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Vertical Integration, Oligopoly and Welfare

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This dissertation collects my latest researches into the theory of Vertical Integration. Vertical integration is defined to be a merger (or even a collusive contract without merger) between a buyer and a seller of an intermediate product. Generally speaking, firms' activities are to transform intermediate products into a final product and some firms' final product may be an intermediate product for other firms except the products that are purchased finally by the consumers. In this sense, the problem of vertical integration is a question that most of the firms may encounter. More specifically, each firm will face the question whether or not the company should make in-house production of its input or choose to buy it on the open market for the production of its own output. One form of this in-house production is a vertical integration; any two vertically related companies - a downstream and an upstream firm may behave as a single company and seek the joint profit maximization. For example, a downstreamer may benefit from buying out, or otherwise controlling an upstramer by a binding contract, insofar as it proves to be less costly than establishing new plant. Then the question arises whether or not the outcome of this behavior in fact benefits the firm itself, and more im-
portantly if it may not harm the consumer. This two-fold question attracted many researchers which in turn produced various approaches. One approach stresses transaction costs involved in negotiating contracts among firms at different vertical levels.\textsuperscript{2} We can not neglect the impact of human nature and uncertainty upon the transaction costs. However, we do not go further to this argument.\textsuperscript{3} Instead, we focus only on the direct issues of vertical integration to answer the forementioned two-fold question.

This dissertation is organized as follows. Chapter 1 surveys the analysis of vertical integration and the related controversy on the effect of vertical integration on the consumer welfare. I find that the results of this question have much to do with the assumptions about the technology and market structures of each downstream and upstream industry. Namely, a positive result on consumer welfare is derived from a combination of the assumptions of production technology of fixed proportions type which does not allow any substitution between inputs and market structures assumption of successive

\textsuperscript{2}The transaction cost approach is emphasized by Williamson, Oliver E. "The modern corporation: origins, evolution, attributes," \textit{Journal of Economic Literature}, 19, No. 4 December 1981, 1537-68.

oligopolies in both a downstream and an upstream industry. On the other hand, a negative effect is derived from the combination of the variable proportions technology and the market structures of an upstream monopolist and a competitive or oligopolistic downstream industry. Chapter 1 generalizes the model to identify which forces are more important to determine the net effect of vertical integration on the final good price. I obtained a surprising result that none of these assumptions is responsible for the welfare result but what appears to matter is the horizontal merger effect which creeps in the model. This result is recently referred to in many models of vertical integration.4

Chapter 2 ponders the welfare effect of vertical integration on the producer’s side. This question is rooted in my economic intuition. If it is good for the consumer as I have proved in the previous chapter, there is no reason for the anti-monopoly authority to disallow the action of the vertical integration. Therefore, if this is also good for the producer as well, then most of the firm would already have vertically integrated and the entire economy would consist of one gigantic firm. But the real world is far from it. So there should be at least some forces against such movements. I explored this problem of

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how to identify the forces in the same framework and found that the answer was in the number of firms in both upstream and downstream industries. In other words, here again the horizontal effect played an important role in deciding the outcome of vertical integration. However, I considered only the incentive for integration in the level of industry i.e., I compared in chapter 2 the total profits of an industry before and after vertical integration without asking if the first-pair integration is profitable.

No story ends without starting. Somebody has to take the first action of integration in order to start the process. At the same time, this action needs to guarantee a greater profit at least for the firm taking the action. If the first-pair integration is proved to be profitable, the second question is whether there is an incentive for the second-pair, third-pair and so on to integrate vertically. Finally we will ask what is the equilibrium market structure. These are the main theme of chapter 3. We explained every possible outcome due to vertical integration only in terms of the number of upstream and downstream firms and obtained number of quite robust propositions.

We found that vertical integration yield a positive producer's welfare in very limited situations. This result approximates the real world situation
quite well. Then what is the ideal market structures for the industry? When we consider this question we come up with the Japanese business practice of Keiretsu. The Keiretsu system is very similar to the system of vertical integration in that each keiretsu company works for the parent company as if it were sacrificing its own interest for some greater cause of the keiretsu family as a whole. However such system is different from the strict vertical integration since each individual firm is an independent entity. So we may not be able to analyze this Keiretsu problem in the same framework. Probably needed is a modelling framework which is more appropriate to explain the problem. Before finding such a model, however, studying a case or two of typical keiretsu may be warranted. For this end, I choose Toyota’s case and its historical analysis is to be included in the final chapter.
ACKNOWLEDGMENTS.

It is my great pleasure to acknowledge in the first place that I owe a great deal of intellectual debt to Professor Dr. Yasuhiro Sakai. It is no exaggeration to say that his advices opened my academic life physically and spiritually. When I was a graduate student at University of TSUKUBA, I was exploring the effect of vertical integration on the final product price. Professor Sakai gave me the very important suggestion that I should separate horizontal effect from the vertical integration literature. Thanks to this suggestion, I was able to solve the controversial problem of what is the main forces to determine the welfare effect from the consumers side, and publish the paper of chapter 1 in Journal of Industrial Economics. I learned from him what the economics thinking should be and how wonderful to study Economics. I also learned the pleasure of presenting papers in international conferences. I presented a paper at the Third Annual Congress of European Economic Association in Bologna 1988 and another paper at the Far Eastern Meeting of Econometric Society in Seoul in 1991, and attended the same Far Eastern Meeting in Taipei 1993 with Professor Sakai and Professor Koji Okuguchi. Through the insightful discussions in these occasions, I really enjoyed the...
academic activities to feel the frontier of Economics thought. I am also very grateful to his wife, Mrs. Tokuko Sakai. Their constant and heartwarming supports helped the development of my research in great details.

I am also greatly indebted to Professor Hiroshi Ohta. My first paper was originally intended to generalize the result of his AER papers (joint with M. L. Greenhut 1976, 1979). I was very much attracted by his research style that explains the complicated economic phenomena using a very simple economic logic and tool. Especially I was fascinated with his vertical integration model. His model suggests that vertical integration always reduces the final product price, and guarantees the greater profit to the merging parties, therefore vertical integration is better for both consumers and producers. This conclusion seemed to me too simple to believe it true. My intuition suggested that something wrong is involved there. This is the initial motivation to tackle this problem. I started my research with this negative feeling and tried to disprove the result by relaxing the assumption he employed. However, the result I obtained in my JIE paper was the generalization of his result contrary to my initial intention. I proved in that paper that vertical integration reduces the final good price no matter what the market structures may be or regardless of the substitution possibility between inputs. This re-
sult strengthens his result from the consumer welfare point of view. Even though I proved his result, I could not wipe out my original doubt about the effect on producer welfare. This leads me to the subsequent researches which I presented in chapters 2 and 3. Thus my work has been started as a critical assessment for of Professor Ohta's work. Nevertheless, Professor Ohta is always supportive to my work. Actually, he read my paper over and over, and gave me many good suggestions besides correcting many typos. Without his support, my research would not have been started. I learned this importance of the friendly but critical mind in academic discourse between researchers from the late Professor Kei Shibata who was the common supervisor for me and Professor Ohta. The late Professor Shibata introduced me an initial motivation for academic society. Without his guide, I could not have survived in this world. In a process of writing my JIE paper, I also got a very good comments from Professor Yoshiyasu Ono and Professor Martin Bronfenbrenner. I really appreciate their supports.

Regarding my recent research development, I want to thank Professor Babu Nahata and Michael Waterson with whom I am now working. My relation with Professor Babu Nahata is especially noteworthy. Our relation started more than ten years ago when I was a graduate student at University
of Tsukuba after I raised some questions on his JPE paper (1980). I was encouraged very much through correspondence since he evaluated my JIE paper highly. We came to communicate more frequently since he was an exchange scholar at Hiroshima Shudo University during 1986-1987 and he visited Fukuoka University in 1992 as a Japan Foundation Fellow. We started a joint research project on an incentive for vertical integration. He invited Michael Waterson for the same project. Mike introduced a game theoretic flavor to our model. Concerning the game theoretic approach, we learned a lot from Professors Ko Nishihara and Koichi Suga. Stephen Martin visited Fukuoka University for summer intensive course in 1994. I received very good comments from him, too.

I presented the paper in chapter 3 at the Eighth Annual Congress of European Economic Association at the University of Helsinki in August 1993 and the twenty-first Annual Conference of the European Association for Research in Industrial Economics at Crete, Greece in September 1994. I would like to thank the participants at the conference for their comments.

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Chapter 1

Vertical Integration, Variable Proportions and Successive Oligopolies
Abstract

This paper\textsuperscript{1} investigates the problem of how the effect of vertical integration upon the final product price is related to the production and/or the market structures. Some ambiguous results of the previous models are shown to involve the compound effect of vertical and horizontal merger. Our model considers purely vertical integration and shows that the (Lerner) index of monopoly remains unaffected before and after merger. Vertical integration is shown accordingly to yield an unambiguous decrease in the final product price - an increase in consumer welfare. And also discussed are implications for antitrust laws.

\textsuperscript{1}This chapter is based on the paper by Abiru(1988)
1.1 Introduction

The theory of vertical integration can be divided into two main streams in terms of the assumptions employed and the welfare implications derived.\(^2\)

One is the stream initiated by Warren-Boulton [1974] which, allowing for substitution between inputs, shows by computer simulation that the integration of a monopolistic input supplier and a perfectly competitive final good producer will result in an increase in the final product price (welfare loss). Mallela and Nahata [1980] improve the analysis in the same framework, and show at the fully analytical level that a merger of this sort may yield a decrease in the final product price when the elasticity of substitution is less than one; otherwise, it results in an increase. From this result alone, however, we cannot deduce many relevant policy implications since the models do not consider market structures, such as oligopolies, which are empirically more likely.

The other approach represented by Greenhut-Ohta [for brevity GO, 1976, 1979] takes market imperfection into account, and advances the powerful policy implication that merger or collusion between an input supplier and a

\(^2\)Concerning the profitability of vertical integration, see Vernon and Graham [1971] and Mallela-Nahata [1980, theorem 4, p. 1022].
final good producer brings about lower prices, greater output and sales, and
greater profits to a merged or colluding firm (welfare gain to both producers
and consumers). This approach, however, is criticized in that the assump-
tions GO employed are too restrictive.\textsuperscript{3} They assume a production function
with fixed input proportions, which eliminates any possibility of substitu-
tion between inputs. (The opposing stream assumes conventional variable
proportions production functions.)

Does it follow therefore that these distinctively different set of assump-
tions are indeed responsible for the mutually opposing results of the two
contrasting models? Which one of the two conflicting assumptions, i.e., the
market structure assumption or the technology-choice assumption, is crucial
in determining the direction of change in the final product price? And which
is more relevant in formulating policy implications?

The purpose of this paper is to answer these and other related questions.
By so doing, we follow the Warren-Boulton model as to production technol-
ogy and demand conditions, but analyze the Warren-Boulton case in more
general market structures in line with GO. Similar work has been presented
by Waterson [1982], who analyzes the effect of vertical integration between

\textsuperscript{3}See Haring and Kaserman [1978].
an input monopolist and an oligopolist by using a C.E.S. production function, and shows by computer simulation that the final product price declines as long as the elasticity of substitution is fairly low; otherwise it often rises.

It may seem from this result that the welfare effect of vertical integration depends on the substitution parameter. This is not the case, however. In fact, under certain particular assumptions we can show that regardless of the elasticity of substitution or the market structure under consideration, the effect of vertical integration is to decrease the price of the final product.

The reason for the dichotomy between this and Waterson’s result can be found in the interpretation of vertical integration. Waterson takes vertical to mean in effect both horizontal and vertical. What he analyzes is a compound effect of vertical and horizontal integration. It is generally accepted that the horizontal merger increases the price of a final good.\textsuperscript{4} It follows that Waterson’s result is not dependent on the degree of substitution but rather on horizontal effects.

\textsuperscript{4}See e.g., Greenhut and Ohta[1976].
Table I

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Our model may be related to and contrasted with the existing literature as shown in Table I. They are classified in terms of underlying assumptions regarding market structures, production technologies, demand conditions and welfare implications. Table I shows on one hand that our model is not sufficiently general in terms of the demand conditions, but the demand conditions seem not to play a decisive role in determining the welfare implications. On the other hand, our model is quite general in terms of both technology and market structure assumptions and these two assumptions appear to be more crucial for the welfare results than are the demand conditions.

Our findings concerning the pure effects of “vertical” integration will be presented in the following manner. In the next section, we will set forth the basic framework. Section III establishes two related propositions (in the form of lemma and theorem) to prove our case. The final section will present some policy implications.
1.2 The Basic Framework: Price Before Integration

As noted in the introduction, while the model presented by Warren-Boulton assumes production technology which appears quite general, their assumptions regarding market structure are rather restrictive. In this section, we will construct an alternative model of vertical integration, i.e., a case of successive Cournot oligopolies, on the basis of final market demand with constant demand elasticity and a C.E.S. production function.

Consider $m$ identical downstream firms and $n$ identical upstream firms. Assume that each downstream firm produces the final good under conditions of constant elasticity of substitution and constant returns to scale. Thus:

$$x_i = Y\left[\delta A_i^{(\sigma-1)/\sigma} + (1 - \delta)B_i^{(\sigma-1)/\sigma\gamma/(\sigma-1)}\right], \quad i = 1, \ldots, m \quad (1.1)$$

where $x_i$ is the $i^{th}$ downstream firm's output, $\sigma(>0)$ the elasticity of substitution, $Y(>0)$ the efficiency parameter and $\delta(1 > \delta > 0)$ the distributive parameter. Input $A_i$ is supplied to the $i^{th}$ downstream firm by $n$ upstream firms at the market price of $P_a$ and is produced with constant marginal cost.
The other input $B_i$ is supplied in the competitive market at a given price $P_b$.

Each downstream firm is assumed to behave as a competitive buyer of inputs, and therefore considers input prices to be given as it tries to minimize the cost of production, $P_a A_i + P_b B_i$, subject to Eq. (1.1). The first order conditions for cost minimization accordingly are:

$$\frac{B_i}{A_i} = \left(\frac{(1 - \delta)P_a}{\delta P_b}\right)^\sigma, \quad i = 1, 2, \ldots, m \quad (1.2)$$

Solving Eqs. (1.1) and (1.2) for $A_i$ and $B_i$ readily yields each downstream firm’s cost function:

$$C_i^b = P_a A_i + P_b B_i = \frac{1}{\sigma}(\delta^\sigma P_a^{1-\sigma} + (1 - \delta)^\sigma P_b^{1-\sigma})^{1/(1-\sigma)} x_i, \quad i = 1, 2, \ldots, m \quad (1.3)$$

where the superscript “b” denotes “before integration”.

5The assumption of constant marginal cost is not needed for our results to hold.
The final market demand is assumed to be of constant elasticity:

\[ X = Z P_x^{-\eta} \]  \hspace{1cm} (1.4)

where \( Z > 0, \eta > 1 \) and \( P_x \) is the unit price of the final product \( X \). In equilibrium the market demand equals the aggregate supply \( x_i \) by \( m \) downstream oligopolists:

\[ X = \sum_{i=1}^{m} x_i \]  \hspace{1cm} (1.5)

Since each Cournot downstream firm considers its rivals' supply as fixed, the first order profit maximization conditions subject to (1.3), (1.4) and (1.5) yield the reaction functions:

\[ P_x - \frac{x_i}{\eta X} P_x = c^b, \quad i = 1, 2, \ldots m \]  \hspace{1cm} (1.6)

where \( c^b \) stand for the marginal (and average) cost of producing \( x_i \). Since (1.6) applies equally to all \( i \), it follows that \( X = \sum_{i=1}^{m} x_i = mx_i \). Therefore Eq. (1.6) becomes:

\[ P_x \left( 1 - \frac{1}{mn} \right) = c^b \]  \hspace{1cm} (1.7)
where $c^b$, from Eq. (1.3), is given by:

$$c^b = \frac{1}{Y} (\delta^\sigma P_a^{1-\sigma} + (1 - \delta)^\sigma P_b^{1-\sigma})^{1/(1-\sigma)}$$  \tag{1.8}

Thus, the product price is clearly seen to depend upon, among other parameters, the input price $P_a$. Before vertical merger, this input price is given to the $m$ downstream firms in the market in which $n$ upstream firms compete with one another as Cournot oligopolists. The input price, while given to the downstream firms, itself depends upon the Cournot behavior of the upstream sellers. Each upstream Cournot oligopolist will sell inputs so as to maximize profit. The first order conditions for profit maximization in the Cournot upstream market require:

$$P_a \left( 1 - \frac{1}{nE} \right) = M_a$$  \tag{1.9}

where $M_a$ stands for the marginal cost of producing $A$, $n$ for the number of upstream Cournot oligopolists, and $E$ for the elasticity of derived demand for $A$, which must be greater than unity since we assume positive marginal

\footnote{There is a typographic error in Eq. (1.6) of Waterson [1982,p.132]; the correct expression for $c^b$ is given by our Eq. (1.8).}
cost. The elasticity $E$ is further specifiable as:

$$E = k_a \eta + (1 - k_a) \sigma, \quad 0 \leq k_a \leq 1 \quad (1.10)$$

where $k_a$ is the share of the cost of $A$ in the total cost of producing $X$. (The parameter $k_a$ is not a constant; it can be shown to be a unique function of $P_a$: $k_a = (1 + ((1 - \delta)/\delta)^\sigma(P_a/P_b)^{\sigma-1})^{-1}$. See e.g. Waterson [1982].)

The product price before merger is thus uniquely determined by solving equations (1.7), (1.8), (1.9) and (1.10) for $P_x$. This $P_x$, henceforth, will be referred to as $P_x^b$, superscript “b” standing for “before merger”.

## 1.3 The Welfare Effects of Vertical Integration

### 1.3.1 Price after vertical integration

Before we proceed to examine the impacts of vertical integration, we stress that our basic model is fully general as set out above: it contains each of the

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Waterson [1980] proves that this formula [Hicks, 1963], derived under conditions of perfect competition, also holds in the case of oligopoly.
previous models as a special case. When the number of upstream firms falls to one \((n = 1)\), the model becomes identical with Waterson’s [1982]. When the elasticity of substitution is equal to zero \((\sigma = 0)\), this model approximates the GO model [1979].\(^8\) Further, if the number of upstream firms is one and the number of downstream firms tends to infinity \((n = 1 \text{ and } m \to \infty)\), our model reduces to the models of Warren-Boulton [1974] and Mallela-Nahata [1980]. However, we do not intend to generalize the model for the sake of generality; instead we provide a possible framework to settle the controversy about the effect of vertical integration on the final product price. In this context, the interpretation of “vertical” integration plays the crucial role.

The cost function of a downstream firm after merger with an upstream firm is obtained directly by replacing \(P_a\) with \(M_a\) in Eq. (1.3):

\[
C_i^a = \frac{1}{Y} (\delta^\sigma M_a^{1-\sigma} + (1-\delta)^\sigma P_b^{1-\sigma})^{1/(1-\sigma)} x_i, \quad i = 1, 2, ..., \mu
\]  

(1.11)

where the superscript “a” denotes “after integration” and \(\mu\) “number of firms participating in vertical integration”. Taking the same procedures above, we

\(^8\)Our model with \(\sigma = 0\) is not exactly the same as GO’s since GO [1979] assumes the demand function which is less restrictive than our constant elasticity demand.
obtain each integrated firm’s reaction function after merger:

\[ P_x^a \left( 1 - \frac{1}{\mu \eta} \right) = c^a = \frac{1}{Y} (\delta^\sigma M^1_{x}^{1-\sigma} + (1 - \delta)^\sigma P^1_{b}^{1-\sigma})^{1/(1-\sigma)} \]  

(1.12)

where \( P_x^a \) and \( c^a \) stand respectively for the final price and the marginal cost of producing \( X \) after integration. This gives, in general, the final product price when each firm attempts to integrate vertically with a downstream or upstream firm. Moreover, this can readily be compared with the treatment of “vertical” integration in the previous models. (The analysis by Waterson [1982], for example, limited itself to the case in which \( \mu = 1 \) in the above equation. However, this in effect involves the compound effect of vertical and horizontal integration because \( m \) downstream firms initially assumed to exist horizontally before integration are required to be merged into one and only one firm after vertical integration. Since we are interested only in the effect of vertical integration by itself, any horizontal effect has to be eliminated.)

One way to pursue the pure theory is simply to consider the case of \( \mu = m = n \). When \( m \neq n \), the vertical merger could involve musical chairs and resultant elimination or horizontal take-over of some slow firms which fail to integrate vertically. Avoiding such complication, we will initially consider
the special case with $m = n$, to be followed by a more general case where $m \neq n$.

