbf{x} \in V_{I^{n^-+1}_i} \mid F_{n^-}(\mathbf{x}) \leq \alpha\}$ is given by the *intersection* of the circles centered at $\mathbf{p}_u$ $(u \in I^{n^-+1}_i)$ with radius α, so any level set within $V_{I^{n^-+1}_i}$ has to be convex. The level sets of the partial anti-center problem with $n^- = 1$, and the order-two Voronoi diagram are illustrated in Figure 1, where five inhabitants $\mathbf{p}_1, \cdots, \mathbf{p}_5$ are denoted by filled circles. Accordingly, we can obtain the geometrically interesting result that the *reflex* vertices of any level set occur only along ∂V^{n^-+1}. This implies that the solution $\mathbf{a}^*_{n^-}$ has to lie at 1) a vertex of ∂V^{n^-}, or 2) a vertex of $\partial\Omega$, as implicitly pointed out by Plastria and Carrizosa (1999).

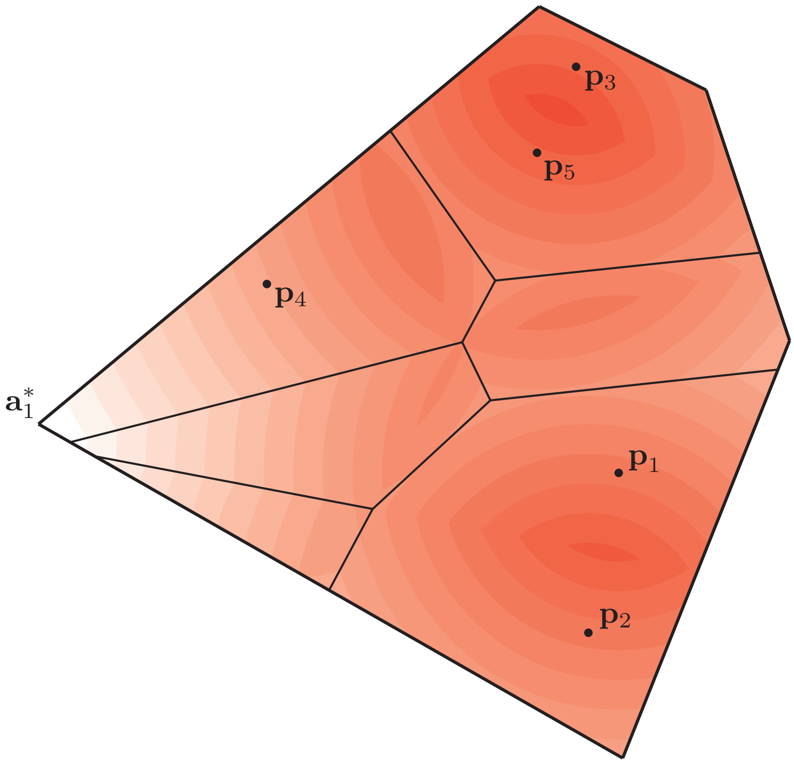

Figure 1: Partial anti-center location with $n^- = 1$ and order-two Voronoi diagram

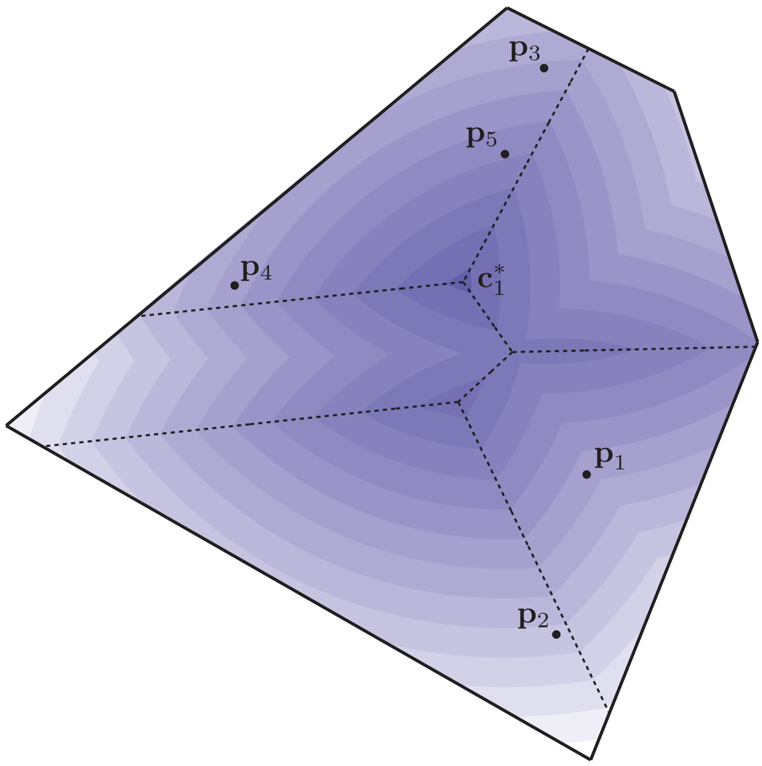

Figure 2: Partial center location with $n^+ = 1$ and order-three Voronoi diagram

It is evident from the definition of higher-order Voronoi diagrams (3) that the farthest order-k Voronoi diagram coincides with the order-($|I| - k$) Voronoi diagram. Hence, similarly as for $F_{n^-}(\mathbf{x})$, $G_{n^+}(\mathbf{x}) = \min_{u \in I - I_j^{|I|-n^+-1}} \|\mathbf{x} - \mathbf{p}_u\|$ for $\mathbf{x} \in V_{I_j^{|I|-n^+-1}}$. So the level set of $G_{n^+}(\mathbf{x})$ within $V_{I_j^{|I|-n^+-1}}$ consists of the *union* of the circles centered at $\mathbf{p}_v$ ($v \in I_j^{|I|-n^+-1}$) with the same radius. This implies that any vertex of a level set within $V_{I_j^{|I|-n^+-1}}$ cannot be cusped. Figure 2 shows the level sets of the partial anti-center problem with $n^+ = 1$, and the order-three (farthest order-two) Voronoi diagram. Hence we see that the cusped vertices of any level set occur only along a segment within $\partial V^{|I|-n^+-1}$. This implies that the solution $\mathbf{c}_{n^+}^*$ must be at 1) a vertex of $\partial V^{|I|-n^+-1}$, 2) a midpoint on $\partial V^{|I|-n^+-1}$ between two inhabitants, 3) a vertex of $\partial\Omega$, or 4) a foot of the perpendicular to $\partial\Omega$ from an inhabitant, as in Ohsawa (2000).

3 Biobjective Models

3.1 Formulation

The biobjective problem generated by combining the objective functions (1) with (2), which are both neither convex nor concave, is called the *push-pull partial covering model*. We assume that $n^- + n^+ < |I|$ because none of the neglected inhabitants can simultaneously remain unserved. As a special case, we term the problem with $n^- = n^+ = 0$ *push-pull full covering model*, which has already been explored in Ohsawa (2000).

As usual, $\mathbf{x} \in \Omega$ is efficient if and only if there does not exist $\mathbf{y} \in \Omega$ such that either 1) $F_{n^-}(\mathbf{y}) > F_{n^-}(\mathbf{x})$ and $G_{n^+}(\mathbf{y}) \leq G_{n^+}(\mathbf{x})$ or 2) $F_{n^-}(\mathbf{y}) \geq F_{n^-}(\mathbf{x})$ and $G_{n^+}(\mathbf{y}) < G_{n^+}(\mathbf{x})$. Throughout this paper, we consider the objective space with the horizontal (vertical) axis measuring the values of $F_{n^-}(\mathbf{x})$ ($G_{n^+}(\mathbf{x})$). Since the right and lower directions on the objective space are better in terms of $F_{n^-}(\mathbf{x})$ and $G_{n^+}(\mathbf{x})$, respectively, any efficient location has no alternative in southeasterly quadrant direction. Let E_{n^-,n^+}^* be the efficient set, and t_{n^-,n^+}^* be the biobjecitve values corresponding to E_{n^-,n^+}^* in objective space. Since t_{n^-,n^+}^* shows the performance of all alternatives within E_{n^-,n^+}^*, we call t_{n^-,n^+}^* the tradeoff between $F_{n^-}(\mathbf{x})$ and $G_{n^+}(\mathbf{x})$.