1.3.2 The pure effect of complete vertical integration:

The case $\mu = m = n$

We are now in position to prove our fundamental proposition set forth at the outset as we consider the case $\mu = m = n$. Initially the following conditions are derived from Eqs. (1.7) and (1.12):

$$\frac{P^*_x - c^*}{P^*_x} = \frac{P^a_x - c^a}{P^*_x} = \frac{1}{\mu \eta}$$  \hspace{1cm} (1.13)

where the left end and middle part of Eq. (1.13) represents the so-called Lerner index of monopoly, before integration and after integration, respectively. Since both are equal $1/\mu \eta$ we see that the Lerner index of monopoly is left unchanged by vertical integration. This proves the following LEMMA.

Lemma. Suppose that $m$ homogeneous Cournot firms in the downstream industry are faced by market demand with constant elasticity. Then the Lerner index of monopoly remains unchanged if these downstream firms integrate
one-to-one vertically with homogeneous upstream firms.

Our fundamental proposition is now immediately derivable from this Lemma and the following observation. A comparison of the final product prices before and after integration is reduced to the comparison of the marginal costs through Eqs. (1.8) and (1.12), which comparison in turn depends on that of $P_a$ and $M_a$. Namely,

$$P^a_x \leq P^b_x, \quad \text{if and only if} \quad M_a \leq P_a$$  \hspace{1cm} (1.14)

which will be true if and only if $nE > 1$ from Eq. (1.9). Since $n \geq 1$ and $E$ is required to be greater than unity under a positive marginal cost, it follows that $P^a_x < P^b_x$, thus establishing the following THEOREM.

**Theorem.** Suppose that the downstream industry is characterized by the constant elasticity of substitution in production besides the conditions of the Lemma. Then the effect of vertical integration of downstream firms with upstream firms is to lower the final product price.

It goes without saying that the reduction in the final good price due to a vertical merger causes an increase in the equilibrium quantity according to
the demand function as shown in Eq. (1.4).

1.3.3 The pure effects of partial vertical integration:

The case $\mu \leq m < n$

As we noted above, not all firms can effect one-to-one vertical integration when $m \neq n$. Nevertheless, we note that insofar as the downstream firms are fewer in number than the upstream firms, the Lerner index of monopoly remains unaltered before and after complete vertical merger, being given by $1/m\eta$ via Eq. (1.7) and $1/\mu\eta$ via Eq. (1.12), where $\mu = m$.\(^9\) Thus, because marginal cost after vertical integration can be shown to be strictly lower, the welfare effect of vertical integration remains strictly positive, and moreover dominates the negative horizontal effects, if any, of liquidated upstream firms.

Our lemma does not apply, however, to the case in which the smaller number $\mu$ of firms is in the upstream industry, that is, $\mu = n < m$. The Lerner index of monopoly after complete vertical merger is clearly higher than before since $1/m\eta < 1/n\eta$ insofar as $n < m$. It is worthy of note here that our proposition no longer holds in this case $n < m$ due to the unavoidable

\(^9\)In contrast to Waterson [1982, section III], we assume that, after merger, the integrated firms do not trade with nonintegrated firms.

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horizontal effect that arises even under conditions of partial integration, that is, \( \mu < n < m \).

Let us consider a case in which at least one firm remains unintegrated in each of the vertically related industries and keeps behaving as a Cournot successive oligopolist so that \( \mu < m < n \). This is the case considered, although not definitively, by GO[1979]. We consider the GO results, and moreover go beyond them by relaxing their assumption of \( \sigma = 0 \). (Our demand function is more restrictive than GO’s, however.)

At the upstream stage \( \mu \) input oligopolists are eliminated from the market because of the vertical integration. Therefore, \((n - \mu)\) nonintegrated upstream firms now supply the input to the \((m - \mu)\) nonintegrated downstream oligopolists. The optimization conditions for the \((n - \mu)\) upstream nonintegrated firms are then given:

\[
P^n_a \left(1 - \frac{1}{(n - \mu)E}\right) = M_a
\]

where \( P^n_a \) stands for input price for the nonintegrated firms. Immediately

\(^{10}\)Quirmbach [1986A, 1986B] also refers to partial integration. However, Quirmbach’s partial integration is not exactly the same as ours. In our model it is not possible, after merger, for the upstream monopolist to sell the input to the independent downstream competitors as we assumed above.
from the comparison of this equation and Eq. (1.9), we see that the input price for nonintegrated downstream firms is not equal to the price before integration as long as \( \mu > 0 \). More precisely, depending upon the parameter value, \( P^n_a \geq P_a \) or \( P^n_a < P_a \).

The first-order profit-maximization conditions for these \( \mu \) integrated firms are:

\[
P^a_x - \frac{x_i}{\eta X} P^a_x = c^a, \quad i = 1, 2, \ldots, \mu
\]

(1.16)

The optimization conditions for the remaining \((m - \mu)\) nonintegrated firms are subject to Eq. (1.6) over \( j = \mu + 1, \ldots, m \), to obtain:

\[
P^a_x - \frac{x_j}{\eta X} P^a_x = c^n, \quad j = \mu + 1, \mu + 2, \ldots, m
\]

(1.17)

where \( c^n \) stands for marginal cost of the nonintegrated firms. Note \( c^n \) is now distinguished from \( c^b \) in Eq. (1.6) as seen below:

\[
c^n = \left( \delta^\sigma P^{n1-\sigma} + (1 - \delta)^\sigma P^{1-\sigma}_b \right)^{1/(1-\sigma)}
\]

(1.18)

Aggregating Eqs (1.16) and (1.17), respectively, over \( i = 1, \ldots, \mu \) and \( j = \ldots, m \).
\(\mu + 1, \ldots, m\), then yields:

\[
\mu P_x^a - \frac{\mu x_i}{\eta X} P_x^a = \mu c^a
\]  

(1.19)

and

\[
(m - \mu) P_x^a - \frac{(m - \mu)x_j}{\eta X} P_x^a = (m - \mu) c^n
\]  

(1.20)

These two equations provide, respectively, the aggregate inputs of the \(\mu\) integrated and \((m - \mu)\) nonintegrated firms. Since market equilibrium requires \(X = \mu x_i + (m - \mu)x_j\), adding Eq. (1.19) to Eq. (1.20) and dividing both sides by \(m\) yields:\(^{11}\)

\[
P_x^a \left(1 - \frac{1}{m\eta}\right) = c^n - \frac{\mu}{m}(c^n - c^a)
\]  

(1.21)

\(^{11}\)The nonintegrated firms may not be able to survive due to their inability to cover the average costs. In order to cover the costs, the final market price must exceed the nonintegrated firms’ marginal cost; \(P_x^a > c^a\). This condition can be written via Eq. (1.8), Eq. (1.15), Eq. (1.12) and Eq. (1.20) as: When \(0 < \sigma < 1\),

\[
\delta^{1-\sigma} P_a^{1-\sigma} \left[ \left(1 - \frac{1}{(n - \mu)E}\right)^{1-\sigma} - \left(1 - \frac{1}{\mu\eta}\right)^{1-\sigma} \right] + (1 - \delta)^{1-\sigma} P_b^{1-\sigma} \left[ 1 - \left(1 - \frac{1}{\mu\eta}\right)^{1-\sigma} \right] > 0
\]

And when \(\sigma > 1\),

\[
\delta^{1-\sigma} P_a^{1-\sigma} \left[ \left(1 - \frac{1}{(n - \mu)E}\right)^{1-\sigma} - \left(1 - \frac{1}{\mu\eta}\right)^{1-\sigma} \right] + (1 - \delta)^{1-\sigma} P_b^{1-\sigma} \left[ 1 - \left(1 - \frac{1}{\mu\eta}\right)^{1-\sigma} \right] < 0
\]

A sufficient condition for these inequalities to hold is given by \(\mu\eta < (n - \mu)E\). Partial integration will be feasible in this case. Otherwise, i.e., when \(P_x^a < c^a\), nonintegrated firms have no choice but take part in the vertical integration.
Thus the marginal cost after merger is expressed as the weighted average of marginal costs for both integrated and nonintegrated firms. The left hand sides of Eq. (1.7) and Eq. (1.21) are the same form, so we have only to compare the right hand sides of these equations to prove the theorem. The right hand side of Eq. (1.21) can be shown to be strictly less than that of Eq. (1.7) or $c^b$ when $\sigma = 0$. The same relation holds generally even when $\sigma > 0$ insofar as both $n$ and $E$ are sufficiently large, or one of $1/nE$, $1/(n - \mu)E$ nearly zero.\(^\text{12}\) This conditions, however, is put only to facilitate the analytical proof. In fact we have also run a computer simulation with $m, n, \mu, \delta, \sigma$ and $P_b$ as parameters. We obtained the result that the above relation holds even when both $n$ and $E$ are as small as $n = 2$ and $E = 1.5$. Provided these conditions are acceptable, we finally have:

$$P^a \left(1 - \frac{1}{m\eta}\right) < c^b$$

(1.22)

Note that Eq. (1.21) gives the final good price when only $\mu$ firms integrate vertically while the remaining firms behave independently as before. The Lerner indices before and after merger in the light of Eqs. (1.7) and (1.21)

\(^{12}\)Details of the proof can be obtained from the author upon request.
are both seen to be $1/m\eta$ (LEMMA). It then follows from Eqs. (1.7) and (1.22) that the final market prices after vertical integration are strictly lower than those before merger (THEOREM).

\section*{1.4 Policy implications}

Throughout the discussion of this paper, we have generalized selected models of vertical integration which have appeared since 1970s in an attempt to answer the question whether the effect of vertical integration on the final product price is sensitive to the degree of substitution and/or the market structures as suggested in the controversy. Remarkably the result is not what has been believed: Greenhut and Ohta’s conclusion on welfare may be extended to cover not only cases involving moderate substitution [Waterson 1982] but also the cases far beyond. We also show that previous vertical integration models involve both vertical effects and horizontal effects. However, these two effects are offsetting each other; e.g., see Mallela-Nahata [1980].

The propositions proved in this paper are worth repeating because of the robustness of their implications for antitrust regulation. Under Cournot assumptions with constant elasticity of final product demand and a smaller
number of firms in the downstream industry:

(i) The Lerner index of monopoly remains unaffected by purely vertical integration;

(ii) Vertical integration decreases the price of the final product.

Antitrust authorities have led the battle against conglomerate takeovers on grounds that vertical integration would actually (or even potentially, prior to 1973) reduce competition, restrict output and raise the final output prices.\textsuperscript{13} Our theorems show, however, that these views are not quite valid. It is true, as we noted above, that antitrust authorities’ assumptions hold for the case of horizontal integration. Therefore, we propose that antitrust authorities should first make a clear distinction between “vertical” and “horizontal” integration, and then focus their attention on ways to eliminate the “horizontal” effect.

\textsuperscript{13}See e.g., Spengler [1950,p.347] and Mallela and Nahata [1980,p.1024].
1.5 References


Mallela, P., and Nahata, B., “Theory of Vertical Control with Variable


Waterson Michael, "Vertical Integration, Variable Proportions and Oligopoly,"

Chapter 2

Profit Incentive in Pure Vertical Integration
Abstract

The paper shows that in a pure vertical merger there is no profit incentive to integrate. The model considers \( n \) identical Cournot upstream oligopolists and \( m \) identical downstream Cournot oligopolists. The production function for the final product is that of C.E.S. with constant returns to scale, and the demand function is of constant elasticity form. When \( m = n \) and each upstream firm merges with each downstream firm (a pure vertical merger), there is no profit incentive. When \( m < n \), and \( \min(m, n) \) firms vertically integrate, then also there is no profit incentive. However, when \( m > n \), and \( \min(m, n) \) firms integrate, there always exists a profit incentive if merger result in an increase in the price. However, when merger results in a decrease in price there may or may not be an incentive to integrate depending on the cost and demand parameters and the number of firms before merger in each industry. The analysis, therefore, suggests that if profit is the incentive for merger, a pure vertical merger should not take place.

\(^1\)This chapter is rewritten using the papers by Abiru (1987A, 1987B and 1991)
2.1 Introduction

In economic literature, over a span of more than twenty-five years, the topic of vertical integration has been analysed in considerable detail. Two quite recent papers by Quirmbach (1986A, 1986B) and one by Lee (1987) and another by Abiru (1988) indicate there is continuing interest in the topic. It is well known in the literature that an input monopolist has no profit incentive to integrate in a downstream industry when inputs are used in fixed proportion to produce a final good. Spengler's (1950) analysis for this case concluded that vertical integration is neutral; that is, the pre- and post-merger price, output, and profits are identical. However, a paper by Vernon and Graham (1971), which led the way to future analyses, clearly demonstrated that if inputs are used in variable proportion, there is always a profit incentive to integrate forward. A stream of analyses that followed concentrated on analyzing the anticompetitive effects of vertical integration by comparing the pre- and post-merger prices of final product and evaluating the welfare changes.\(^2\)

The purpose of this paper is to reconsider the question of profit incentive in the case of "pure" vertical integration. We use the term pure because the primary focus of our analysis is to isolate the vertical effects of merger. In existing analyses of vertical merger, both vertical and horizontal effects are intertwined because the way vertical merger is treated involves both horizontal and vertical effects. Since the prevailing usage of the term vertical includes both vertical and horizontal effects, the adjective pure seems to be more appropriate in order to distinguish our usage of the term vertical from the existing one.

If the horizontal effects of merger are eliminated and only the vertical effects are considered, the analysis that follows shows a surprising and counterintuitive result. It is shown that there is no incentive to integrate vertically except in the case of successive monopolists. In other words, the only situation where pure vertical integration offers a profit incentive is when pre-merger market structures of both input and final products are monopoly. In other situations, horizontal effects must accompany vertical effects for a profit incentive to exist.

Before we develop the basic framework of our analysis, it is appropriate to discuss briefly the reasons for this perverse result. In the analysis by
Vernon and Graham, in order to show that a profit incentive exists, it is assumed that vertical integration by the input monopolist results in monopolization of a downstream industry. It must be stressed that monopolization of a final product industry involves horizontal effects. Similarly, the analysis of vertical control by Mallela and Nahata (1980, Theorem 4, p. 1022) also includes horizontal effects. Strictly speaking, a pure vertical merger should not involve any horizontal effects because monopolization of a downstream industry is not necessary for pure vertical merger. In other words, a pure vertical merger should leave unchanged the market structure of the downstream industry. Vernon and Graham's analysis can still be applied in a pure vertical integration situation when vertical merger takes place between an input monopolist and a monopolist producing a final product. In fact, in the case of successive monopolists, profit incentive to merge exists even when the production function for the final product is one of fixed proportion.

The focus of our analysis is to explore the question of profit incentive in the case of successive oligopolies. Section II develops the basic framework for analysis. The model considers $m$ identical Cournot downstream oligopolists and $n$ identical Cournot upstream oligopolists. The production function for the final product is that of CES, and the demand function is of the constant
elasticity form. These specifications have become quite standard and have
been employed in several previous analyses. Section III considers the case of
pure vertical merger, \( m = n \) by allowing each downstream firm to merge with
each upstream firm so that there is no horizontal effect accompanying the
vertical merger. The main result, showing that there is no profit incentive, is
proved as Theorem 1. The case \( m < n \) and \( m > n \) are analyzed next. In both
of these cases, vertical merger accompanies horizontal effects. It has been
proved that when \( m < n \) there is again no profit incentive, and the result
is stated as Theorem 2. The analysis of case \( m > n \), which generalizes the
previous analyses, clearly demonstrates that when \( m > n \geq 2 \) the increase
in post-merger price is sufficient for profit incentive to exist. This result is
stated as Theorem 3. However, when merger results in a decrease in price,
profit incentive may or may not exist depending on the parameter values.
This surprising result contrasts to earlier results where it has been shown
(Mallela and Nahata (1980), Theorem 4) that \( n = 1 \) and vertical merger
leads to complete monopolization of the final good industry, then the profit
incentive exists, regardless of the direction of price changes after merger.
Conclusion summarizes the important implications of the analysis.
2.2 Analytical Framework

Although we used the same specifications for production and demand functions as employed by Warren-Boulton (1974), Mallela and Nahata (1980), and Waterson (1982), our model is different and considers oligopolies in both input and product markets allowing vertical integration between input and product market oligopolists.

Consider $n$ identical upstream Cournot oligopolists producing input $A$ which is being used with input $B$ to produce the final product $X$. The production function of the final product $X$ is in the form of constant elasticity of substitution and constant returns to scale.

\[
X = Y[\delta A^{-\rho} + (1 - \delta)B^{-\rho}]^{-1/\rho}
\]  

(2.1)

Where the elasticity of substitution, $\sigma = 1/(1 + \rho)$, $\sigma > 0, Y > 0$, and $0 < \delta < 1$. The input $A$ is produced with constant marginal cost, $M_a$, in the relevant range. It is further assumed that there is no demand for $A$ other than the derived demand, arising as an input for producing the final product $X$. The other input, $B$, is supplied competitively at constant price, $P_b$.  

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The final product $X$ is produced by $m$ identical downstream Cournot oligopolists. The market demand for $X$ is of the constant elasticity form,

$$X = ZP_x^{-\eta}$$  \hspace{1cm} (2.2)$$

where $Z > 0$, $m\eta > 1$ and $P_x$ is the unit price of $X$. Since both upstream and downstream firms are Cournot oligopolists, the conjectural variation is zero, and the price, $P_a$ is the unit price of $A$ and $P_x$ is determined by equating marginal revenue with marginal cost in the respective markets.

The profits, $\pi_a$, of the upstream industry can be written as $\pi_a = P_aA - M_aA$. The profit-maximizing price, $P_a$, before merger is given by:

$$P_a = \frac{nEM_a}{nE - 1}$$  \hspace{1cm} (2.3)$$

where $E$ is the elasticity of derived demand for input $A$ and is given by

$$E = \frac{\sigma(\eta + e) + k_a\epsilon(\eta - \sigma)}{\eta + e - k_a(\eta - \sigma)}$$  \hspace{1cm} (2.4)$$

where $e$ is the elasticity of supply of input $B$ and $k_a$ is the share of the cost of $A$ in the total cost of producing $X$. Since $B$ is assumed to be available at
an infinitely elastic supply, putting $e \to \infty$ in equation (2.4) will yield

$$E = \eta k_a + \sigma (1 - k_a) \tag{2.5}$$

Note that $0 \leq k_a \leq 1$ implies $\sigma \leq E \leq \eta$ or $\eta \leq E \leq \sigma$ depending on whether $\sigma \leq \eta$ or $\eta \leq \sigma$ and that whenever $0 < k_a < 1$, $E = \eta = \sigma$ if and only if $\eta = \sigma$.