3.2 Geometrical Solution

Since $n^- + n^+ < |I|$, $F_{n^-}(\mathbf{x}) < G_{n^+}(\mathbf{x})$. So at any point $\mathbf{x} \in \Omega$ the curvature of the boundary of the level set of $F_{n^-}(\mathbf{x})$ through $\mathbf{x}$ is greater than that of $G_{n^+}(\mathbf{x})$ through it. Hence, based on Ohsawa and Tamura (2003), efficient solutions may only be found in $\partial V^{n^-+1} \cup \partial V^{|I|-n^+-1} \cup \partial\Omega$. Thus, we obtain the following proposition:

PROPOSITION 1 $E_{n^-,n^+}^* \subseteq \partial V^{n^-+1} \cup \partial V^{|I|-n^+-1} \cup \partial\Omega$.

When $n^- = n^+ = 0$ this proposition reduces to the result by Ohsawa (2000). In geometric terms this is equivalent to saying that the southeasterly envelope of the loci $(F_{n^-}(\mathbf{x}), G_{n^+}(\mathbf{x}))$ for the two-dimensional area Ω, that is, the tradeoff, is given by the southeasterly envelope of only the loci for the set of line-segments $\partial V^{n^-+1} \cup \partial V^{|I|-n^+-1} \cup \partial\Omega$.

As a consequence of Proposition 1, an algorithm for construction of the efficient solutions and the tradeoffs can be given by modifying the method by Ohsawa and Tamura (2003) as follows:

Step 1. Delineate the order-($n^- + 1$) and order-($|I| - n^+ - 1$) Voronoi diagrams.

Step 2. Split the links within ∂V^{n^-+1}, $\partial V^{|I|-n^+-1}$ and $\partial\Omega$ into sublinks along which the (n^-+1)-th and $(|I| - n^+ - 1)$-th nearest-point are both constant.

Step 3. Draw the loci $(F_{n^-}(\mathbf{x}), G_{n^+}(\mathbf{x}))$ in objective space for all $\mathbf{x}$ in each of these sublinks.

Step 4. Detect the south-eastward envelope of these loci.

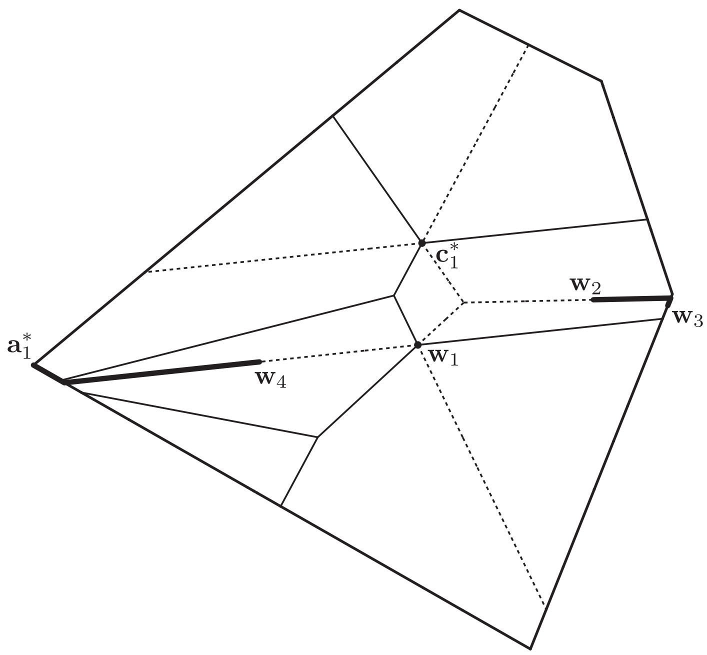

Figure 3: Efficient set for push-pull partial covering with $n^- = 1$ and $n^+ = 1$

Step 5. Specify the parts of sublinks corresponding to the envelope in geographical space.