The total cost, $C_x$, of producing $X$ is

$$C_x = P_a A + P_b B \tag{2.6}$$

The first-order conditions of cost minimization for $X$ subject to equation (2.1) yield

$$\frac{B}{A} = \left[ \frac{(1 - \delta)P_a}{\delta P_b} \right]^{\sigma} \tag{2.7}$$

Using equations (2.6) and (2.7) and noting that $k_a = \frac{P_a A}{C_x}$, we get

$$\frac{C_x}{P_a A} = \frac{1}{k_a} = 1 + \left( \frac{1 - \delta}{\delta} \right)^{\sigma} \left( \frac{P_a}{P_b} \right)^{\sigma-1} \tag{2.8}$$
Solving for \( \frac{1}{k_a} \) from (2.5), we get

\[
\frac{1}{K_a} = \frac{\eta - \sigma}{E - \sigma} = 1 + \frac{\eta - E}{E - \sigma}
\]  

(2.9)

Equating the right-hand sides of equations (2.8) and (2.9), we get

\[
\frac{\eta - E}{E - \sigma} = \left( \frac{1 - \delta}{\delta} \right)^{\sigma} \left( \frac{P_a}{P_b} \right)^{\sigma-1}
\]  

(2.10)

Now the expression for \( P_a \), the pre-merger industry price for input \( A \), is obtained by solving for \( E \) from equation (2.3) and then substituting the result for \( E \) in (2.10):

\[
(n\eta - 1)P_a - n\eta M_a + \left( \frac{1 - \delta}{\delta} \right)^{\sigma} \left( \frac{P_a}{P_b} \right)^{\sigma-1} [n\sigma(P_a - M_a) - P_a] = 0
\]  

(2.11)

\[2.2.1 \text{ Pre- and Post-Merger Profit}\]

The total pre-merger profits of Cournot oligopolists in an upstream industry can be written as

\[
\Pi_a = P_a A - M_a A = P_a A \left( 1 - \frac{M_a}{P_a} \right) = \frac{P_a A}{nE} = \frac{k_a C_x}{nE}
\]  

(2.12)
The pre-merger profits of downstream firms can be written by computing the pre-merger price and cost; the pre-merger price of final product $X$ is given by

$$P_x = \frac{m\eta MC_x}{m\eta - 1} \quad (2.13)$$

where $MC_x$ is the marginal cost (=average cost) of producing $X$ and can be written as

$$MC_x = Y^{-1}[\delta^\sigma P_1^{1-\sigma} + (1 - \delta)^\sigma P_b^{1-\sigma}]^{1/(1-\sigma)} \quad (2.14)$$

Now the pre-merger total profits, $\Pi_x$, of a downstream industry can be obtained by

$$\Pi_x = P_x X - MC_x X = \frac{P_x}{m\eta} = \frac{Z P_1^{1-\eta}}{m\eta} \quad (2.15)$$

Total pre-merger profits of upstream and downstream industries can be written as

$$\Pi_u + \Pi_x = \frac{P_u A}{nE} + \frac{P_x X}{m\eta} \quad (2.16)$$

---

3 For derivation, see Mallela and Nahata (1980).

4 Note that in all previous analyses market structure of the downstream industry was assumed to be competitive and therefore $\Pi_x = 0$. Vertical integration leads to complete monopolization of downstream industry and therefore involves horizontal effects, therefore $\Pi_u < \Pi_x$ and there is always a profit incentive to integrate.
Similarly, the post-merger price, $\overline{P}_z$, can be written as

$$\overline{P}_z = \frac{m \eta}{m \eta - 1} \overline{MC}_x$$  \hfill (2.17)

where $\overline{MC}_x$ can be written as

$$\overline{MC}_x = Y^{-1}\left[\delta^\sigma M_a^{1-\sigma} + (1 - \delta)^\sigma P_b^{1-\sigma}\right]^{1/(1-\sigma)}$$  \hfill (2.18)

The post-merger total profits, $\overline{\Pi}_z$, of merged firms in downstream industry are given by

$$\overline{\Pi}_z = \overline{P}_z \overline{X} - \overline{MC}_x \overline{X} = \frac{\overline{P}_z \overline{X}}{m \eta} = \frac{\overline{P}_{1-\eta}^z}{m \eta}$$  \hfill (2.19)

### 2.2.2 Comparison of pre- and post-merger profits

We first consider the case when $m = n$ and each upstream firm integrates vertically with a downstream firm. Since the number of firms in both downstream and upstream industries are the same, each upstream firm can integrate forward and after integration there are $m(= n)$ integrated firms. This case represents a pure vertical merger because there are no horizontal effects associated with vertical integration. We will consider the case when $m \neq n$
later. Without loss of generality, we assume \( Y = Z = 1 \).

Pure vertical merger offers no profit incentive if

\[
\Pi_x < \Pi_a + \Pi_x
\]

or alternatively,

\[
\Pi_x - \Pi_x < \Pi_a
\]

Substituting for \( \Pi_x, \Pi_x, \Pi_a \) from eqns. (2.19), (2.15) and (2.12) and noting

\[ m = n \], we get

\[
\frac{\overline{P}_x \overline{X}}{m \eta} - \frac{P_x X}{m \eta} < \frac{P_a A}{m E} \tag{2.20}
\]

or

\[
\frac{\overline{P}_x \overline{X}}{P_x X} - 1 < \frac{\eta P_a A}{E P_x X}
\]

Substituting for \( X \) and \( \overline{X} \) from eqn. (2.2) and for \( P_x \) from eqn. (2.13) and noting that \( \frac{P_a A}{MC_x X} = k_a \), we get

\[
\left( \frac{\overline{P}_x}{P_x} \right)^{1-\eta} < 1 + \frac{\eta m \eta - 1}{m \eta} k_a \tag{2.21}
\]

Using eqns. (2.13), (2.14), (2.17) and (2.18) simultaneously on LHS and eqn.
(2.9) for $k_a$ in RHS, we get

$$\left( \frac{\delta \sigma M_a^{1-\sigma} + (1 - \delta)^\sigma P_b^{1-\sigma}}{\delta \sigma P_a^{1-\sigma} + (1 - \delta)^\sigma P_b^{1-\sigma}} \right)^{\frac{1-\eta}{1-\sigma}} < 1 + \frac{m\eta - 1}{m\eta} \left( \frac{\eta E - \eta \sigma}{\eta E - E \sigma} \right)$$

Note that the above inequality contains $P_a$ on LHS and $P_a$ cannot be solved for explicitly (see eqn. 11); therefore, this inequality cannot be used for analytical proofs. However, following Mallela and Nahata (1980), we overcome this problem by rewriting it as follows:

Dividing both the numerator and denominator on LHS by $\delta \sigma P_a^{1-\sigma}$ and using eqns. (2.3) and (2.10), we get

$$\left( \frac{E - \sigma \left( \frac{mE - 1}{m \eta} \right)^{1-\sigma} + \eta - E}{\eta - \sigma} \right)^{\frac{1-\eta}{1-\sigma}} < 1 + \frac{m\eta - 1}{m\eta} \frac{E - \sigma}{\eta - \sigma} \quad (2.22)$$

Proving the above inequality for all values of $E$, $E$ as a variable the range of which is determined by the range of $\eta$ and $\sigma$, is equivalent to proving that pure vertical integration offers no profit incentive. This is so because $E$ is uniquely determined for any given $\delta, \sigma, \eta, P_b, m$ and $M_a$. Thus, if the above inequality holds for all values of $E$, it holds for a particular value of $E$ appearing in (2.22). The spirit of this argument will be adhered to in all
the proofs that follow in the paper.

\section{Main Results}

As stated earlier, in this section we prove the main result of the paper. The basic results are stated as theorems.

\textbf{Theorem 1.} If each of $n$ upstream Cournot oligopolists vertically integrate with each of $n$ downstream Cournot oligopolists so that after integration there are $n$ vertically integrated firms, then there is no profit incentive for vertical integration if $n \geq 2$, $0 < \eta < \infty$, and $0 < \sigma \leq 2$.

\textbf{Proof:} We prove the theorem in four parts: (i) $0 < \eta < 1$, (ii) $\eta > 1$ and $0 < \sigma < 1$, (iii) $\eta > 1$ and $\sigma = 1$, and (iv) $\eta > 1$ and $1 < \sigma \leq 2$.

\subsection*{Case $0 < \eta < 1$}

It must be noted that the industry price elasticity need not be greater than one. In fact, empirical estimates of elasticity that can be classified as oligopolies seem to suggest that for a large number of industries the the elasticities values are less than one. However, at industry equilibrium $m\eta$ must exceed one.
The proof for this case is rather straightforward. Observe from eqn. (2.20) that the necessary condition for profit incentive is that $\Pi_x > \Pi_x$ (because $\Pi_a > 0$). Since $MC_a < MC_x$ implies that $\bar{P}_x < P_x$ and $\Pi_x = \frac{\bar{P}_x^{\frac{1}{1-\eta}}}{m\eta}$ and $\Pi_x = \frac{\bar{P}_x^{\frac{1}{1-\eta}}}{m\eta}$, it is obvious that if $\eta < 1$, then $\Pi_x < \Pi_x$. Thus, the necessary condition for profit incentive is that $\eta$ must exceed one. Note that this result is true regardless of the value of $\sigma$.

Q.E.D.

2.3.2 Case $\eta > 1$ and $0 < \sigma < 1$

The proof is somewhat cumbersome and lengthy. In order to conserve space, only the outline to the proof is provided. The detailed version of the proof can be obtained by writing the author.

Inequality (2.22) can be rewritten as follows:

$$1 < \left[ 1 + \frac{m\eta - 1}{mE} \left( \frac{E - \sigma}{\eta - \sigma} \left( 1 - \left( \frac{mE - 1}{mE} \right)^{1-\sigma} \right) \right)^{\frac{n-1}{1-\sigma}} \right]$$

Multiplying both sides by the second term on RHS without an exponent, we
Using binomial inequality \((1 + x)^\alpha \geq 1 + \alpha x\) and \((1 + x)^\alpha < 1 + \alpha x\) according as \(\alpha \geq 1\) and \(\alpha < 1\) and with algebraic manipulations, the inequality reduces to proving

\[
\frac{E \left[1 - \left(\frac{mE-1}{mE}\right)^{1-\sigma}\right]}{(1 - E) + (E - \sigma) \left(\frac{mE-1}{mE}\right)^{1-\sigma}} \leq \frac{m\eta - 1}{m\eta - N}
\]  \hspace{1cm} (2.25)

Noting that for \(m \geq 2\) the RHS > 1, it is enough to prove that LHS < 1 which requires (after some algebraic manipulations) proving

\[
\left(1 + \frac{1}{mE - 1}\right)^{1-\sigma} \leq \frac{2E - \sigma}{2E - 1}
\]  \hspace{1cm} (2.26)

Using binomial inequality again, the proof follows.

Q.E.D.
2.3.3 Case $\sigma = 1$ and $\eta > 1$

It is well known that when $\sigma = 1$, the CES production function reduces to Cobb-Douglas and

$$X = A^\alpha B^\beta$$  \hspace{1cm} (2.27)

The pre- and post-merger marginal cost functions can be easily derived as (see Ferguson, 1975, pp. 163-165)

$$MC_x = P_b \left( \frac{BP_a}{\alpha P_b} \right)^\alpha$$  \hspace{1cm} (2.28)

$$\overline{MC}_x = P_b \left( \frac{BM_a}{\alpha P_b} \right)^\alpha$$  \hspace{1cm} (2.29)

Substituting for $k_a = \alpha$ and $\overline{P}_x$ (eqn. 17), and $P_x$ (eqn. 13), $MC_x$ (eqn.2.28), $\overline{MC}_x$ (eqn.2.29) in (2.21), there is no profit incentive if

$$\left( \frac{P_a}{M_a} \right)^{\alpha(\eta-1)} < 1 + \frac{\alpha(m\eta - 1)}{mE}$$  \hspace{1cm} (2.30)

Substituting for $P_a$ from (2.3) and noting $m = n$, we get

$$\left( 1 + \frac{1}{mE - 1} \right)^{\alpha(\eta-1)} < 1 + \frac{\alpha(m\eta - 1)}{mE}$$  \hspace{1cm} (2.31)
Taking logarithms on both sides and using \( \frac{x}{1 + x} < \ln(1 + x) < x \), (31) reduces to proving

\[
\frac{\alpha(\eta - 1)}{mE - 1} \leq \frac{\alpha(m\eta - 1)}{1 + \frac{\alpha(m\eta - 1)}{mE}}
\]

which on simplification reduces to

\[
\frac{\eta - 1}{m\eta - 1} \leq \frac{mE - 1}{mE + \alpha(m\eta - 1)}
\]  \hspace{1cm} (2.32)

when \( \sigma = 1 \) and \( k_a = \alpha \) from eqn. (2.5) \( mE = m\eta \alpha + m(1 - \alpha) \). Substituting for \( E \) in eqn. (2.32) and rearranging the terms eqn. (2.32) reduces to

\[
\alpha(m\eta - 1)(\eta - 1) \leq m\eta(m - 1)[\alpha(\eta - 1) + 1]
\]  \hspace{1cm} (2.33)

or, it is enough to prove

\[
\alpha(m\eta - 1)(\eta - 1) \leq m\eta \alpha(m - 1)(\eta - 1)
\]  \hspace{1cm} (2.34)

which reduces to proving:

\[
2m\eta \leq m^2\eta + 1
\]
which is true because \( m \geq 2 \).

Q.E.D.

2.3.4 Case \( \sigma > 1 \) and \( \eta > 1 \)

We now consider the situation when \( \sigma > 1 \). It should be noted from eqn. (2.5) that when \( \sigma > 1 \), then \( E \geq \eta \) and \( E < \eta \) according to \( \sigma \geq \eta \) and \( \sigma < \eta \).

We first consider \( 1 < \sigma \leq 2 \) and show that there is no profit incentive. We consider the situation \( 1 < \eta < E < \sigma \leq 2 \):

Inequality (2.22) for this case can be written as

\[
\left[1 + \frac{\sigma - E}{\sigma - \eta} \left(\frac{mE}{mE - 1}\right)^{\sigma - 1} - 1\right]^{\frac{m-1}{\sigma-1}} < 1 + \frac{m\eta - 1}{mE} \frac{\sigma - E}{\sigma - \eta} \tag{2.35}
\]

Noting that \( \sigma - 1 < 1 \) using binomial inequality \((1 + x)^\alpha \leq 1 + \alpha x\) when \( \alpha \leq 1 \), it is enough to prove

\[
\left[1 + \frac{(\sigma - E)(\sigma - 1)}{(\sigma - \eta)(mE - 1)}\right]^{\frac{m-1}{\sigma-1}} \leq 1 + \frac{m\eta - 1}{mE} \frac{\sigma - E}{\sigma - \eta} \tag{2.36}
\]

Taking logarithms on both sides and using \( \frac{x}{1+x} < \ln(1+x) < x \), it is enough
to prove

\[ \frac{(\eta - 1)(\sigma - E)}{(\sigma - \eta)(mE - 1)} \leq \frac{(m\eta - 1)(\sigma - E)}{mE(\sigma - \eta) + (m\eta - 1)(\sigma - E)} \]  \hspace{1cm} (2.37)

Cross-multiplying and rearranging the terms yield

\[ \eta(m\eta - 1)(\sigma - 1) - E[m\eta(\sigma - \eta)(m - 1) + (\eta - 1)(m\eta - 1)] \leq 0 \]  \hspace{1cm} (2.38)

Note for \( \eta \leq E \leq \sigma \leq 2 \) it is an equation of a line, therefore it is enough to prove that \( f(E = \eta) < 0 \) and \( f(E = \sigma) < 0 \).

Proving \( f(E = \eta) < 0 \) is equivalent to

\[ \frac{m\eta - 1}{m\eta} \leq m - 1 \]  \hspace{1cm} (2.39)

which is true because LHS < 1 and RHS \( \geq 1 \).

And \( f(E = \sigma) \leq 0 \) is equivalent to proving

\[ \frac{m\eta - 1}{m\eta} \leq \sigma(m - 1) \]  \hspace{1cm} (2.40)

which is again true because LHS < 1 and RHS > 1.
The proof for case $1 < \sigma < E < \eta$ for $\sigma \leq 2$ is identical to the above proof. Thus, regardless of whether $\sigma > \eta$ or $\sigma < \eta$, there is no profit incentive as long as $\sigma \leq 2$.

Q.E.D.

2.3.5 Case $\sigma > 2$

For $\sigma > 2$, a formal mathematical proof to show that there is no profit incentive appears to be extremely difficult. However, using extensive simulation with different parameter combinations, we found no instance that shows a profit incentive. The results of the simulation are shown in Figures 1 through 4.
Incentive for Merger  $M = 2$  $N = 2$  $M_A = 1$  $D = .6$  $PB = 2$

Figure-1

Incentive for Merger  $n = 1.1$  $M_A = 1$  $D = .6$  $PB = 2$

Figure-2
Incentive for Merger

Figure-3

Incentive for Merger

Figure-4
2.4 Horizontal Effects Accompanying Vertical Integration

We now consider the situation when the number of upstream firms are not equal to the number of downstream firms; i.e., \( m \neq n \). We have two possible cases: (1) \( n > m \) and (2) \( n < m \). Both of these cases are analyzed below.

2.4.1 Case \( n > m \)

In this case \( \min(m, n) \) or only \( m \) firms can vertically integrate. As a result of vertical integration, the number of firms in the downstream industry remains unchanged but \( (n - m) \) upstream firms have to drop out because after integration there is no demand for their input. This elimination of \( (n - m) \) firms can be viewed as a horizontal effect associated with vertical merger. We prove that in this case also there is no profit incentive to integrate. We state this result as Theorem 2.

\textbf{Theorem 2.} If the number of upstream firms \((n)\) is greater than the number of downstream firms \((m \geq 2)\) so that only \( \min(m, n) = m \) firms can vertically integrate, then there is no profit incentive provided \( 0 < \sigma \leq 2 \).
**Proof:** The proof of theorem 2 is similar to the proof of Theorem 1 outlined earlier.

**Case (i) 0 < \sigma < 1**

From inequality (2.20) it is clear that there is no incentive if

\[
\frac{P_x X}{m\eta} - \frac{P_x X}{m\eta} < \frac{P_a A}{nE} \tag{2.41}
\]

As shown earlier, the above inequality is equivalent to proving

\[
1 < \left[1 + \frac{E - \sigma}{\eta - \sigma} \frac{m\eta - 1}{nE} \right] \left[1 - \frac{E - \sigma}{\eta - \sigma} \left(1 - \left(\frac{nE - 1}{nE}\right)^{1-\sigma}\right)\right]^{\frac{n-1}{1-\sigma}} \tag{2.42}
\]

Following the steps outlined in the proof of Theorem 1, the above inequality reduces to proving

\[
\frac{E - E \left(\frac{nE - 1}{nE}\right)^{1-\sigma}}{1 - E + (E - \sigma) \left(\frac{nE - 1}{nE}\right)^{1-\sigma}} \leq \frac{m\eta - 1}{n\eta - n} \tag{2.43}
\]

Cross-multiplication simplification reduces (2.43) to

\[
\left(\frac{nE}{nE - 1}\right)^{1-\sigma} \leq \frac{n\eta E - nE + m\eta E - m\eta\sigma - E + \sigma}{n\eta E - nE + m\eta E - m\eta - E + 1} \tag{2.44}
\]

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Using binomial inequality \((1 + x)^\alpha \leq 1 + \alpha x\) when \(\alpha < 1\), it is enough to prove

\[
1 + \frac{1 - \sigma}{nE - 1} \leq 1 + \frac{m\eta - m\eta \sigma - 1 + \sigma}{n\eta E - nE + m\eta E - m\eta - E + 1}
\]  

(2.45)

The above inequality on simplification is equivalent to

\[
\frac{n}{n - 1} \leq \frac{m\eta - 1}{\eta - 1}
\]

or

\[
n\eta \leq m\eta(n - 1) + 1
\]  

(2.46)

It is enough to prove that

\[
n\eta \leq m\eta(n - 1)
\]

or

\[
\frac{n}{n - 1} \leq m
\]  

(2.47)

The above inequality always holds because for \(n > m \geq 2\), the LHS < 2 and RHS \(\geq 2\).

Q.E.D.
Case $\sigma = 1$

For $\sigma = 1$ the CES production function reduces to Cobb-Douglas and $n > m$, inequality (2.30) becomes

$$
\left( \frac{P_a}{M_a} \right)^{\alpha(\eta-1)} = \left( \frac{nE}{nE-1} \right)^{\alpha(\eta-1)} < 1 + \frac{\alpha(m\eta - 1)}{nE}
$$

(2.48)

Taking logarithms on both sides and using $\frac{x}{1+x} < \ln(1+x) < x$, it is enough to show that

$$
\frac{\alpha(\eta-1)}{nE-1} \leq \frac{\alpha(m\eta - 1)}{nE + \alpha(m\eta - 1)}
$$

(2.49)

since $E = \eta \alpha + (1 - \alpha) = \alpha(\eta - 1) + 1$ (from eqn.2.5). Substituting for $\eta - 1 = \frac{E - 1}{\alpha}$ on LHS of (2.49), we get

$$
\frac{E - 1}{\alpha(nE - 1)} \leq \frac{m\eta - 1}{nE + \alpha(m\eta - 1)}
$$

(2.50)

which on simplification reduces to proving

$$
\frac{n}{n - 1} \leq m
$$

(2.51)
The above inequality always hold as shown above.

\[ \text{Q.E.D.} \]

**Case 1 < \sigma < 2**

As mentioned earlier in this case, \( \eta > \sigma \) or \( \eta < \sigma \). Since the proof follows similar steps, in order to conserve space we consider the situation \( \eta > \sigma \) (\( \sigma > \eta \) case was considered for Theorem 1). For \( \eta > \sigma > 1 \) and \( n > m \), inequality (2.22) is equivalent to proving

\[
\left( 1 + \frac{E - \sigma}{\eta - \sigma} \left( \frac{nE}{nE - 1} \right)^{\sigma - 1} - 1 \right)^{\frac{n-1}{\sigma-1}} < 1 + \frac{m\eta - 1}{nE} \frac{E - \sigma}{\eta - \sigma} \quad (2.52)
\]

since \( \sigma - 1 \leq 1 \) (1 < \( \sigma \leq 2 \)) using binomial inequality, it is enough to prove

\[
\frac{\eta - 1}{\sigma - 1} \ln \left[ 1 + \frac{E - \sigma}{\eta - \sigma} \left( 1 + \frac{\sigma - 1}{nE - 1} - 1 \right) \right] \leq \ln \left( 1 + \frac{m\eta - 1}{nE} \frac{E - \sigma}{\eta - \sigma} \right) \quad (2.53)
\]

Using logarithmic inequality \( \frac{x}{1 + x} < \ln(1 + x) < x \) and simplifying reduces (2.53) to proving

\[
\frac{\eta - 1}{nE - 1} \leq \frac{(m\eta - 1)(\eta - \sigma)}{nE(\eta - \sigma) + (m\eta - 1)(E - \sigma)} \quad (2.54)
\]
Cross-multiplication and simplification reduces (2.54) to proving

\[0 \leq \eta(m\eta - 1)(\sigma - 1) - E[(\eta - 1)(m\eta - 1) - n(\eta - \sigma)(m\eta - \eta)] \]  (2.55)

Inequality (2.55) represents an equation of a line in variable \(E\). Thus, for \(\sigma \leq E \leq \eta\) it is enough to prove \(f(E = \sigma)\) and \(f(E = \eta)\) is greater than zero. \(f(\sigma) > 0\) is equivalent to proving \(N - 1 > \frac{m\eta - 1}{n\eta\sigma}\) which is true because LHS > 1 and RHS < 1. Similarly, \(f(\eta) > 0\) is equivalent to proving \(n > \frac{m\eta - 1}{m\eta - \eta}\) which is true because for \(n > m \geq 2\), the LHS is > 2, and RHS is < 2.

For \(\sigma > 2\) a formal mathematical proof is difficult. However, the simulation results show that there is no incentive to integrate. The simulation results are similar to the case \(m = n\) and in order to conserve space are not included.

### 2.4.2 Case \(n < m\)

When the number of firms in the upstream industry is less than those in the downstream industry, vertical integration will result in horizontal effects in the downstream industry. The number of firms in downstream industry will be reduced by \(m - n\). The case \(n < m\) generalizes all previous analyses.
For example, when $n = 1$ and $m \to \infty$, we have the Warren-Boulton (1974) model; when $n = 1$ and $m$ is a finite number, we have the case analyzed by Waterson (1982). Since in all the earlier models $n = 1$ and merger resulted in monopolization of the downstream industry, there was always a profit incentive to merge. What we show next is that when $m > n \geq 2$, an incentive to integrate does not always exist. Note that unlike situations considered earlier (i.e., $m = n$ and $m < n$) the price after integration may rise or fall when $m > n$ depending on the parameter values. It should be noted that when $n = 1$ the price falls only if $\sigma < 1$ (see Mallela and Nahata (1980) and Westfield (1981) and Lee (1987). However, when $n \geq 2$, the price can fall when $\sigma < 1$ or $\sigma > 1$. When $n = 1$ and the price falls there is always a profit incentive to integrate. This is no longer the case when $n \geq 2$. When $P_x < P_*$, we demonstrate with the help of numerical examples that there may or may not be profit incentive. Infinite number of parameter combinations exists. In the examples below we keep all other parameters constant and vary either $\sigma$ or $m$. In all examples the following parameter values are assumed $\eta = 1.1$, $\delta = 0.6$, $n = 2$, $P_b = 2$, $M_a = 1$.

**Example 1:** Let $\sigma = 0.4$ and $m = 5$. The following values are computed before and after merger

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\[ P_a = 2.63051, \quad MC_x = 4.62662, \quad P_x = 5.65476, \quad X = 0.148712, \quad A = 0.151949 \]

\[ B = 0.144166, \quad E = 0.80665, \quad \Pi_a = 0.247753, \quad \Pi_x = 0.152895, \quad \Pi_a + \Pi_x = 0.400648, \quad \overline{MC_x} = 2.827766, \quad \overline{P_x} = 5.18424, \quad \overline{X} = 0.163624, \quad \overline{\Pi_x} = 0.385575, \]

\[ \overline{P}_x < P_x \text{ and } \overline{\Pi}_x < \Pi_a + \Pi_x. \text{ No profit incentive to integrate.} \]

**Example 2:** Let \( \sigma = 0.8 \) and \( m = 5. \)

\[ P_a = 2.05373, \quad MC_x = 3.996946, \quad P_x = 4.88515, \quad X = 0.174677, \quad A = 0.197744 \]

\[ B = 0.14603, \quad E = 0.9745, \quad \Pi_a = 0.208369, \quad \Pi_x = 0.1551498, \quad \Pi_a + \Pi_x = 0.363518, \quad \overline{MC_x} = 2.6634, \quad \overline{P_x} = 4.88292, \quad \overline{X} = 0.174764, \quad \overline{\Pi_x} = 0.38789, \]

\[ \overline{P}_x < P_x \text{ and } \overline{\Pi}_x > \Pi_a + \Pi_x. \text{ Profit incentive to integrate.} \]

**Example 3:** Let \( \sigma = 1.2 \) and \( m = 3. \)

\[ P_a = 1.78427, \quad MC_x = 3.63731, \quad P_x = 5.21875, \quad X = 0.162434, \quad A = 0.206843 \]

\[ B = 0.110879, \quad E = 1.13753, \quad \Pi_a = 0.1622215, \quad \Pi_x = 0.25688, \quad \Pi_a + \Pi_x = 0.41901, \quad \overline{MC_x} = 2.51376, \quad \overline{P_x} = 4.60855, \quad \overline{X} = 0.186243, \quad \overline{\Pi_x} = 0.39014, \]

\[ \overline{P}_x < P_x \text{ and } \overline{\Pi}_x < \Pi_a + \Pi_x. \text{ No profit incentive to integrate.} \]

**Example 4:** Let \( \sigma = 1.2 \) and \( m = 4. \)

\[ P_a = 1.78427, \quad MC_x = 3.63731, \quad P_x = 4.70711, \quad X = 0.181958, \quad A = 0.231705 \]

\[ B = 0.124206, \quad E = 1.13753, \quad \Pi_a = 0.18172, \quad \Pi_x = 0.194658, \quad \Pi_a + \Pi_x = 0.376377, \quad \overline{MC_x} = 2.51376, \quad \overline{P_x} = 4.60855, \quad \overline{X} = 0.186243, \quad \overline{\Pi_x} = 0.39014, \]

\[ \overline{P}_x < P_x \text{ and } \overline{\Pi}_x > \Pi_a + \Pi_x. \text{ There is profit incentive to integrate.} \]
**Theorem 3.** When the number of upstream firms \((n \geq 2)\) is less than the number of downstream firms \((m > n)\) and \(\min (m, n) = n\) firms vertically integrate and \(m - n\) firms drop out (horizontal effect), then there always exists a profit incentive if merger results in an increase in price of the final product.

**PROOF:** When \(P_x > P_x\). There exists a profit incentive if and only if

\[
\Pi_a < \Pi_x - \Pi_x
\]

Note that \(\Pi_a = P_a A - M_a A = MC_x X - P_b B - M_a A = MC_x X - \overline{MC}_x X\).

Note, \(\Pi_a\) represents the amount of input distortion resulting from a price higher than the marginal cost for input \(A\) before merger. \(\Pi_a\) is equal to the difference in the cost of producing the pre-merger output, \(X\). Substituting for \(MC_x\) and \(\overline{MC}_x\) and \(\Pi_x\), and \(\Pi_x\), it is enough to prove

\[
X \left(\frac{m\eta - 1}{m\eta} P_x - \frac{n\eta - 1}{n\eta} P_x\right) < \frac{P_x X}{n\eta} - \frac{P_x X}{m\eta} \tag{2.56}
\]
which upon simplification reduces to

\[
\frac{P_x - P_x}{P_x} < \frac{X - X}{n\eta X}
\]

Substituting for \( X \) for \( \overline{X} \), the above inequality reduces to

\[
n\eta \left( \frac{P_x - P_x}{P_x} \right) < \left( \frac{P_x}{P_x} \right)^\eta - 1
\]

or

\[
1 - \left( \frac{P_x}{P_x} \right)^\eta < n\eta \left( \frac{P_x - P_x}{P_x} \right)
\]  \hspace{1cm} (2.57)

Note that when \( P_x > P_x \) and \( \eta \) is less than one, then \( \left( \frac{P_x}{P_x} \right)^\eta > \frac{P_x}{P_x} \); hence, it is enough to prove that

\[
1 - \frac{P_x}{P_x} < n\eta \left( \frac{P_x - P_x}{P_x} \right)
\]

which is true because \( n\eta > 1 \). However, when \( \eta > 1 \), then (57) can be written as

\[
1 - \left( 1 - \frac{P_x - P_x}{P_x} \right)^\eta < n\eta \left( \frac{P_x - P_x}{P_x} \right)
\]
Using binomial inequality and noting that \( \eta > 1 \), it is enough to prove that

\[
1 - 1 + \eta \left( \frac{P_x - P_z}{P_x} \right) < n \eta \left( \frac{P_x - P_z}{P_x} \right)
\]

The above inequality always holds because \( n > 1 \).

Q.E.D.

### 2.5 Conclusions

The important conclusions and implications of the analysis can be summarized as follows.

1. Pure vertical integration does not offer any profit incentive in the case of successive oligopolies.

2. For profit incentive to exist, horizontal effects must accompany a vertical merger. However, presence of horizontal effect is only a necessary condition for profit incentive.

3. If horizontal effects dominate the vertical effects, that is, the post-merger price exceeds the pre-merger price there is always a profit incentive.
4. If vertical effects dominate the horizontal effects (pre-merger price higher than the post-merger price) the question of profit incentive cannot be answered without a priori knowledge of demand and cost conditions and the number of firms in both the upstream and downstream industries.

5. Since pure vertical merger neither increases market power nor offers any profit incentive, it should not concern the antitrust authorities. In fact, if profit incentive is the motivation for the merger then it should not take place.

6. When horizontal effects accompany a vertical merger the policy implications are not quite clear. Since with horizontal effects the post merger price can either rise or fall, the presence of horizontal effects is not enough to disallow a merger. As earlier studies have shown that even with a complete monopolization of downstream industry, there may be a welfare gain.

7. When the horizontal effects dominate and post-merger price rises, there is a loss in consumer surplus, but at the same time there is a gain in productive efficiency. In this case, the appropriate policy depends on
the interpretation of the antitrust laws. Since there is no consensus among economists and legal scholars regarding the goals of antitrust (see, Lande 1982), it is hard to prescribe policy guidelines. Clearly, the policy would depend on whether efficiency should be considered as a primary goal or wealth transfer.
2.6 References


Chapter 3

On the Profitability of Vertical Integration
Abstract

The central purpose of this paper\textsuperscript{1} is to analyze the private profitability of vertical integration within the context of models commonly used to investigate social welfare implications of integration. Particularly when upstream numbers are relatively small, it turns out that integration is commonly not profitable. The paper includes a rigorous analysis of the requirements for private profitability, and an examination of the profitability of the outside sales.

\textsuperscript{1}This chapter is a revised version of Abiru et al.(1992)
3.1 Introduction

Over the past few years, there have been a number of examinations of the effects of vertical mergers; see e.g., Greenhut and Ohta (1976, 1978, 1979), Perry (1978), Salinger (1988), Ordover, Saloner and Salop (1990), Hart and Tirole (1990). For the most part, these have taken a welfare economic stance. As a result, they have failed to provide a complete consideration of the centrally important issue of the profitability of vertical merger. We step back to ask: Within a straightforward two stage oligopoly model, is vertical integration privately desirable? Clearly some vertical mergers are perceived as profitable ex ante. In some cases they will come up for antitrust authority attention. But to examine welfare issues without regard to profitability may lead to empty theorising. Surprisingly perhaps, under commonly-used assumptions many are unprofitable.

2The following opening quote from Greenhut and Ohta (1979) is instructive: “Vertical integration of successive monopolists has long been known to provide merging monopolists with greater profit and their customers with greater outputs at lower prices... similar welfare attributes apply to mergers between monopolist input suppliers and Cournot-type oligopolists”. (p.137, emphasis ours).

3There is an additional question of why, if some mergers would have a profitable outcome, they have not already taken place. At least two explanations can be suggested. First, the antitrust regime of a country might move from a constrained to a more laissez-faire stance on vertical merger. Second, there may be a structural change in the characteristics of an industry which could facilitate vertical integration if it were profitable (for example plants which previously had to locate some distance apart may find a proximate location
The central purpose of this paper is to examine the effects on profitability of vertical merger within the context of a model along the lines of the one in Salinger's (1988) paper. Indeed he looks at the issue briefly, but rather incompletely – almost as an afterthought: “Mergers in this model do not necessarily increase the joint profits of the merging firms. This result seems odd – ”(p.353)

In fact, the question is rather complex. There are at least five forces impacting upon the profitability of a vertical merger without going into transaction cost issues (which we ignore here, in common with much of the literature cited above). Three act so as to favour integration. Integration removes the output restricting effects of double marginalization and also removes input distortions due to non marginal cost pricing of input, where inputs may be used in variable proportions. It also has a transient effect discussed in a moment.

Two, less often identified, forces have the opposite impact. First, for reasons discussed by Bonanno and Vickers (1988) amongst others, with equal numbers at each stage of production, vertical separation can be more profitable because it induces more 'friendly' behaviour at the upstream end. This feasible).
force is particularly important in situations where there is price competition between the (differentiated) products at the final stage, but it is also relevant to homogeneous product market cases like ours (Lin, 1988). Second, and so far as we know not previously identified explicitly, there is an unequal numbers effect. If one assumes, with Salinger (1988) for example, that integrated firms do not sell to outsiders, then integration can reduce output, in contrast to what might be expected. Of course, the first integration gains greatly through having production cost advantages; this is the third favourable force. Certainly this is true, but on the other hand if there are more downstream than upstream firms then the remaining unintegrated upstream firms sell to relatively more firms in the downstream industry after integration than before. For example if, prior to integration, there were three upstream and 12 downstream firms, and one upstream integrates with one downstream, then if there is no exit, the remaining two upstream firms sell to 11 downstream firms. As a result, the integrated firm would be upstream supplier to $\frac{1}{12}$ of the downstream firms (though more than $\frac{1}{12}$ of the output). The integrated firm’s market share from the perspective of the integrating upstream firm is reduced, ceteris paribus. Hence integrations are penalized.

In the literature, in order to reduce the complexity of the analysis, the
variable proportions effects are often left out (see, e.g., Greenhut and Ohta, 1979). Also, it will be apparent that the impact of the double marginalization force is strongest when small numbers are involved. Hence in relatively large number cases, and especially where there are more downstream than upstream suppliers, forces acting so as to favour disintegration will be relatively strong. Just how strong is something we examine in section 2, after having developed the model in section 1.

It is also evident from our limited discussion of the transient force so far that the eventual impact upon profitability of integration may well be less favourable than the impact of the first integration. So what is relevant to our considerations? Again there has been some confusion in the literature on this point. It seems clear to us that a necessary condition for the first integration to take place is that it results in a profitability gain. Whether that is also sufficient depends upon how sophisticated thinking is believed to be. If the first firm is farsighted enough to appreciate that further integration may be profitable to the firms involved given the first integration but that eventually the industry ends up with lower profitability all round (for vertical

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4Greenhut and Ohta (1979, p140) agree “that vertical integration requires profit incentives ... However, such merger does not require greater industry profits.”

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separation reasons), then the first integration may not take place. In section 3 we consider these strategic issues using game-theoretic tools standard in the analysis of horizontal integration.

Sections 4 and 5 consider extensions to the framework. The former reconsiders the assumption that an integrated firm does not sell to the outsiders and shows that outside sale can be profitable. Section 5 introduces variable proportions. Finally, we draw together some concluding remarks in section 6.
3.2 Basic Model - fixed proportions

The purpose of this section is essentially to set out the framework and some key equations used in later sections. Consider successive oligopolies where there are $N$ identical Cournot oligopolists in the upstream stage and $m$ identical Cournot oligopolists in the downstream stage (all notations are defined in Table 1). Before merger, each upstream firm buys the raw material from a competitive market at given unit price $M_a$, and sells a homogeneous input $A$ at price $P_a$ in oligopolistic market. Each downstream firm produces a final product using an input $B$ available at given unit price $P_b$ in competitive market and the input $A$. The downstream firm has no oligopsony power over the input. It chooses its final product level $x_n$ so as to maximize profit, taking input price as a parameter. Following Salinger (1988) we specify linear industry demand and fixed-proportions technology (we relax the latter assumption later) where one unit of output requires one unit of each of the two inputs.

Indeed, equations (3.1)-(3.16) below largely parallel Salinger’s (1), (2a),..., (8), though with more explicit statement of profit.\(^5\) We assume that an

\(^5\)Salinger’s equation (4) contains an innocuous misprint.
arbitrary number of firms (\(= \mu \)) are integrated and the integrated firms do not supply the input to nonintegrated downstream firms (later we evaluate when such an arrangement is feasible or profitable).

Thus the final demand function is assumed to be:

\[
P_x = a - bX, \quad a, b > 0, \quad \eta = \frac{(a - bX)}{bX}. \quad (3.1)
\]

### 3.2.1 Profit-maximization in downstream industry.

The profit of each integrated firm is defined by:

\[
\Pi^{int}(\mu) = P_x^a x_i - (M_a + P_b)x_i; \quad i = 1, 2, \ldots \mu. \quad (3.2)
\]

The profit of each non-integrated firm is defined by:

\[
\Pi^d(\mu) = P_x^a x_n - (P_a + P_b)x_n; \quad n = \mu + 1, \mu + 2, \ldots m. \quad (3.3)
\]
Table 1: Notations

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ and $B$</td>
<td>the two inputs used</td>
</tr>
<tr>
<td>$A_n(\mu)$</td>
<td>quantity of input sold by a nonintegrated upstream firm to a nonintegrated downstream producer</td>
</tr>
<tr>
<td>$X(\mu)$</td>
<td>total output produced by downstream industry</td>
</tr>
<tr>
<td>$X_n(\mu)$</td>
<td>total output produced by all nonintegrated downstream producers</td>
</tr>
<tr>
<td>$x_i(\mu)$</td>
<td>output of an integrated firm $\frac{X-X_n}{\mu}$</td>
</tr>
<tr>
<td>$x_n(\mu)$</td>
<td>output of a nonintegrated downstream firm $\frac{X_n}{m-\mu}$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>marginal (=average) cost of an integrated firm</td>
</tr>
<tr>
<td>$c_n(\mu)$</td>
<td>marginal (=average) cost of a nonintegrated firm</td>
</tr>
<tr>
<td>$P_f(\mu)$</td>
<td>price of the final product when $\mu$ firms are integrated</td>
</tr>
<tr>
<td>$m$</td>
<td>number of firms in the downstream industry</td>
</tr>
<tr>
<td>$\eta$</td>
<td>the price elasticity of demand $-\frac{dX P}{dP X} = \frac{2-P\mu}{b\mu}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>number of integrated firms</td>
</tr>
<tr>
<td>$N$</td>
<td>number of upstream firms producing input $A$</td>
</tr>
<tr>
<td>$M_a$</td>
<td>marginal cost of input $A$, assumed to be constant</td>
</tr>
<tr>
<td>$P_a(\mu)$</td>
<td>profit-maximizing price for input $A$ charged by the nonintegrated upstream firms to the nonintegrated downstream firms</td>
</tr>
<tr>
<td>$P_b$</td>
<td>price of input $B$ which is supplied by a competitive industry</td>
</tr>
<tr>
<td>$\Pi^u(\mu)$</td>
<td>profits of each nonintegrated upstream producer when $\mu$ firms are integrated</td>
</tr>
<tr>
<td>$\Pi^d(\mu)$</td>
<td>profits of each nonintegrated downstream firm when $\mu$ firms are integrated</td>
</tr>
<tr>
<td>$\Pi^{int}(\mu)$</td>
<td>profits of each integrated firm when $\mu$ firms are integrated</td>
</tr>
</tbody>
</table>

Note that all variables are written as a function of $\mu$ and parameters are independent of $\mu$. 