Any higher-order Voronoi diagram for $|I|$ inhabitants can be constructed in at most $O(|I|^2 \log |I|)$ time: see Agarwal et al. (1998), so Step 1 takes $O(|I|^2 \log |I|)$. As shown in Lee (1982), the number of regions and edges of these higher-order Voronoi diagrams are both bounded by $O(|I|^2)$. In addition, any link can be partitioned into sublinks along which the (n^-+1)-th and $(|I|-n^+-1)$-th nearest-point are constant in $O(|I|^2)$ time. Hence, Step 2 can be done in $O(|I|^4 + |\partial\Omega||I|^2)$. Since Ω is convex, the planar graph $\partial V^{n^-+1} \cup \partial V^{|I|-n^+-1} \cup \partial\Omega$ has $O(|I|^4 + |\partial\Omega|)$ edges. As shown in Ohsawa (2000), $G_{n^+}(\mathbf{x})$ can be expressed analytically in function of $F_{n^-}(\mathbf{x})$ for any point $\mathbf{x}$ on a line segment l whose $(n^- + 1)$-th nearest inhabitant is $\mathbf{p}_i$ and $(|I| - n^+ - 1)$-th nearest one, is $\mathbf{p}_j$. So Step 3 runs in $O(|I|^4 + |\partial\Omega|)$ time. Step 4 requires $O((|I|^4 + |\partial\Omega|) \log(|I|^4 + |\partial\Omega|))$ effort: see Ohsawa (2000). Step 5 can be done in $O(|I|^4 + |\partial\Omega|)$ time. In total, our solution requires $O((|I|^4 + |\partial\Omega|) \log(|I|^4 + |\partial\Omega|) + |\partial\Omega||I|^2)$.

PROPOSITION 2 *The efficient set $E^*_{n^-,n^+}$ and its tradeoff $t^*_{n^-,n^+}$ can be found in $O((|I|^4 + |\partial\Omega|) \log(|I|^4 + |\partial\Omega|) + |\partial\Omega||I|^2)$ time.*

For non-convex study area Ω, our algorithm is valid, but the time complexity increases.

For the same inhabitant set ($|I| = 5$) as in Figures 1 and 2, Figure 3 shows the efficient set $E^*_{1,1}$, consisting of two points $\mathbf{c}_1^*$, $\mathbf{w}_1$, the (two-piece) path between $\mathbf{w}_2$ and $\mathbf{w}_3$, and the (two-piece) path between $\mathbf{w}_4$ and $\mathbf{a}_1^*$. Figure 4 shows the loci corresponding to the boundaries $\partial V^2 \cup \partial V^3 \cup \partial\Omega$. The boundaries ∂V^2, ∂V^3 and $\partial\Omega$ in Figure 3, and the plots corresponding to them in Figure 4 are indicated by thin, broken and thick lines, respectively. Note that the tradeoff can not be defined over the open interval $(F_1(\mathbf{c}_1^*), F_1(\mathbf{w}_1))$. This shows by example that, contrary to the full covering case (see Ohsawa, 2000)), the tradeoffs do not form a connected set.

3.3 Sensitivity Analysis

We have explored the partial problems only for fixed n^- and n^+. Undoubtedly, as more inhabitants are neglected, both objective functions (1) and (2) are improved. As a result, in the partial

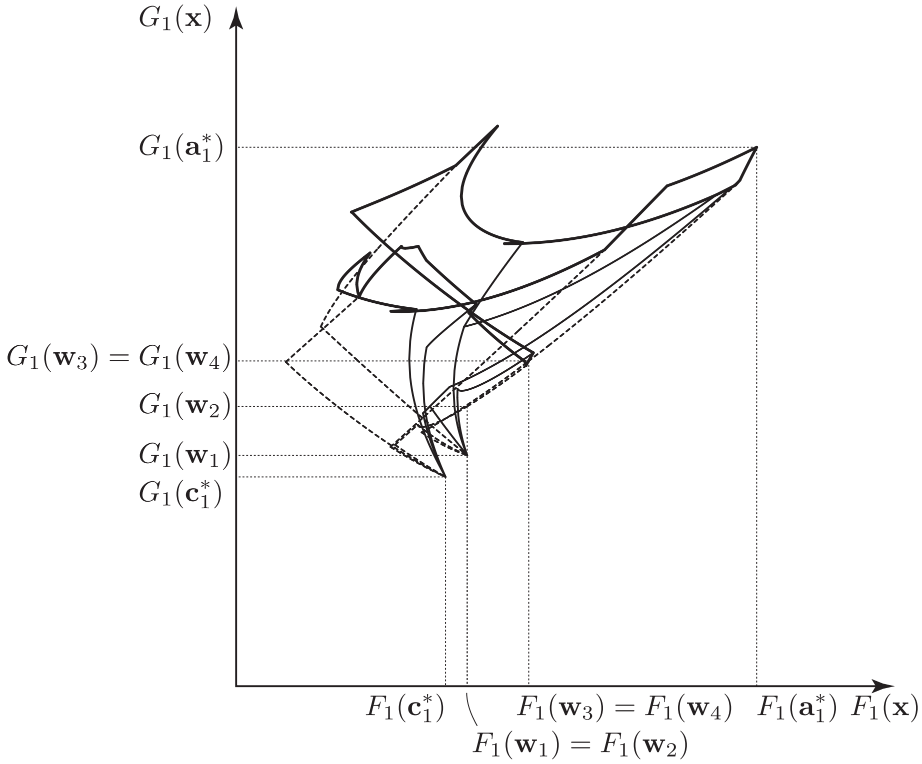

Figure 4: Tradeoff for push-pull partial covering with $n^- = 1$ and $n^+ = 1$

anti-center problem, the obnoxious level and the resettlement fee to move out the inhabitants contradict. The tradeoff between them is examined in Plastria and Carrizosa (1999). In the partial center problem, there is the tradeoff between the service level and the beneficial fee to unserved inhabitants, which is explored in Carrizosa and Plastria (1998). To see the impacts caused by varying n^- and n^+ over a narrow range, we take up following simple schemes via our example as shown in Figures 1 and 2: 1) a full covering, i.e., $n^- = n^+ = 0$; 2) one inhabitant to hate the facility is neglected, i.e., $n^- = 1$, $n^+ = 0$; 3) one inhabitant to desire the facility is neglected i.e., $n^- = 0$, $n^+ = 1$; and 4) one inhabitant to hate the facility and one inhabitant to desire it are neglected, i.e., $n^- = n^+ = 1$: see Figures 3 and 4.

The effect of varying n^- and n^+ can be observed in Figures 5 and 6, where four efficient sets E^*_{00}, E^*_{10}, E^*_{01} and E^*_{11} and their corresponding tradeoffs t^*_{00}, t^*_{10}, t^*_{01} and t^*_{11} are identified by thin, single-dot-dash, broken and solid lines, respectively. The set E^*_{00} is formed of the path between $\mathbf{c}^*_0$ and $\mathbf{s}_1$, and the one between $\mathbf{s}_2$ and $\mathbf{a}^*_0$. The set E^*_{10} consists of the path between $\mathbf{c}^*_0$ and $\mathbf{v}_1$, the path between $\mathbf{v}_2$ and $\mathbf{w}_1$, and the one between $\mathbf{v}_3$ and $\mathbf{a}^*_1$. The set E^*_{01} comprises the path between $\mathbf{c}^*_1$ and $\mathbf{u}_1$, and the one between $\mathbf{u}_2$ and $\mathbf{a}^*_0$. Therefore, it can be concluded that the efficient set changes significantly according to n^- and n^+, even though the efficient sets E^*_{00} and E^*_{01} overlap each other from $\mathbf{s}_2$ to $\mathbf{a}^*_0$.

We understand from Figure 6 that the tradeoff of partial-covering t^*_{10} (t^*_{01}, t^*_{11}) is located at the east (south, southeast) of that of full covering t^*_{00}. An increase in n^- and n^+ will shift the tradeoff to the right and the bottom, respectively, as we would expect. Thus, such sensitivity analysis within a partial covering framework enables us to perceive clearly and quickly to what extent the biobjective functions can be improved by relaxing the constraint that all inhabitants can be covered.