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The profit maximization condition for each integrated firm is:

\[ P_a^a - \frac{x_i}{\eta x} P_p^a = c_i = M_a + P_h; \quad i = 1, 2, \ldots, \mu. \]  

(3.4)

The profit maximization condition for each non-integrated firm is:

\[ P_a^a - \frac{x_n}{\eta X} P_p^a = c_n = P_a + P_h; \quad n = \mu + 1, \mu + 2, \ldots, m. \]  

(3.5)

Aggregating in equations (3.4) and (3.5) we obtain

\[ \mu P_a^a - \frac{\mu x_i}{\eta X} P_p^a = \mu(M_a + P_h). \]  

(3.6)

\[ (m - \mu) P_a^a - \frac{(m - \mu)x_n}{\eta X} P_p^a = (m - \mu)(P_a + P_h). \]  

(3.7)

Summing equations (3.6) and (3.7) and noting \( \mu x_i + (m - \mu)x_n = X \) we get

\[ P_a^a \left(1 - \frac{1}{m\eta}\right) = c_n - \frac{\mu}{m}(c_n - c_i) = (P_a + P_h) - \frac{\mu}{m}(P_a - M_a). \]  

(3.8)

which holds when partial integration applies i.e., \( \mu < N < m \).
From equations (3.1) and (3.8) we obtain the derived demand for the intermediate good in terms of the parameters and the total output of the final product $X$:

$$P_a = \frac{m(a - P_b) - \mu M_a}{m - \mu} - \frac{b(m + 1)}{m - \mu} X \quad (3.9)$$

which holds only when $\mu < N$. We can also derive the expression for derived demand in terms of total output of nonintegrated firms, i.e., $(m - \mu)x_n$ after obtaining $X$ from equations (3.7) and (3.1) and substituting it into equation (3.9) as follows:

$$P_a = \frac{a - P_b + \mu M_a}{\mu + 1} - \frac{b(m + 1)}{(\mu + 1)(m - \mu)} X_n \quad (3.10)$$

where $X_n = (m - \mu)x_n = (N - \mu)A_n$.

### 3.2.2 Profit-maximization in the upstream industry

Profit of each non-merging upstream firm is defined by:

$$\Pi^u(\mu) = P_a A_n - M_a A_n; \quad n = \mu + 1, \mu + 2, ... N \quad (3.11)$$

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The profit-maximization condition yields:

\[ P_a \left(1 - \frac{1}{(N - \mu)E}\right) = M_a, \quad E = -\frac{dA}{dP_a} \frac{P_a}{A} \]  

(3.12)

### 3.2.3 Equilibrium

equations (3.10) and (3.12) give the equilibrium input price \( P_a \) as follows:

\[ P_a = \frac{(a - P_b - M_a)}{(\mu + 1)(N - \mu + 1)} + M_a \]  

(3.13)

Now we can solve for the total output \( X \) from equations (3.9) and (3.13):

\[ X = \frac{(a - P_b - M_a)(Nm\mu - m\mu^2 + Nm + \mu)}{b(m + 1)(\mu + 1)(N - \mu + 1)} \]  

(3.14)

Using equations (3.10) and (3.13) we obtain

\[ X_n = \frac{(a - P_b - M_a)(N - \mu)(m - \mu)}{b(m + 1)(N - \mu + 1)} \]  

(3.15)
Final product price is then obtained from equation (3.1) as:

\[
P_x = \frac{a(N\mu + N - \mu^2 + m - \mu + 1) + (P_b + M_a)(Nm\mu - m\mu^2 + Nm + \mu)}{(m + 1)(N - \mu + 1)(\mu + 1)}
\]  

(3.16)

Each upstream firm’s profit is obtained using equations (3.11), (3.13) and (3.15), and noting that \( A_n = \frac{x_n}{N-\mu} \),

\[
\Pi^u(\mu) = \frac{(m - \mu)(a - P_b - M_a)^2}{b(\mu + 1)(m + 1)(N - \mu + 1)^2}
\]  

(3.17)

Each downstream firm’s profit is obtained using equations (3.1), (3.3), (3.8),(3.13) and (3.14) and noting that \( x_n = \frac{x_n}{m-\mu} \)

\[
\Pi^d(\mu) = \frac{(a - P_b - M_a)^2(N - \mu)^2}{b(m + 1)^2(N - \mu + 1)^2}
\]  

(3.18)

Each integrated firm’s profit is obtained using equations (3.1), (3.2), (3.4),(3.8) and (3.14)

\[
\Pi^{int}(\mu) = \frac{1}{b} \left[ \frac{(a - P_b - M_a)[(m + N + 1) + \mu(N - \mu - 1)]}{(\mu + 1)(m + 1)(N - \mu + 1)} \right]^2
\]  

(3.19)
3.3 Profit Incentive for Integration

3.3.1 First-pair incentive

Whether the industry will be integrated, either partially or fully, depends quite importantly on profit incentives for the first-pair. Both Greenhut and Ohta (1979) and Salinger (1988) have implicitly assumed that the first-pair incentive always exists. Yet, the condition we derive below clearly demonstrates that it may in many cases not exist. Therefore, in general one cannot assume that an industry is initially integrated partially and analyze the effect of an additional merger as Salinger has done. This issue has not been explored fully in the literature. Our analysis shows that the first-pair incentive plays a role when there are unequal numbers of firms. In particular, depending upon the relative number of upstream and downstream firms, a first-pair incentive need not exist.

Profit incentive for the first-pair exists, if and only if,

\[ \Pi^{\text{int}}(1) > \Pi^u(0) + \Pi^d(0) \]  

(3.20)

that is, the merged firm’s profit must exceed the sum of the upstream and
the downstream firm’s pre-merger profits.

Each upstream firm’s pre-merger profit can be obtained from equation (3.17) by putting $\mu = 0$.

$$\Pi^u(0) = \frac{m(a - P_b - M_a)^2}{b(m + 1)(N + 1)^2}. \quad (3.21)$$

Equation (3.18) gives a downstream firm’s profit before merger.

$$\Pi^d(0) = \frac{1}{b} \left( \frac{N(a - P_b - M_a)^2}{(N + 1)(m + 1)} \right). \quad (3.22)$$

If only one pair is integrated vertically, then the merged firm’s profit can be obtained from equation (3.19) by substituting $\mu = 1$:

$$\Pi^{int}(1) = \frac{1}{b} \left( \frac{(a - P_b - M_a)(2N + m - 1)}{2N(m + 1)} \right)^2. \quad (3.23)$$

The following Lemma follows immediately from (3.20)- (3.23);

**Lemma 1.** By itself first-pair integration is profitable, if and only if,

$$\left( \frac{2N + m - 1}{2N} \right)^2 > \frac{N^2 + m^2 + m}{(N + 1)^2} \quad (3.24)$$
It is important to note that this inequality does not hold for all $m$ and $N$. Therefore one cannot assume that an incentive for first-pair integration always exists. Because of the lack of first-pair incentive in many instances the industry may have no integrated firm. First, we focus our attention on the case when $m \leq N$.

**Lemma 2.** If $m \leq N$ first-pair integration is always profitable.

Algebraic Proofs of this and all later Lemmas may be found in the Appendix.

### 3.3.2 $m \leq N$: Incentive for second-pair and full integration

Suppose that it is profitable for the first-pair to integrate and it has integrated. Does an incentive for subsequent integrations exist? This is an important issue which to date appears not to have been examined in any detail. In this section we explore the profit incentive for the second and third pairs and so on until the process stops when no more integration is feasible. The result is surprisingly clear-cut within our modeling framework.
Integration incentive for the \((\mu + 1)\)st-pair exists if and only if,

\[
\Pi^{\text{int}}(\mu + 1) > \Pi^e(\mu) + \Pi^d(\mu)
\]

This condition can be written using equations (3.17), (3.18) and (3.19) as:

\[
\left(\frac{m + 1 + (\mu + 2)(N - \mu - 1)}{(\mu + 2)(N - \mu)}\right)^2 > \frac{(\mu + 1)(N - \mu)^2 + (m - \mu)(m + 1)}{(\mu + 1)(N - \mu + 1)^2}
\]

(3.25)

The above inequality provides conditions for profit incentive for subsequent integration. For example, when one pair has integrated (i.e., \(\mu = 1\)) the incentive for second-pair exists (put \(\mu = 1\) in equation (3.25)), if and only if,

\[
\left(\frac{m + 1 + 3(N - 2)}{3(N - 1)}\right)^2 > \frac{2(N - 1)^2 + (m - 1)(m + 1)}{2N^2}
\]

Similarly, the condition for the third-pair incentive can be obtained by putting \(\mu = 2\) and so on. The condition for the final-pair integration can be obtained by putting \(\mu = N - 1\), when \(m > N\), and \(\mu = m - 1\) when \(m < N\).

For a given number of upstream firms \(N\), Table 2 shows the number of
downstream firms, \( m \), necessary for a first-pair incentive to exist. It also shows that whenever there exists an incentive for the first-pair to integrate, then there also exists an incentive for subsequent integration. Because there is always an incentive for subsequent integration, the process results in a fully integrated industry. Table 2, obtained from inequality (3.25) above, clearly shows that partial integration is not a feasible structure in the long run within the present framework.\(^6\)

**Table 2. Profit Incentive for Subsequent and Full Integration**

<table>
<thead>
<tr>
<th>N</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Pair ( \mu = 0 )</td>
<td>( m \leq 5 )</td>
<td>( m \leq 6 )</td>
<td>( m \leq 7 )</td>
<td>( m \leq 9 )</td>
<td>( m \leq 9 )</td>
<td>( m \leq 10 )</td>
<td>( m \leq 11 )</td>
</tr>
<tr>
<td>Second-Pair ( \mu = 1 )</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Third-Pair ( \mu = 2 )</td>
<td>-</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Fourth-Pair ( \mu = 3 )</td>
<td>-</td>
<td>-</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Fifth-Pair ( \mu = 4 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sixth-Pair ( \mu = 5 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Seventh-Pair ( \mu = 6 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Eighth-Pair ( \mu = 7 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Lemma 3.** For \( m \leq N \) there is also a profit incentive for the second-pair integration, the \((\mu + 1)st\) pair integration and so on until no further integration

\(^6\)This statement turns out to be false after we derive Lemma 5 and 6 later. In the previous version we derived the lemma 3 based on the Table 2. We now proved the result without relying on computer simulations.
is feasible.

It is important to note here that Lemma 3 does not imply that each merging firm’s profits are greater than the initial state of no-integration ($\Pi^a(0) + \Pi^d(0)$). Once the process of integration begins, the incentive for the second and third pairs exists in the sense that if they do not integrate their profits decline more rapidly than the profits of those who do integrate. Thus Lemma 3 describes the firms’ myopic behavior resulting in full integration. Lemma 4 below compares the profits between the initial state of no integration and the state when all firms become integrated.

**Lemma 4.** When $m = N$ and all firms become integrated then the profits of an integrated pair are always lower than their combined profits at the initial state of no integration. For $m < N$ full integration is profitable compared to the initial state if and only if $N > \frac{m^2 + m - 1}{2}$.

The profitability of full integration holds for many combinations of $N$ and $m$. For example, $m = 2$ and $N \geq 3$, $m = 3$ and $N \geq 6$ and so on. Thus when $m \leq N$, we have shown that there is always the first-pair incentive and at the same time an incentive for the subsequent integration that leads to full integration. However, compared with the initial state of no integration, a
fully integrated firm may find itself worse off eventually. In the next section, we examine how the results of these Lemmas 2-4 change when \( m > N \).

### 3.3.3 \( m > N \): First-Pair and Complete Integration

**Incentive**

In this section we consider the case when \( m > N \). The reason this case is analyzed separately is that the conclusions differ significantly compared to the case when \( m \leq N \). The following Lemmas summarize the main results.

**Lemma 5.** When \( m > N \) the first pair incentive exists only if

\[
m < \frac{36N^3 + 18N^2 + 24N + 10 + 36N\sqrt{N^4 + 4N^3 - 4N^2 - 2N + 2}}{18(3N^2 - 2N + 1)}
\]

(3.26)

**Lemma 6.** When \( m > N \) the incentive for final pair integration exists only if

\[
m < \frac{(N^3 + 5N - 2) + \sqrt{(N^3 + 5N - 2)^2 - 4(N - 1)^4}}{2(N - 1)^2}
\]

(3.27)

Comparison between Lemmas 2 and 5, and 3 and 6 shows how unequal number of firms impact upon the profitability of vertical integration.

First, when \( m \leq N \) there is always an incentive for the first-pair to
integrate but this is not the case when $m > N$. An example will help to explain this result intuitively. Suppose that $N = 16$ and $m = 4$. Before any integration each upstream firm supplies one-fourth of the requirements of the downstream firms and each downstream firm produces one quarter of the final product. The first integration benefits the integrated firm because it obtains a larger share of the final product market because of lower cost and also there is no loss of sales from the upstream branch (the upstream division will produce more after integration). Thus there is always an incentive for the first-pair to integrate. Now take the opposite case where $N = 4$ and $m = 16$. In this situation before integration each upstream firm supplies to four downstream firms and each downstream firm produces one-sixteenth of the final product output. It is true that the integrated downstream firm gains through the larger share of the final product market due to lower cost, but at the same time the integrated firm also loses the outside sales from its upstream division (the unintegrated upstream firms now supply to five downstream firms compared to four before). It is quite possible that the upstream division of the integrated firm may actually be producing less after integration than before. When this loss in sales of its upstream division is relatively large compared to the gain in the final product market then the
first integration may not be profitable. This is precisely the reason that the first-pair integration is not always profitable when \( m > N \). In order for the first-pair integration to be profitable, the difference between the numbers of downstream and upstream firms has an upper bound.

Second, for \( m \leq N \) not only does the incentive for first-pair always exist but after first-pair has integrated, the incentive for complete integration also always exists. Again, this can be explained using the same example as above. When \( N = 16 \) and \( m = 4 \) there is an incentive for first integration. After first integration the unintegrated downstream firms lose market and their profits also fall. The unintegrated upstream firms also lose market on two counts. First, each unintegrated upstream firm supplies one-fifth of the requirements as opposed to one-fourth before any integration and second, the total requirement for their input is also lower because the remaining unintegrated downstream firms now produce less of the final product. Since both unintegrated downstream and upstream firms lose profits after first integration, it pays for the second-pair to get integrated. The same argument applies to the third-pair and so on until all firms become integrated.

For \( m > N \) the incentive for the final-pair integration need not exist although the incentive for first-pair exists. The reason this may happen is as
follows. The unintegrated upstream firms supply a greater proportion of the unintegrated downstream firms' requirements as integration continues. After some firms have integrated this may eventually result in a disincentive for any further integration because the gain in profits from each subsequent integration decreases (the increase in output due to integration becomes smaller and smaller) and the loss from the outside sales from the upstream division becomes larger and larger. Thus it is quite possible that after enough firms integrate the profits of the remaining unintegrated upstream firms may even be higher than the profits at the initial state of no integration. When this happens the gain due to further integration cannot compensate for the loss in profits of the upstream firms.

Finally we derive Lemma comparing profits under full integration with profits under the initial state of no integration.

**Lemma 7.** *When \( m > N \) and all firms become integrated then the profits of an integrated pair is higher compared to initial state if and only if*

\[
(m + 1) > N^2
\]

(3.28)

Comparison of Lemmas 4 and 7 reveals that complete integration may
not be profitable compared to the initial state of no integration under both situations.

In the following section we evaluate the relevance of the above propositions using a game theoretic approach.

3.4 A Game-theoretic Approach to Integration

Lin (1988) has provided some analysis on the question of profit incentives. He concludes that vertical separation (disintegration) may be desirable in many situations, especially if demands are sufficiently inelastic or if franchising opportunities are available (p.251). Bonnano and Vickers (1988) also arrive at a similar conclusion independently. Both Lin and Bonnano and Vickers consider a duopoly model with differentiated products. In a homogeneous product model such as the one considered here, and also analyzed by Greenhut and Ohta (1979), a Prisoner's Dilemma may arise [see Lin(1988, p.254) and Greenhut and Ohta (1979, p.140)]. As noted by Lin, “The reason why a Prisoner's Dilemma occurs in some parametrizations of the model is not well
understood.” (p.254). In this section we investigate the issue rather more systematically.

The problem of first-pair and second-pair or full integration incentive can be presented alternatively as a 2-person normal form game. We consider two firms in the upstream industry and $m$ firms in the downstream industry. Player 1 is the first-pair firm and Player 2 is the second-pair firm. Player 1, 2 choose their actions simultaneously from the action space $\{i,n\}$. Here $i$ stands for "integration" and $n$ stands for "non-integration". Each player’s payoff in a period depends on the number of downstream firms $m$. The payoff matrices for all cases discussed in this section are obtained using the following parameter values: $N = 2$, $b = 0.9$, $a = 6$, $P_b = 0.6$ and $M_a = 0.4$. Different values of $m$ are used to characterize different scenarios.

3.4.1 Vertical separation

Vertical separation is desirable when there is no incentive for the first-pair to integrate (and also there is no incentive for the complete integration). Although vertical separation may be a desirable outcome when there are equal number of firms in upstream and downstream industry, the incentive
to remain disintegrated is more pronounced when there are unequal number of firms in upstream and downstream stages. The force favoring vertical separation when there are unequal number of firms is not very well recognized in the literature and hence the effect on profitability of vertical mergers has not been fully explored. It is clear from Table 2 and Lemmas 1 and 7, that in many instances it is more profitable not to integrate at all. For example, when $N = 3$, the number of downstream firms should not exceed 6 for profitable first-pair incentive but for complete integration to be profitable the number of downstream firms should be at least 9 (or a monopolist). Clearly when $N = 3$ and $6 < m < 9$ vertical separation should prevail. Similarly, for other cases for $N > 3$ the number of downstream firms implying a separation equilibrium can be obtained from Table 2.

### 3.4.2 Prisoner’s Dilemma

**Table 3. Case 2: Incentive vs. No Incentive (m=2)**

<table>
<thead>
<tr>
<th>Player 1 \ Player 2</th>
<th>i</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$(3.08642, 3.08642)$</td>
<td>$(4.82253, 1.92901)$</td>
</tr>
<tr>
<td>n</td>
<td>$(1.92901, 4.82253)$</td>
<td>$(3.42935, 3.42935)$</td>
</tr>
</tbody>
</table>

A classic Prisoner’s Dilemma argument assumes a one-shot game in which
the solution is that if player 1 integrates then player 2 should also. Although both are worse off than if neither integrated, integration is in the individual but not in the joint interests of the players. Notice in Table 3 that compared to the initial state, the first-pair integration is profitable and the second is not, but that if the first integration had taken place, the second firm would want to integrate. This outcome can be explained using propositions 2, 3, and 4. When \( m = N \) there is always a profit incentive for the first-pair. However, when there is an incentive for the first-pair to integrate there is also an incentive for the second-pair to integrate. But because the condition of proposition 4 can never be satisfied when \( m = N \), the firms find themselves worse off after they all have integrated, resulting in the Prisoner’s Dilemma. Table 3 shows the payoffs of this one-shot game.

However if we consider an infinite period game, in some circumstances (n, n ) will be a solution. It is well known from the Folk Theorem that in a repeated game, several strategies including the Grim strategy and various trigger strategies yield a Nash Equilibrium in which both players cooperate. This is also the case in the game assumed here. Let us consider it in a multi-stage context. At each stage \( t \), players 1, 2 choose their actions simultaneously from the action space \{i, n\}. We define total payoff as the
(discounted) sum of payoff in each period i.e., \( \lim_{T \to \infty} \sum_{j=1}^{T} \delta^t U_j^t \). (where \( \delta \) is the discount factor and \( U_j^t \) denotes \( j \)'s payoff at \( t \)). If we permit a player to choose 'n' after he chose 'i' in the previous stage, then our game is an infinitely repeated game with Prisoner’s Dilemma. We can then apply the Folk Theorem in our context. That is, if each player for example takes Grim strategy (i.e., choosing ‘n’ as long as his opponent chooses ‘n’, but choosing ‘i’ thereafter if his opponent chooses ‘i’), then their strategies constitute Nash equilibrium which yields ( \( n, n \) ) in all stages for a suitable discount factor.

Alternatively, our game may not be exactly the same as the repeated game stated above being instead the case where once a player chose ‘i’, he must remain ‘i’. However suppose that each player takes a strategy that he chooses ‘n’ in so far as the other player continues to play ‘n’, and if he chooses ‘i’, the other player also chooses ‘i’ in the following period, then again both players will choose ‘n’ to avoid the worst situation ( \( i, i \) ) for a suitably high discount factor. (Here \( \delta \geq 0.8 \) approximately)\(^7\) In sum, even if there is a

\(^7\)Integration is unprofitable for player 1 if

\[
\frac{\Pi_1(n,n)}{1-\delta} \geq \Pi_1(i,n) + \frac{\delta}{1-\delta} \Pi_1(i,i)
\]

where profits are written as a function of the first and second player respectively being integrated or not. Inserting the profit values from Table 3 enables us to solve for \( \delta \). The situation is of course symmetric for player 2.
first integration profit advantage, but a final integration profit disadvantage, it can be argued that in many cases integration will not occur.

### 3.4.3 No incentive vs. incentive

There may be cases where although there is no first-pair incentive there is nevertheless an incentive for complete integration. Table 4 gives an example payoff matrix when \( m = 6 \). It is clear from the matrix that the Nash solution is either \((i, i)\) or \((n, n)\). Whichever state the industry happens to be in, it is likely to remain there.

**Table 4. Case 3: No Incentive vs. Incentive (m=6)**

<table>
<thead>
<tr>
<th>Player 1 \ Player 2</th>
<th>i</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>(3.08642, 3.08642)</td>
<td>(2.8699, 2.62188)</td>
</tr>
<tr>
<td>n</td>
<td>(2.62188, 2.8699)</td>
<td>(2.89746, 2.89746)</td>
</tr>
</tbody>
</table>

**Table 5. Case 4: Incentive in both case (m=4)**

<table>
<thead>
<tr>
<th>Player 1 \ Player 2</th>
<th>i</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>(3.08642, 3.08642)</td>
<td>(3.40278, 2.36111)</td>
</tr>
<tr>
<td>n</td>
<td>(2.36111, 3.40278)</td>
<td>(2.96296, 2.96296)</td>
</tr>
</tbody>
</table>
3.4.4 Incentive both for first and final integration

Finally, in some circumstances there is an incentive for the first-pair to integrate and at the same time there is also an incentive for complete integration. This special case occurs in our model where $m > N$ only when $N = 2$ and $m = 4$ or 5. The payoff matrix is given in Table 5 for $m = 4$. In this case the Nash equilibrium solution is (i, i) because compared to initial state the first integration is profitable and so is the second. Only for this case can we say without any reservation that the integration will take place.\(^8\)

3.5 Equilibrium Levels of Integration

From the discussion above it is clear that at the equilibrium any one of the following three situations can emerge: (a) none of the firms are integrated (vertical separation), (b) full integration (all pairs that can integrate have integrated), and (c) partial integration, an intermediate situation where some firms are integrated and some are not.\(^9\) In this section we examine different

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\(^8\)However when $N > m$, in many cases there exist incentives for both first and complete integration. For example, from propositions 2 and 4, this holds when $m = 2$, $N \geq 3$ and $m = 3$, $N \geq 5$.

\(^9\)This section is basically the revised version of the previous section. When we wrote the previous section, we did not find any possibility of partial integration because we did not reach the Lemmas 5-6 which we have now in section 3.3 in previous version.
equilibrium situations using the following framework:

Stage 1: Each player (a pair of upstream and downstream firms) simultaneously decides whether to integrate or not. Once a player integrates, the player must remain integrated.

Stage 2 - The market stage: Trading takes place, integrated firms exclusively exchange internally, unattached firms drop out if none remain on the other side of the market.

Our solution concept is the standard subgame perfect equilibrium one. The conditions for stage 1 Nash equilibrium with $\mu$ firms integrated are the following:

$$\Pi^{\text{int}}(\mu) \geq \Pi^{u}(\mu - 1) + \Pi^{d}(\mu - 1)$$

(3.29)

(integrated firms do not wish to separate).

$$\Pi^{\text{int}}(\mu + 1) \leq \Pi^{u}(\mu) + \Pi^{d}(\mu)$$

(3.30)

(unintegrated firms do not wish to merge).

Using these, together with equations (3.17)-(3.19) and Lemmas 1-7 above

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These are conditions equivalent to those used by d’Aspremont et al (1983) concerning collusion, I am grateful to Steve Martin for drawing this to my attention.
enable us to derive a number of propositions.

We first note the following immediate implications of (3.29) and (3.30). The condition for full integration being an equilibrium is that a final-pair incentive exists. The condition for no integration being an equilibrium is that no first-pair incentive exists. Therefore:

**Proposition 1.** No integration is an equilibrium if the conditions of Lemma 1 fail to hold.

**Proposition 2.** Full integration is an equilibrium if the conditions of Lemma 6 hold.

(Note from lemma 3 that those conditions always hold for \( m \leq N \)) The proofs of these propositions are immediate.

**Proposition 3.** Full integration is the unique market equilibrium for all \( m \leq N \).

**Proof:** From (3.29) and (3.30) the condition for full integration being a unique equilibrium is that

\[
\Pi^{\text{int}}(\mu) \geq \Pi^u(\mu - 1) + \Pi^d(\mu - 1) \quad \text{for all} \ 0 \leq \mu \leq m
\]
Lemmas 2 and 3 together show that this condition holds true. Q.E.D.

It is worth remarking that this does not mean all firms are better off following integration. In fact, the contrary often holds - see Lemma 4.

In order to investigate the situation in a little more detail, in figure 1 we plot the curves $m = N$, the line below which (3.26) (Lemma 5) holds true, and the line to the left of which (3.28) (Lemma 7) holds true. This divides the spectrum into a number of areas.

Below the line $DK$, full integration is the unique equilibrium. In area $DBFK$ full integration is an equilibrium whereas no integration is not. More particularly, along line $CB$ (i.e., where $N = 2$ and $m = 4$ or 5) full integration is the unique equilibrium (since both first and final/second pair incentives exist). Above $AFH$, no integration is an equilibrium whilst full integration cannot be. Simulations suggest that no integration is the unique equilibrium, but a rigorous demonstration appears difficult. In the area bounded by $AFB$ both no integration and full integration (and possibly others) are equilibria. Finally in area $HFJ$, the only possibility is partial integration.

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11 For convenience, the curves are sketched as continuous functions of $m$ and $N$.  
12 The difficulty arises because of nonmonotonicities in the relevant inequality expression.
A few examples will help to clarify the nature of what happens in these last two rather interesting areas and in addition will show the sensitivity to the precise definition of equilibrium. For all examples we take $a = 6, b = 0.9, M_a = 0.4, P_b = 0.6$.

**Table 6 Outcomes where** $N = 2$ $m = 6$

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrate</td>
<td>(3.09, 3.09)</td>
<td>(2.87, 2.62)</td>
</tr>
<tr>
<td>Not</td>
<td>(2.62, 2.87)</td>
<td>(2.90, 2.90)</td>
</tr>
</tbody>
</table>

**Table 7 Outcomes where** $N = 3$ $m = 7$

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 3</th>
<th>(Player 2’s payoff and action first)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Int</td>
<td>(1.74, 1.74, 1.74)</td>
<td>(1.57, 1.46, 1.46)</td>
<td>(1.91, 1.35, 1.35)</td>
</tr>
<tr>
<td>Not</td>
<td>(1.46, 1.57, 1.46)</td>
<td>(1.35, 1.91, 1.35)</td>
<td>(1.81, 1.81, 1.81)</td>
</tr>
</tbody>
</table>

In example 1, $N = 2$, $m = 6$. Table 6 shows the payoffs to the first and second integrated pair. Clearly there are two equilibria in the simultaneous play game. However, consider the following alternative to stage one above, which includes a *sequential decision* framework.
Figure 1.
Stage 1: In each period, \( t = 1, 2, \ldots \), one pair, arbitrarily chosen, decides whether to integrate or not. Once a player integrates, it must remain integrated. Each player evaluates the total payoff without discounting.

With this alternative, it is clear on drawing out the game tree that full integration would take place. Example 2 in Table 7 has \( N = 3, m = 7 \). Here, again there are two equilibria. But on drawing the game tree we find that the sequential equilibrium is in this case no integration. Essentially the reason is that the second example is to the right of line \( CE \) (final profit) in figure 1, whereas the previous one lay to the left.

The final example concerns the case \( N = 6, m = 10 \) in region \( JFH \) shown in Table 8. In example 3 there is an incentive for the first-pair to integrate

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( P_a )</th>
<th>( \mu )</th>
<th>( x_i )</th>
<th>( x_n )</th>
<th>( \Pi^a(\mu) )</th>
<th>( \Pi^d(\mu) )</th>
<th>( \Pi^a(\mu) + \Pi^d(\mu) )</th>
<th>( \Pi^\text{int}(\mu) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.11</td>
<td>0.72</td>
<td>2.10</td>
<td>–</td>
<td>0.43</td>
<td>0.51</td>
<td>0.17</td>
<td>0.68</td>
</tr>
<tr>
<td>1</td>
<td>0.82</td>
<td>0.76</td>
<td>1.80</td>
<td>0.88</td>
<td>0.42</td>
<td>0.31</td>
<td>0.16</td>
<td>0.48</td>
</tr>
<tr>
<td>2</td>
<td>0.73</td>
<td>0.81</td>
<td>1.70</td>
<td>0.77</td>
<td>0.40</td>
<td>0.27</td>
<td>0.15</td>
<td>0.42</td>
</tr>
<tr>
<td>3</td>
<td>0.71</td>
<td>0.88</td>
<td>1.65</td>
<td>0.72</td>
<td>0.38</td>
<td>0.28</td>
<td>0.13</td>
<td>0.41</td>
</tr>
<tr>
<td>4</td>
<td>0.73</td>
<td>1.01</td>
<td>1.64</td>
<td>0.70</td>
<td>0.34</td>
<td>0.34</td>
<td>0.10</td>
<td>0.44</td>
</tr>
<tr>
<td>5</td>
<td>0.82</td>
<td>1.26</td>
<td>1.64</td>
<td>0.72</td>
<td>0.25</td>
<td>0.53</td>
<td>0.06</td>
<td>0.58</td>
</tr>
<tr>
<td>6</td>
<td>–</td>
<td>–</td>
<td>1.71</td>
<td>0.79</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 8: Case of Partial Equilibrium when \( N = 6 \ m = 10 \)

(the condition of Lemma 1 is met). After first-pair integration each upstream
firm supplies more input to the remaining nonintegrated downstream firms despite the fact that each nonintegrated downstream firm produces less and less of the final product. However, the profit of each unintegrated upstream firm does not start rising until three firms have integrated. Because the gain in the profit is small further integration is still profitable. As a result fourth and fifth-pair integration still increase the joint profits. After five firms have integrated, there is only one unintegrated upstream firm remaining which now supplies to five downstream firms. Both the output and the profits of this remaining unintegrated upstream firm increase significantly and any further integration fails to compensate for the loss in the profits of the upstream firm. A simultaneous decision equilibrium exists at $\mu = 5$. However, a difficulty arises in trying to construct how such a pure strategy equilibrium might be reached\(^{13}\) in a sequential decision framework since the first mover would need to find some way of signaling it would not integrate in order for others in turn to undergo the process of integration.

Doubtless it would be easy to generate further regions of partial integration by incorporating some cost of integrating. However, we feel it is

\(^{13}\)A parallel issue emerges in some models of research joint venture where a subset of the industry forms a research sharing group (see e.g., the discussion in Katz, 1986).
interesting that it can arise even in the absence of such frictions.

3.5.1 Effect on the Final Product Price

In this section we evaluate the effect of vertical integration on the price of the final product. We compare prices at two states. The initial state is assumed to involve no integration. The final equilibrium state is defined to be one where no private profitability incentive exists for any further integration. Note from the earlier analysis that when $m \leq N$, the final equilibrium state is always full integration. For $m > N$ final equilibrium state could be a full integration, no integration or a partially integrated industry (see example 1).

We state the following Proposition.

Proposition 4. Compared to the initial state of no integration, the price of the final product is never higher at the final equilibrium.

Notice this does not say that price falls continually on the path towards equilibrium.\footnote{Examples can be constructed in which price rises when an additional integration takes place (see e.g. Salinger (1988))} However, it is true that at all points on the path towards equilibrium, price is lower than at the initial state. A proof is in the Appendix

Proposition 4 also enables us to infer the following.
Proposition 5. Compared to initial position, the Marshallian measure of welfare is higher in the final equilibrium.

Propositions 4 and 5 are essentially restatements of results in Greenhut and Ohta (1979).

As pointed out earlier, the firms in the industry are often worse off as a result of integration. However Proposition 5 says that their loss is more than outweighed by consumers' gain. Figure 2 illustrates the point. \( P_x(0) \) is the initial price and \( P_x(\mu) \) the final price. \( C^*_x \) is the marginal cost of the final product when both inputs \( A \) and \( B \) are priced at marginal cost. If input \( A \) is priced above marginal cost, downstream firm's marginal costs are higher but there is no producer deadweight loss, merely a transfer to upstream firms, since we assume fixed proportions in production. Hence the sum of profits at the initial and final positions is given by \((P_x(0) - C^*_x)X(0)\) and \((P_x(\mu) - C^*_x)X(\mu)\). As a result of the price fall, there is a net gain in social surplus represented by the shaded area.
Figure 2. Marshallian measure of welfare
3.6 Vertical Integration with Outside Sales

So far, we have not considered the possibility that a firm, once integrated, continues to supply outsiders. The purpose of this section is simply to explore this issue a little. Matters are only taken for enough to establish that it is feasible under certain circumstances (not investigated in detail) for outside sales to be profitable and, on occasion, to make what would otherwise be an unprofitable integration more profitable. This question has received rather little attention in the literature to date, although Reiffen (1992) raises the issue in a comment on Ordover, Saloner and Salop (1990). It is sometimes implied, indeed that the practice would be illogical within the present context (Salinger, 1988, 347-9). Yet the practice is relatively common - European motor industry examples include Peugeot’s supply of diesel engines to Ford and Volkswagen’s supply of gearboxes to Rover - and deserves examination.

In order to pursue matters, the model is stripped down to consider only the profitability of the first integration. For tractability, we assume that prior to integration the $N$ upstream firms each sell to $\frac{N}{N}$ downstream firms, the latter number also being an integer. Upon integration, the integrator considers continuing to sell to “its” $\frac{N}{N} - 1$ old customers downstream. It
offers to sell at the same price other downstream firms obtain from their suppliers (which would, in a large-number case, be the “market” or “never knowingly undersold” price). Again, this is simply a tractable and plausible assumption amongst possible cases. Thus the integrator’s profit is:

\[ \Pi^s = P_x^a x_i - (M_a + P_b)x_i + (P_a - M_a)\left(\frac{m}{N} - 1\right)x_n \]  

(3.31)

Note that all unintegrated downstream firms will be of equal size in our case, since they all face identical factor prices and competitive conditions. From equation (5), derived demand \( x_n \) may be written:

\[ x_n = \frac{(P_x^a - P_a - P_b)}{b} \]  

(3.32)

Substituting into equation (3.31), the first order condition for profit maximization, treating \( P_a \) as a parameter (supplied by unintegrated upstream firms), is

\[ \frac{\partial \Pi^s}{\partial x_i} = (P_x^a - M_a - P_b) - x_i b - (P_a - M_a)\left(\frac{m}{N} - 1\right) = 0 \]
Hence

\[ x_i = \frac{1}{b} \left( P_x^n - P_a \left( \frac{m}{N} - 1 \right) - M_a \left( 2 - \frac{m}{N} \right) - P_b \right) \]  

(3.33)

Now if \( \frac{m}{N} \geq 3 \), \( x_i < x_n \). This is unlikely to prove profitable - the integrated firm cuts back so much on its own downstream output that it sells less than those to whom it sells input. Therefore we focus on the case where \( \frac{m}{N} = 2 \) for the remainder of the section. Inspection of equation (3.33) for this case reveals immediately on comparison with equation (3.32) that \( x_i = x_n \). Hence setting \( X \equiv x_i + (m - 1)x_n = mx_n \) yields:

\[ P_a = a - P_b - b \left( \frac{m + 1}{m} \right) X \]  

(3.34)

using equations (3.32) and (3.1).

Turning to the derived demand facing the \( N - 1 \) unintegrated upstream firms, \( X_n \), we have:

\[ X_n = x_n \left( m - \frac{m}{N} \right) = x_n m \left( 1 - \frac{1}{N} \right) = X \left( \frac{N - 1}{N} \right) \]  

(3.35)

They supply \( m - \frac{m}{N} \) i.e. \( m - 2 \) of the downstreamers, their previous cus-
tomers. Thus from equation (3.32):

\[ P_x^a - P_x^a - P_x^b = \frac{bX_n}{m - \frac{m}{N}} \]  
(3.36)

From equations (3.34) and (3.35):

\[ P_a = a - P_b - b\left(\frac{m + 1}{m}\right)\left(\frac{N}{N - 1}\right)X_n \]  
(3.37)

Profit of each upstream firm is

\[ \Pi^a = P_a A_n - M_a A_n; \quad n = 2, ..., N. \]

The first-order condition for profit maximization with respect to output yields:

\[ P_a - A_n b\left(\frac{m + 1}{m}\right)\left(\frac{N}{N - 1}\right) = M_a \]

Summing over the \(N - 1\) firms, all alike, yields:

\[ (N - 1)P_a - X_n b\left(\frac{m + 1}{m}\right)\left(\frac{N}{N - 1}\right) = (N - 1)M_a \]  
(3.38)

111
Using equation (3.37) we can solve for $X_n$ and $P_a$:

$$X_n = \frac{(N - 1)^2}{N^2} \left( \frac{a - P_b - M_a}{b} \right) \frac{m}{m + 1} \quad (3.39)$$

$$P_a = \frac{a - P_b}{N} + \frac{N - 1}{N} M_a \quad (3.40)$$

and, using equation (3.35),

$$x_n = \left( \frac{N - 1}{N} \right) \frac{1}{m + 1} \left( \frac{a - P_b - M_a}{b} \right) = x_i \text{ in this case} \quad (3.41)$$

Then, from equations (3.36) and (3.39):

$$P_x = P_a + P_b + \left( \frac{N - 1}{N} \right) \frac{1}{m + 1} (a - P_b - M_a) \quad (3.42)$$

Finally, these values can be substituted into equation (3.31) to solve for $\Pi^*$ in terms of parameters of the model. After some manipulation (and recalling that all these formulae are generated under the specific assumption that $\frac{m}{N} = 2$), we find:

$$\Pi^* = \frac{(a - P_b - M_a)^2}{bN^2(2N + 1)^2} (N - 1)(1 + 5N) \quad (3.43)$$
We can now draw comparisons between the profits obtained when integrating and supplying outside with the profits obtained from integrating but not supplying outside and the combined profits of an upstream and downstream firm separately, to evaluate the incentive to integrate. From equations (3.17) and (3.18) we obtain:

\[
\Pi^u(0) + \Pi^d(0)|_{m=2N} = \frac{(a - P_b - M_a)^2}{bN^2(2N + 1)^2} \left( \frac{N^2(5N^2 + 2N)}{(N + 1)^2} \right)
\]  

and from equation (3.19):

\[
\Pi^{int}(1)|_{m=2N} = \frac{(a - P_b - M_a)^2}{bN^2(2N + 1)^2} \left( \frac{(4N - 1)^2}{4} \right)
\]  

Some illustrative comparisons based on these formulae are listed in table 9 below.

Based upon this analysis, we may state:

**Proposition 6.** Vertical integration with outside sales can be more profitable than vertical integration with foreclosure. It can enable integration to take place which would not be profitable in the absence of outside sales.

These points are illustrated by the row for \( N = 3 \) and the row for \( N = 5 \) in
Table 9. Comparisons between $\Pi^a$, $\Pi^u(0) + \Pi^d(0)$ and $\Pi^{int}(1)$

\[ a = 6; \ b = .9; \ M_a = .4; \ P_b = .6. \]

<table>
<thead>
<tr>
<th>$N$</th>
<th>$m$</th>
<th>$\Pi^a$</th>
<th>$\Pi^u(0) + \Pi^d(0)$</th>
<th>$\Pi^{int}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>3.06</td>
<td>2.96</td>
<td>3.40</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2.02</td>
<td>1.81</td>
<td>1.91</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>1.35</td>
<td>1.21</td>
<td>1.21</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.96</td>
<td>0.86</td>
<td>0.83</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>0.71</td>
<td>0.64</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 9. Note nevertheless that outside sales are not inevitably profitable - see the row for $N = 2$. It should also be recalled that complete integration is not profitable either for $N \geq 3$ here (see Lemma 7). However, given the work of the previous section, it is not necessary for an industry to become integrated that complete integration be profitable - this will depend upon the firms' discount factors. Thus the analysis in this section has demonstrated that, if outside sales are contemplated, the number of cases for which integration will occur is greater than if consideration of outside sales is omitted. The reason is that outside sales are not perfect substitutes for final good sales, even with fixed proportions.
3.7 Variable Proportions Technology

In this section we focus on the effects of vertical integration when the production technology allows substitution between the two inputs. In particular, we consider the CES production function commonly used in earlier papers in the area. As mentioned earlier, the force favoring vertical integration is stronger when the production function is of variable proportions type because vertical integration eliminates not only double marginalization but it also creates efficiency by eliminating input distortions. With two forces favoring integration instead of only one under fixed proportions, one would expect some results to change. We bring out the differences and the similarities after deriving the equilibrium conditions and presenting the results based on simulations. As will become apparent, equilibrium conditions result in expressions that are nonlinear in nature and therefore cannot be solved analytically. To overcome this difficulty we use extensive simulations using many different combinations of the demand and the cost parameters.