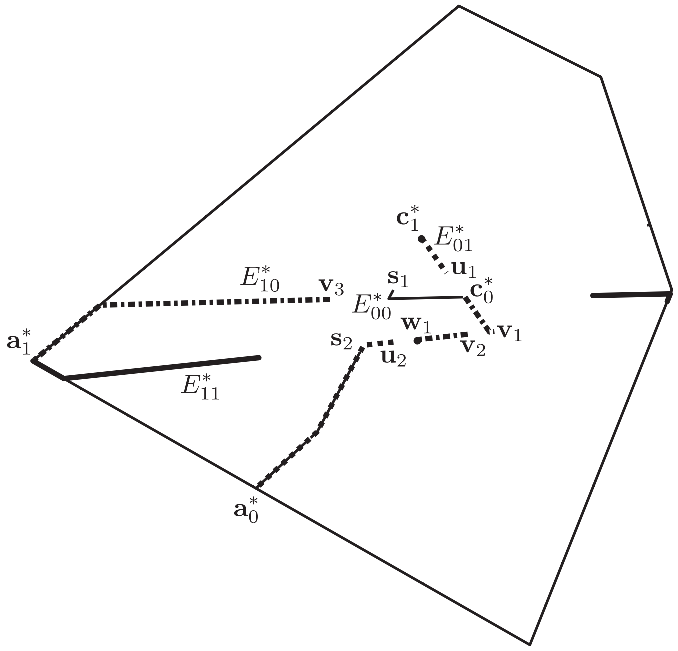

Figure 5: Four efficient sets

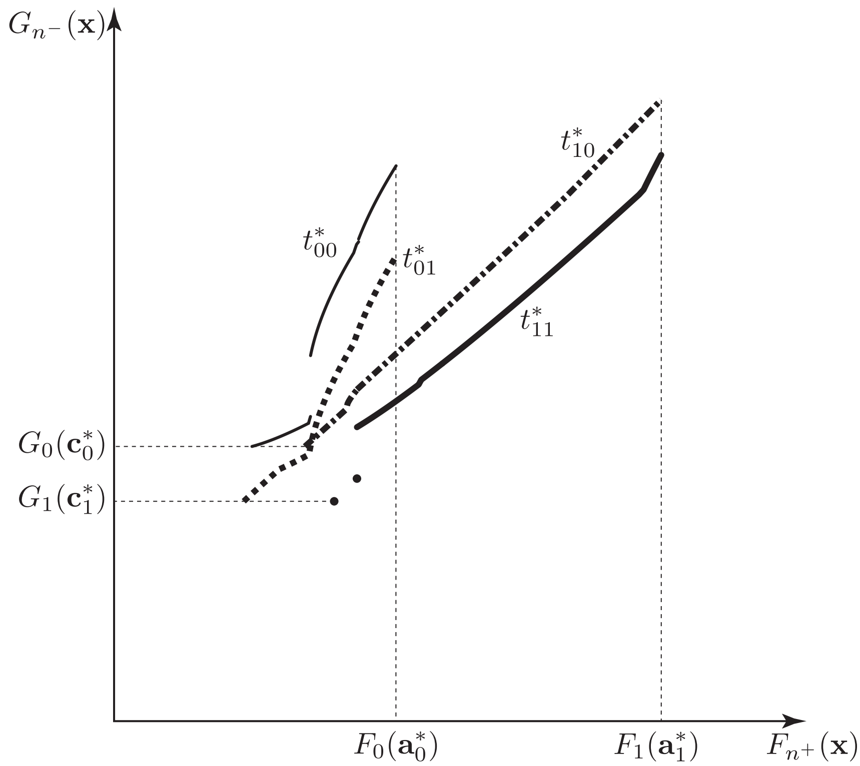

Figure 6: Four tradeoffs

4 Conclusions and Future Research

This paper has formulated a new more realistic bicriteria location problem where partial covering is introduced to model the siting of a semi-obnoxious facility. We have developed polynomial-time algorithm to delineate the efficient location and the tradeoff associated with this problem. We have shown that comparing the tradeoff for the full covering with the others enable decision makers to understand what to extent the maximin and minimax criteria are improved at the expense of uncovering through some illustrative examples.

Although a theoretical basis for the push-pull partial covering has been established, the full sensitivity analysis against all possible combination of resettled inhabitants and unserved inhabitants is possible topic for future research.

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