The demand for the final product is linear and is given by $P_x = a - bX$ where,

$$X = [\delta A^{(\sigma-1)/\sigma} + (1 - \delta)B^{(\sigma-1)/\sigma}]^{(\sigma-1)}$$  \hspace{1cm} (3.46)
The marginal cost functions \( c_i \) and \( c_n \) are (see Abiru, 1988)

\[
c_i = \left[ \delta^{\sigma} M_a^{1-\sigma} + (1 - \delta)^{\sigma} P_b^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{3.47}
\]

\[
c_n = \left[ \delta^{\sigma} P_a^{1-\sigma} + (1 - \delta)^{\sigma} P_b^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{3.48}
\]

### 3.7.1 Equilibrium

The equilibrium conditions for the downstream and upstream firms can be obtained as:

\[
P^a_x = \frac{m \eta}{m \eta - 1} \left[ c_n - \frac{\mu}{m} (c_n - c_i) \right] \tag{3.49}
\]

\[
P_a = \frac{(N - \mu) E}{(N - \mu) E - 1} M_a \tag{3.50}
\]

where the derived demand elasticity, when \( B \) is supplied competitively, is

\[
E = \eta K_a + \sigma(1 - K_a)
\]

and,
\[ K_a = \frac{1}{1 + \left(\frac{1-\delta}{\delta}\right)^\sigma \left(\frac{P_a}{P_b}\right)^{\sigma-1}} \]

Using the expressions for \( E \) and \( K_a \) simultaneously, in equation (3.49), \( P_a \) can be found using the following equation.

\[
\begin{align*}
((N - \mu) \eta - 1) P_a + ((N - \mu) \sigma - 1) P_a \left(\frac{1 - \delta}{\delta}\right)^\sigma \left(\frac{P_a}{P_b}\right)^{\sigma-1} \\
- (N - \mu) \sigma M_a \left(\frac{1 - \delta}{\delta}\right)^\sigma \left(\frac{P_a}{P_b}\right)^{\sigma-1} - (N - \mu) \eta M_a = 0
\end{align*}
\]

(3.51)

Since \( P_a \) cannot be solved explicitly, \( c_n \) and \( P_{x_a} \) cannot be obtained explicitly in terms of given parameters. Therefore, to obtain the profit-maximizing values, equations for \( P_a \), \( P_{x_a} \) and \( c_n \) must be solved simultaneously.

The expressions for profits can be written as follows.

\[
\Pi^d(\mu) = (P_{x_a} - c_n)x_n
\]

(3.52)

\[
\Pi^{int}(\mu) = (P_{x_a} - c_i)x_i
\]

(3.53)

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where,

\[ x_i = \frac{P_x^i - c_i}{b} \]  

(3.54)

\[ x_n = \frac{P_x^n - c_n}{b} \]  

(3.55)

\[ \Pi^u(\mu) = (P_a - M_a)A_n \]  

(3.56)

where \( A_n = (\delta + (1 - \delta)(\frac{1 - \delta}{\delta}P_a)\sigma - 1) \frac{\sigma}{\sigma - 1} x_n \).

### 3.7.2 Profit incentive

In this section we evaluate the profit incentives by obtaining numerical solutions using the expressions derived above. The simulation results are based on many different combinations of various parameters. We varied \( \sigma \) from 0.01 to 5 in 0.01 steps, \( \delta \) from 0.1 to .9 in 0.01 steps, \( a \) from 2 to 6 in 0.1 steps, \( M_a \) from .4 to 1 in 0.1 steps, \( P_b \) from .4 to 1 in 0.1 steps, \( b \) from .1 to .9 in .1 steps, \( N \) from 2 to 6 in 1 steps, \( m \) from 3 to 9 in 1 steps. Table 10 gives a sample result of the simulations. Table 10 is based on the set of parameters specified at the top of the table and numerical values of all the variables are listed. We consider three relationships between \( N \) and \( m \). The initial state is when \( \mu = 0 \) and the final state is when \( \mu \) is either 2 or 3 depending on the
values of $N$ and $m$. $\Pi^{int}$ denotes the profits of an integrated pair and varies depending on the number of integrated pairs. Based on the simulation runs for the ranges of the parameters mentioned above and the results presented in Table 10 we state the following three propositions.

**Table 10. Profit Incentive Under Variable proportions**

$a = 3; b = .9; M_a = A; P_b = .6; \sigma = .2; \delta = .5; c_i = .996$

<table>
<thead>
<tr>
<th>$N, m$</th>
<th>$\mu$</th>
<th>$P_a$</th>
<th>$A_a$</th>
<th>$c_m$</th>
<th>$P^2$</th>
<th>$x_n$</th>
<th>$x_i$</th>
<th>$\Pi^a$</th>
<th>$\Pi^d$</th>
<th>$\Pi^a + \Pi^d$</th>
<th>$\Pi^{int}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = m = 2$</td>
<td>0</td>
<td>.764</td>
<td>.593</td>
<td>1.362</td>
<td>1.908</td>
<td>.607</td>
<td>–</td>
<td>.216</td>
<td>.331</td>
<td>.547</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>2.718</td>
<td>0</td>
<td>3.168</td>
<td>1.998</td>
<td>0</td>
<td>1.113</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.116</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.664</td>
<td>–</td>
<td>.742</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>.496</td>
<td></td>
</tr>
<tr>
<td>$3 = N &gt; m = 2$</td>
<td>0</td>
<td>.638</td>
<td>.649</td>
<td>1.238</td>
<td>1.825</td>
<td>.653</td>
<td>–</td>
<td>.154</td>
<td>.383</td>
<td>.538</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>.843</td>
<td>.401</td>
<td>1.439</td>
<td>1.812</td>
<td>.414</td>
<td>.906</td>
<td>.177</td>
<td>.155</td>
<td>.322</td>
<td>.739</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.664</td>
<td>–</td>
<td>.742</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>.496</td>
<td></td>
</tr>
<tr>
<td>$2 = N &lt; m = 3$</td>
<td>0</td>
<td>.833</td>
<td>.432</td>
<td>1.429</td>
<td>1.822</td>
<td>.436</td>
<td>–</td>
<td>.183</td>
<td>.171</td>
<td>.355</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>3.928</td>
<td>0</td>
<td>4.246</td>
<td>1.998</td>
<td>0</td>
<td>1.113</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.116</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.664</td>
<td>–</td>
<td>.742</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>.496</td>
<td></td>
</tr>
<tr>
<td>$N = m = 3$</td>
<td>0</td>
<td>.687</td>
<td>.470</td>
<td>1.286</td>
<td>1.715</td>
<td>.476</td>
<td>–</td>
<td>.135</td>
<td>.204</td>
<td>.339</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>.905</td>
<td>.267</td>
<td>1.499</td>
<td>1.748</td>
<td>.277</td>
<td>.836</td>
<td>.135</td>
<td>.069</td>
<td>.204</td>
<td>.629</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8.717</td>
<td>0</td>
<td>8.422</td>
<td>1.664</td>
<td>0</td>
<td>.742</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>.496</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.497</td>
<td>–</td>
<td>.557</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>.279</td>
<td></td>
</tr>
<tr>
<td>$4 = N &gt; m = 3$</td>
<td>0</td>
<td>.612</td>
<td>.496</td>
<td>1.212</td>
<td>1.659</td>
<td>.497</td>
<td>–</td>
<td>.105</td>
<td>.222</td>
<td>.327</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>.717</td>
<td>.372</td>
<td>1.316</td>
<td>1.657</td>
<td>.379</td>
<td>.735</td>
<td>.118</td>
<td>.129</td>
<td>.247</td>
<td>.486</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.332</td>
<td>0</td>
<td>1.716</td>
<td>1.664</td>
<td>0</td>
<td>.742</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>.496</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.497</td>
<td>–</td>
<td>.557</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>.279</td>
<td></td>
</tr>
<tr>
<td>$3 = N &lt; m = 4$</td>
<td>0</td>
<td>.719</td>
<td>.367</td>
<td>1.318</td>
<td>1.654</td>
<td>.374</td>
<td>–</td>
<td>.117</td>
<td>.126</td>
<td>.243</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>.940</td>
<td>.199</td>
<td>1.532</td>
<td>1.718</td>
<td>.207</td>
<td>.802</td>
<td>.107</td>
<td>.039</td>
<td>.146</td>
<td>.580</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10.287</td>
<td>0</td>
<td>9.778</td>
<td>1.664</td>
<td>0</td>
<td>.742</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>.496</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.497</td>
<td>–</td>
<td>.557</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>.279</td>
<td></td>
</tr>
</tbody>
</table>

**Proposition 7.** Regardless of the number of upstream and downstream firms there is always an incentive for the first-pair and subsequent integra-
tions up to complete integration.

**Proposition 8.** When the number of upstream and downstream firms are equal \( N = m \) then complete integration is less profitable compared to the initial state of no integration.

**Proposition 9.** When \( N > m \) full integration is always less profitable compared with the initial state, but when \( m > N \) full integration is always more profitable compared with the initial state.

Again it is only in circumstances where both first pair and final profit increases are present that integration will be guaranteed.

### 3.7.3 Comparison between fixed and variable proportions

In this sub-section we compare and contrast the similarities and differences between the results under these two cases.
3.7.4 First-pair and subsequent integrations

In the fixed-proportions case a first-pair incentive always exists when \( N \geq m \). Thus it is not at all surprising that in the variable proportions case there is also an incentive. When \( m > N \) first-pair incentive requires a constraint on the number of downstream firms in the fixed proportions case because the elimination of double marginalization is not enough to compensate for the foregone profit of the upstream firm. Therefore, a first-pair incentive may not exist. In the case of variable proportions elimination of double marginalization together with the reduction in input distortions turns out to be more than enough to compensate for the profit of the upstream firm, hence when \( m > N \) there is always an incentive for the first-pair to integrate. The same argument can be applied to subsequent integrations. The remaining unintegrated firms become worse off as a result of the first integration. In order to improve their profitability they too must integrate. As was the case in fixed proportions partial integration is not in the interest of the remaining firms in the variable proportions case.

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3.7.5 Complete integration

(a) equal numbers case  When the numbers of firms at the upstream and
the downstream stages are equal there is no incentive for full integration.
This result is the same in both cases. Although once integration has begun
it pays for the remaining firms to follow, they all become worse off after full
integration. This Prisoner's Dilemma situation also occurred in the fixed
proportions case.\textsuperscript{15}

(b) unequal numbers case  It is when the number of firms at the two
stages are unequal, that we find the most striking difference between the two
cases. In order to explain the differences we construct examples in Table
11. This shows the profits at the initial and the final state of full integration
under fixed and variable proportions. The table shows the effects of changes
in $N$ and $m$ as well as variations in $\sigma$ and $\delta$. The values of $\sigma$ and $\delta$ affect
the profits only in the variable proportions case of course. Based on the sim-
ulation runs we find that at the initial state of no integration the higher the
values of $\sigma$, the greater the profits of the downstream firms, \textit{ceteris paribus}.

\textsuperscript{15}For a constant elasticity demand it can be shown that when $N = m$ there is never an
incentive for full integration. A formal proof can be obtained from the authors.
The intuition behind this result is rather simple. Higher values of $\sigma$ allows increased substitution between the inputs thus lowering the cost of the downstream firms, *ceteris paribus*. The relationship between $\sigma$ and the profits of an upstream firm depends on the initial magnitude of $\delta$. As long as the value of $\delta$ is not very high (close to one), the higher the value of $\sigma$, the lower the profits of the upstream firms. But, when $\delta$ approaches one the input $A$ becomes relatively more important in the production of the final good. In this case the effect of higher $\delta$ (which increases the profits) dominates the effect of higher $\sigma$ (which decreases the profits) resulting in a net increase in the profits of the upstream firms. However, in all cases the simulation results show that the downstream firms' profits dominate the profits of the upstream firms with respect to all parameters. For any given $\sigma$ and $\delta$, as both $N$ and $m$ increase simultaneously the profits of each upstream and downstream firm decrease because the industries at both the stages become relatively more competitive.

Observe from Table 11 that at the initial state for given parametric values of demand and cost conditions the profits of an upstream firm under fixed proportions are *always* higher than an upstream firm's initial profits under variable proportions. The reason is that input substitution is not possible
and thus upstream firms' monopoly power is relatively high which enables them to earn higher profits. However, a downstream firm's profits under fixed proportions are always lower than a downstream firm's profits under variable proportions. This is because the initial cost of a downstream firm under variable proportions is lower (it is more cost efficient because of input substitution) than the cost of a downstream firm under fixed proportions. These inequalities always hold for all our simulation runs. The combined profits at the initial state under the two cases depend on whether the number of upstream firms are greater or smaller than the number of the downstream firms. When $N > m$, the combined profits under variable proportions are always higher than the combined profits under fixed proportions. The opposite is the case when $N < m$. This reversal in the direction of these inequalities is the key determinant of whether the profit incentive exists or not at full integration when there are unequal numbers of firms.

The intuitive economic explanation for $\Pi^u + \Pi^d$ being higher under variable proportions when $N > m$ is as follows. Observe from Table 11 that at the initial state the second term ($\Pi^d$) of the combined profits is much larger than the corresponding term under fixed proportions. Although the first term under fixed proportions is higher than the corresponding term
under variable proportions, the dominance of the second term (because of fewer downstream firms) results in higher combined profits under variable proportions. Since the initial combined profits under variable proportions are higher to begin with, full integration does not generate enough profits to make up for the profits of the upstream firm. Therefore there is never an incentive for full integration under variable proportions. Depending on the difference between the number of upstream and downstream firms (which in turn determines the impact of double marginalization at the initial state), there may exist an incentive for full integration in the fixed proportions case (see Lemma 4).

Table 11. Profits at the Initial and Final States

\[ a = 3; \ b = .9; \ M_a = .4; \ P_b = .6. \]

<table>
<thead>
<tr>
<th>( N,m )</th>
<th>( N = 3,m = 2 )</th>
<th>Fixed proportions</th>
<th>Variable proportions</th>
<th>( \Pi^u(0) )</th>
<th>( \Pi^d(0) )</th>
<th>( \Pi^u + \Pi^d )</th>
<th>( \Pi^{int} )</th>
<th>( \sigma, \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 3,m = 2 )</td>
<td>( .185 )</td>
<td>( .278 )</td>
<td>( .463 )</td>
<td>( .494 )</td>
<td>( .154 )</td>
<td>( .383 )</td>
<td>( .538 )</td>
<td>( .496 )</td>
</tr>
<tr>
<td>( N = 4,m = 3 )</td>
<td>( .133 )</td>
<td>( .178 )</td>
<td>( .311 )</td>
<td>( .278 )</td>
<td>( .105 )</td>
<td>( .222 )</td>
<td>( .327 )</td>
<td>( .279 )</td>
</tr>
<tr>
<td>( N = 2,m = 3 )</td>
<td>( .370 )</td>
<td>( .123 )</td>
<td>( .493 )</td>
<td>( .494 )</td>
<td>( .183 )</td>
<td>( .171 )</td>
<td>( .355 )</td>
<td>( .496 )</td>
</tr>
<tr>
<td>( N = 3,m = 4 )</td>
<td>( .222 )</td>
<td>( .100 )</td>
<td>( .322 )</td>
<td>( .278 )</td>
<td>( .054 )</td>
<td>( .254 )</td>
<td>( .308 )</td>
<td>( .514 )</td>
</tr>
<tr>
<td>( N = 4,m = 5 )</td>
<td>( .222 )</td>
<td>( .100 )</td>
<td>( .322 )</td>
<td>( .278 )</td>
<td>( .054 )</td>
<td>( .254 )</td>
<td>( .308 )</td>
<td>( .514 )</td>
</tr>
</tbody>
</table>

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For $N < m$ the first term ($\Pi^u$) of the combined profits under fixed proportions is much larger than the corresponding term under variable proportions. The dominance of the first term makes the initial combined profits under fixed proportions larger than the initial combined profits under variable proportions. Since the initial profits under fixed proportions are higher to begin with there is never an incentive for full integration except when $N = 2$ and $m = 4$ or 5. In the fixed proportions case, with the exception of $N = 2$ and $m = 4$ or 5; there is no other combination of $N, m$ for which the first-pair and full integration incentives exist at the same time. With variable proportions because of initial lower combined profits there is no constraint on either the first integration or full integration and hence full integration is always profitable compared to the initial state.

We summarize the similarities and differences between the two cases in Table 12. The general implication of these results is that even a limited amount of substitution in production can be important to the profitability of vertical integration.
Table 12. Comparison Between Fixed and Variable Proportions

<table>
<thead>
<tr>
<th>Profit Incentive for</th>
<th>when</th>
<th>fixed proportions</th>
<th>variable proportions</th>
</tr>
</thead>
<tbody>
<tr>
<td>first-pair</td>
<td>$N \geq m$</td>
<td>always</td>
<td>always</td>
</tr>
<tr>
<td></td>
<td>$N &lt; m$</td>
<td>not always</td>
<td>always (see Table 4)</td>
</tr>
<tr>
<td>subsequent</td>
<td>$N \geq m$</td>
<td>always</td>
<td>always</td>
</tr>
<tr>
<td>integrations</td>
<td>or $N &lt; m$</td>
<td>always</td>
<td></td>
</tr>
<tr>
<td>complete integration</td>
<td>$N &gt; m$</td>
<td>if $N \geq \frac{m^2 + m - 1}{2}$</td>
<td>never</td>
</tr>
<tr>
<td>compared to initial state</td>
<td>$N = m$</td>
<td>never</td>
<td>never</td>
</tr>
<tr>
<td></td>
<td>$N &lt; m$</td>
<td>only if $N = 2$ and $m = 4$ or $m = 5$</td>
<td>always</td>
</tr>
</tbody>
</table>

3.8 Conclusions

Ever since the publication of the paper by Vernon and Graham (1971), it has been common for analyses of vertical integration to assume that a profit incentive exists for merging parties. This ignores the effects of some of the forces operating upon integration, in particular the challenge of the vertical separation literature and the straightforward competition effects understood assumptions. As we hope we have shown, these forces are important in particular in cases where factor substitution is impossible, downstream numbers are relatively large and upstream numbers relatively small. But

\footnote{There is an analogy here with the literature on the profitability of horizontal merger, see e.g. Salant, Switzer and Reynolds (1983) and Perry and Porter (1985).}
moreover, we believe we have clarified the conditions for profitability of vertical mergers in terms of initial and subsequent effects. Welfare analyses which do not take profitability effects into account will be seriously misguided.

Our analysis also casts light on the much debated foreclosure issue. In contrast to what is often assumed, a merging firm may wish to continue supplying its own competitors for private profitability reasons, without the need for pressure by outside bodies (see e.g. Monopolies Commission, 1966). The foreclosure issue is certainly an important one, but too much emphasis on foreclosure without exploring the incentives seems misguided. Effort might more profitably be extended in examining features of vertical separation such as exhibited by the Japanese Keiretsu, to investigate their profitability and incentive effects by comparison with integration.
3.9 Appendix

Proof of Lemma 2: Since all values are positive, inequality (3.24) can be rewritten as,

\[
1 + \frac{m - 1}{2N} > \frac{N}{N + 1} \left(1 + \frac{m^2 + m}{N^2}\right)\]

Using the binomial inequality \((1 + y)^{\alpha} \leq 1 + \alpha y\) for \(0 < \alpha < 1\) and \(y \geq 0\) it is sufficient to prove that

\[
1 + \frac{m - 1}{2N} \geq \frac{N}{N + 1} \left(1 + \frac{m^2 + m}{2N^2}\right)
\]

or,

\[
\frac{N + 1}{N} + \frac{(m - 1)(N + 1)}{2N^2} \geq 1 + \frac{m^2 + m}{2N^2}
\]

or,

\[
\frac{1}{N} \geq \frac{m(m + 1)}{2N^2} - \frac{(m - 1)(N + 1)}{2N^2}
\]
Since $m \leq N$ it is enough to show that

$$2N \geq m(N + 1) - (m - 1)(N + 1)$$

or

$$N \geq 1 \quad \text{which is true. Q.E.D.}$$

**Proof of Lemma 3:** The Lemma is true if it can be shown that for any $m \leq N$, inequality (3.25) holds for $1 \leq \mu + 1 \leq m$. Inequality (3.25) can be written as

$$\frac{(m + 1)}{(\mu + 2)(N - \mu)} + \frac{N - \mu - 1}{N - \mu} > \frac{N - \mu}{N - \mu + 1} \left[1 + \frac{(m - \mu)(m + 1)}{(\mu + 1)(N - \mu)^2}\right]^{\frac{1}{2}}$$

Again using the binomial inequality mentioned earlier, it is enough to show that

$$\frac{(m + 1)}{(\mu + 2)(N - \mu)} + \frac{N - \mu - 1}{N - \mu} \geq \frac{N - \mu}{N - \mu + 1} \left[1 + \frac{(m - \mu)(m + 1)}{2(\mu + 1)(N - \mu)^2}\right]$$
or,

\[ \frac{(m + 1)(N - \mu)}{(\mu + 2)(N - \mu)} - \frac{(m - \mu)(m + 1)}{2(\mu + 1)(N - \mu)(N - \mu + 1)} \geq \frac{N - \mu}{N - \mu + 1} - \frac{N - \mu - 1}{N - \mu} \]

simplification yields

\[ (m + 1)[2(\mu + 1)(N - \mu + 1) - (m - \mu)(\mu + 2)] \geq 2(\mu + 1)(\mu + 2) \]

Since \( N - \mu \geq m - \mu \) it is enough to show that

\[ (m + 1)[2(\mu + 1)(N - \mu + 1) - (N - \mu)(\mu + 2)] \geq 2(\mu + 1)(\mu + 2) \]

or,

\[ (m + 1)[2(\mu + 1)(N - \mu) + 2(\mu + 1) - (N - \mu)(\mu + 2)] \geq 2(\mu + 1)(\mu + 2) \]

or,

\[ \mu(N - \mu)(m + 1) \geq -2(\mu + 1)(m - 1 - \mu) \]

the LHS\( > 0 \) and the RHS\( \leq 0 \) for \( 1 \leq (\mu + 1) \leq m. \) Q.E.D.
**Proof of Lemma 4:** When $m = N$, full integration is less profitable than no integration, if and only if, $\Pi^{\text{int}}(\mu) < \Pi^u(0) + \Pi^d(0)$. Using equations (3.17), (3.18) and (3.19) with $m = N$ and simplifying we get,

$$1 < \frac{m}{m+1} + \frac{m^2}{(m+1)^2}$$

or, $1 < m^2 - m$ which is always true for $m \geq 2$.

When $m < N$, full integration is profitable, if and only if $\Pi^{\text{int}}(\mu) > \Pi^u(0) + \Pi^d(0)$ which can be written as (by putting $\mu = m$)

$$\left(\frac{m + N + 1 + m(N - m - 1)}{(m + 1)^2(N - m + 1)}\right)^2 > \frac{m}{(m+1)(N+1)^2} + \frac{N^2}{(m+1)^2(N+1)^2}$$

which after simplification reduces to

$$1 > \frac{m^2 + m + N^2}{(N+1)^2}$$

or

$$N > \frac{m^2 + m - 1}{2}$$

Q.E.D.
**Proof of Lemma 5:** Inequality (3.24) can be written as\(^{17}\)

\[(3N^2 - 2N - 1)m^2 - 2(2N^3 + N^2 - 1)m - (4N^3 - 3N^2 - 2N + 1) < 0\]

For any given value of \(N\) there are two roots for \(m\). The positive root represents the value of \(m\) for the first-pair incentive. Q.E.D.

**Proof of Lemma 6:** Incentive for the final integration exists if equality (3.25) holds for \(\mu + 1 = N\). By substituting \(\mu = N - 1\) in (3.25) we get

\[(N - 1)^2m^2 - m(N^3 + 5N - 2) + (N - 1)^2 < 0\]

For any given numbers of upstream firms, \(N\), the solution to the above inequality gives two roots for \(m\). The positive root represents the value of \(m\) that is necessary for the incentive for final pair integration. Q.E.D.

**Proof of Lemma 7:** When \(m > N\), the condition for complete integration to be more profitable than no integration, \(\Pi^nt(\mu) > \Pi^u(0) + \Pi^d(0)\), can

\(^{17}\)Clearly this inequality represents the first pair incentive for all combinations of \(m\) and \(N\). However the point is that whilst it always holds true for \(m \leq N\), it does not for \(m > N\). See Figure 1.

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be written using equations (3.17), (3.18) and (3.19) with $\mu = N$ as:

$$\frac{1}{(N + 1)^2} > \frac{m}{(N + 1)^2(m + 1)} + \frac{N^2}{(N + 1)^2(m + 1)^2}$$

or

$$m + 1 > N^2$$

Q.E.D.

**Proof of Proposition 4:** It is equivalent to prove that $X(\mu = 0) < X(\mu)$. From equation (3.14) the expressions result in the following inequality.

$$\frac{(a - P_b - M_a)mN}{b(m + 1)(N + 1)} < \frac{(a - P_b - M_a)(mN\mu - m\mu^2 + mN + \mu)}{b(m + 1)(\mu + 1)(N - \mu + 1)}$$

$$mN(\mu + 1)(N - \mu + 1) < (N + 1)(mN\mu - m\mu^2 + mN + \mu)$$

or

$$0 < (N + 1) + m(N - \mu)$$

which is always true. Q.E.D.
3.10 References


1978, 68, 228-30.


Chapter 4

The Toyota Keiretsu: An Historical Analysis

4.1 Introduction

In the last four years,¹ Toyota Motors came first in the league of most earners amongst Japanese companies. Usually, no company is born large; they become so as a result of competition with respect to Research and Development (R & D), gearing their production system towards high quality goods at lower costs, and having the ability to forecast correctly the future demand

¹This paper is based on Abiru (1992).
for their products. In this respect, Toyota is no exception.

In terms of growth and expansion, Toyota is often compared with Nissan Motors. However, although Toyota grows by separating into divisions (vertical separation) which form independent entities, they remain closely connected with the main company through the Keiretsu system, but in the case of Nissan growth takes the form of merging independent companies into the Nissan itself (vertical integration).² My aim in this short paper is to concentrate on only the Toyota Keiretsu system and on only its historical development hoping that this will help us understand it.

Before I get down to this task, I should mention that the history of Toyota can be divided into three phases. Just like human life, it goes through a period of infancy when it is dependent, both biologically and economically, on its parents, develops into youth with independent character but still heavily economically reliant on them, and becomes adult.

For Toyota, infancy is actually the period before the establishment of the company. During this phase, the parents are the Toyota Auto Loom, the Japanese Government and the technology provided by US car makers. The youth phase extends from the establishment of the company to the

beginning of the Korean War. Within this period, especially during World War II, Toyota was under government control, and also many of the Keiretsu companies were formed. Adulthood came after the Korean War.

In the next section, I shall follow the infant to see how it has fared. The youth shall be examined in the following section where I shall look into his family relationships, especially with brothers and sisters under parental guidance and support (the Toyota Group), and his friendships (Kyohokai). However, I shall not go near the adult. The paper finishes with concluding remarks.

4.2 Rise of Toyota Motor Company

When one talks about the birth of Toyota, one cannot neglect the people associated with this establishment. The founder of all the Toyota companies was Sakichi Toyoda, who in 1918 established the Toyoda Boshoku company which developed the spinning and weaving machine for the first time in Japan. After the success of this company, which was mainly due to its continued R&D efforts, he built the Toyoda Auto Loom in 1926; thus making it possible to introduce mass-production in the textile industry which in turns contributed
to the Japanese economy by acquiring foreign currency which was scarce at that time. A symbolic indication of the technological prowess of Toyoda Auto Loom was the purchase of patent rights by Pratt, the world leading spinning company.

Because both companies were successful, they had enough resources to advance into new fields. For Sakichi, the new field was automobile production, and this became the objective following their visit to the United States in 1910, where they witnessed first hand the highly developed automobile society there. This led them to believe that the following year was to commence the automobile age. The idea was close to the heart of his successor, his senior son Kiichiro Toyoda who became known as the founder of Toyota Motors.

Kiichiro Toyoda joined the Toyoda Boshoku company in 1921 before moving to Toyoda Auto Loom when it was established. He started preparing for a new automobile company, and as an initial step he wanted to learn from United States. So, he went there in 1929 and was taken by surprise at the affluence of American society.

The annual sales of automobiles in the US at this time exceeded the 5 million mark, but in Japan it was no more than a few hundreds as the figure
in the following table clearly indicate.

Table: Japanese annual car sales from 1928 to 1931

<table>
<thead>
<tr>
<th>Year</th>
<th>Foreign Car</th>
<th>Japanese Car</th>
</tr>
</thead>
<tbody>
<tr>
<td>1928</td>
<td>32,224</td>
<td>347</td>
</tr>
<tr>
<td>1929</td>
<td>34,356</td>
<td>437</td>
</tr>
<tr>
<td>1930</td>
<td>22,269</td>
<td>458</td>
</tr>
<tr>
<td>1931</td>
<td>22,086</td>
<td>436</td>
</tr>
</tbody>
</table>

Source: 40 Yearbook of Toyota Motors p.36

The first Japanese car was made in 1902 by Shintaro Yoshida and Komanosuke Uchida and was named ‘Takury’. But the production did not last long because of the poor quality of the car as well as the financial problems facing the company at the time. Then, in 1918, a company named ‘Kaishinsha’ (the root of Nissan Motors) started the production of car named ‘Dat’. Recognizing the importance of cars during World War I, the Japanese government started to encouraging the domestic production of automobiles via the Law of Military Car Subsidy in 1918. In addition to this, the Kanto earthquake in 1923 which damaged the public transportation system for central Japan, created an urgent boost in the demand for cars. However, Japanese made cars in those days were far from competitive with their foreign counterpart. Therefore, immediate need was satisfied mostly through imports from
the US. Convinced that Japan was the prospective future car market, Ford established Ford of Japan in February 1925; this started by merely assembling imported parts from the US in March 1925. In 1926, General Motors also established GM of Japan. The large presence of US car makers in Japan gave domestic car makers a hard time: some went bankrupt while others got taken over. In 1926, Japanese car makers were Tokyo Ishikawajima Zosen-sho, Tokyo Gas Denki Kogyo and Dat Automobile Production; these just managed to survive due to government support. As already indicated, it was during this year that Toyoda Auto Loom was established.

After returning from the US, Kiichiro personally decided to start automobile production. He began by studying and developing a small test engine at his office in Toyota Auto Loom in 1930. This was based on the Smith motor which was used in many motor bikes in those days. Having closely studied the mechanism and tested for how it could be adapted for a car engine, he succeeded in producing the first test engine in 1931. After that, he founded an automobile division in Toyoda Auto Loom in 1933 to begin commercial production of automobiles.

His strategy for automobile production was as follows:3

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3See, Toyota Motors (1977), p. 41
The mass production of a family-type car was suitable to Japanese customers and able to compete with Chevrolet and Ford in terms of both price and quality.

A production system should be invented to suit Japanese people, but they need to learn from American mass production system first.

Kiichiro first bought a 1933 model of a Chevrolet sedan, brought it to the store room in the company, and then began a thorough examination of its mechanism, and in the process tried to develop his own test model. His design policy was the following:4

1. In the beginning, the desired engine must be designed in such a way as to fit into the basic structure of a Chevrolet.

2. The chasses and driving parts must also do likewise.

3. The frame and body-connecting parts must also do the same.

4. The body was to be stamped by imitating the streamlined shape of a 1934 Crisler model of Desote. However, Toyota did not own stamping machine then, so it planned to make it by hand.

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4See, Toyota Motors(B) (1967), p. 1
5. Other materials were to be manufactured after purchasing *Chevrolet* pure parts and examining them thoroughly.

6. *Electric devices* were to be bought from pure *Delco* parts and manufactured in-house after they had been carefully examined.

After a long period of trial and error, they succeeded in making the first test car of the A1 model in May of 1935, but what they actually manufactured was only a body, a cylinder head and block, a transmission case and a clutch housing; all the important and pure parts came from the US.\(^5\) However, what the government considered to be in urgent need was not a passenger car, rather a truck. Hence, the strategy switched to truck-making. The first truck, a test G1 model, was completed in August 1935; this was based mainly on a 1934 Ford model.

In spite of the fact that mass-production was based on mass-sales, Kiichiro had no idea about the sales network system. Therefore, in November 1935, he asked Shotaro Kamiya to take charge of this business. Although Kamiya joined GM of Japan in 1908, was promoted to Director of sales promotion at the Osaka headquarters in 1910, and, as a consequence, he gained a great

\(^{5}\) See, *Toyota Motors* (1977), p. 53
deal of experiences on how to establish a effective distribution system, he had no hesitation in immediately (on the same day) accepting Kiichiro’s offer, simply because he could not adjust fully to GM’s cool relationship with its dealer. Ford of Japan and GM of Japan had already established a dealer system with at least one dealer in each prefecture; indeed, between 1925 and 1935, they had more than 100 successful dealers out of a total of 300; a clear indication of how easy it was to go bankrupt. Contracts were signed for only one year, and the dealer with a bad performance was unilaterally dismissed and the manager who attracted that contract was summarily replaced. With this in mind, Kamiya tried to establish a Japanese-style distribution system; one based on long-term contracts and mutual benefits for both maker and dealer. The day after his acceptance, he visited Noboru Yamaguchi, manager of Hinode Motors, a GM car dealer, to ask him to switch to a Toyoda car dealer. Yamaguchi agreed by readily accepting, and began to attract most of the promising GM car dealers for this purpose.\(^7\)

After acquiring Hinode Motors, Toyoda Auto Loom started selling the G1 model named *Toyoda Truck* in November 1935. The price they set for it

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\(^7\)‘Toyota’ was adopted as a name stating in 1937. See, *Toyota Motors* (1977), p. 71-72
was far below Ford and Chevrolet. The company did not expect to make a positive profit by that price until it could achieve mass-production. When the model was put on sale, they were not confident about its quality. The following was what Yamaguchi said at the sales policy meeting:  

We have only two strong points concerning this truck. One is that it is made in Japan; the other that it is manufactured in a local region. Therefore, for our first customers, we have to aim for people who are both patriotic and love their home place. This is because, at the beginning, we expect some inconveniences from the car due to mechanical problems. Therefore, we have to provide constant support for the car when the troubles do occur, hence there is a need to restrict our sales to those who live close by. (My translation)

Hinode motors selected only six customers satisfying these criteria. However, the truck got a worse reputation than was expected since a series of defects had to be fixed; most of defects concerning imitation parts domestically manufactured. However, the experience gained from this actually

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8See, Toyota Motors (1987), p. 99-100
resulted in the improvement of more than 800 parts each year.

Although Toyoda devoted great efforts to the improvement of the model and its selling, it was keen on receiving government authorization as a car manufacturer. The background to this is that the government, fearing the dominance of foreign car manufacturers of the domestic market, when Ford of Japan bought a piece of land for expansion for its Yokohama factory in 1935, decided to extend support to domestic maker. As a result, the Cabinet wanted to enact a law for the automobile industry, with the aim of promoting domestic production, but only by a few manufacturers, as well as excluding foreign capital. This decision became the Law of Automobile Production Enterprise on 29 May 1936. Toyoda was anxious to be selected as one, since otherwise it stood no chance of competing in the industry, and would therefore have had to quit.

This law played an important role not only in the development of the automobile industry but also in the promotion of other industries, such as machinery, chemicals and textiles. Later, government policy switched from one of direction and supervision to one of extending financial support to these
Main contents of this law were as follows:

1. Permission is required for the production of a car of more than 750 cc and annual output of 3000 cars.

2. The permitted company has to have a majority of Japanese shareholders.

3. The permitted company is exempt for five years from paying the corporation tax, the tariff on imported machinery and necessary parts, and can also receive special treatment with regard to equity finance.

4. The permitted company has to be subject to the supervision of the government, and has to obey directions concerning the production of military vehicles and other appropriate military-related needs.

5. The vested interests of those already operating automobile production is admitted and is not subject to the first clause. However, vested interests applied to the time before 9 August 1935.

Toyoda Auto Loom, together with Nissan Motors, succeeded in being selected as permitted company on 19 September 1936. At the same time, GM of Japan

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9 See, Toyota Motors (1987), p. 94
10 See, Toyota Motors (1977), p. 55
and Ford of Japan were obliged to start restricting their production because the fifth clause of the law was effective retrospectively for nine months, which ensured that their planned expansion through the newly purchased land was impossible. Thus, Toyoda Auto Loom was provided with the basis for further development without having to worry about foreign competitors.

However, in those days, the average Japanese was not rich enough to afford buying a car without resort to some financial backing. To solve this problem, again, Toyoda Auto Loom had to resort to learning from the its US counterpart: both GM of Japan and Ford of Japan had been offering a payment-by-installment system through their finance company which was established in 1929. Their share of monthly installments in total sales amounted to 70-80 percent in 1935. Kiichiro decided to follow their example, and founded the Toyota Finance (root of Toyota Tsusho) in 1936 by enticing Tomotsune Jiromaru away from GM of Japan’s finance company.¹¹

Toyoda Auto Loom also began to export the G1 truck to Manchuria (for the use of the Japanese army which occupied the region after the Manchurian Incident) in July 1936. It is important to note here that the now famous ‘just-in-time’ system was seriously contemplated during this year. At that time,

¹¹See, Toyota Motors (1977), p. 72
the revenue from sales was insignificant in comparison with total production costs; thus, the major problem was how to reduce these costs. In spite of this, Kiichiro was determined not to reduce expenditure on technology development since he believed that improved machinery would contribute to better quality. Instead, he wanted to reduce costs pertaining to other elements of the production process. If there were some defects in the components, and shortages or excesses in part supplies, it would not have been possible to maintain a constant level of efficient production. In any case, inefficient production needs space for inventory storage, so he suggested the elimination for such a need since that would automatically cut down costs. Kiichiro wrote ‘just-in-time’ on a sheet of paper, and stuck it on a wall at the Kariya assemble plant. He repeatedly said the following:12

If you arrive at the station if only just a second after a train, you will have missed the train. However, it makes no sense to bring what is not necessary just in time. You have to bring only what is necessary just in time.

Hence, car production at Toyoda Auto Loom progressed steadily. The num-

12See, Toyota Motors (1987), p. 106
ber of workers in the assembly plant increased from 500 in 1932 to 3500 in 1936, and in March 1937, the company decided to establish Toyota Motors.

4.3 Rise of the Keiretsu Company

Toyota Motors was established on 16 August 1937, with Risaburo Toyoda as its president and Kiichiro Toyoda as its vice-president. This was the time just about one month after the ‘China Incident’ (it broke out in July 1937), so the control of the military was strengthened. At the same time, imports from the US ceased due to the restrictions on foreign exchange. Also, both Ford of Japan and GM of Japan were forced to discontinue production. Consequently, the production of Japanese makers boomed. Toyota could not meet the demand for military transport vehicles even after they built the new Koromo plant in 1938. So Toyota needed to expand production by increasing their number of machine tools. However, due to the war, both the import and domestic production of machine tools was becoming increasingly difficult. Thus, Toyota needed domestic suppliers for the thousands of needed to make a car. Thus, the Keiretsu companies were invented to cater for these problems; Keiretsu companies consisted of Kyoroku kei (independ-
dent groups of parts' suppliers) and *Choku kei* (a group of company derived from or established by Toyota Motors Company or Toyota Auto Loom). In the following sub-sections, I shall consider these two in turn.

### 4.3.1 Association of Parts' Suppliers, *Kyoho Kai*

As we have seen, at the initial production stage, Toyota acquired most of its parts from US makers. However, after 1939, the *Law of Automobile Production Enterprise* prohibited the importation of parts.\(^\text{13}\) What Toyota did was to manufacture only the costly and important parts that were not available on the domestic market, but tried to purchase most of the other parts domestically. However, in those days, Japanese parts' suppliers did not have enough experience in this area, hence most of them could not pass the quality standards set by Toyota Motors. Kiichiro proposed the '14 principles of purchasing policy' which established the principle that Toyota had to order each part from two suppliers, thus ensuring competition between parts' suppliers and improvement in the quality of parts. These principles also included the assessment of the performance of the suppliers close scrutiny of their manage-

\(^\text{13}\)See, *Toyota Motors* (1987), p. 104
ment and market conditions as well as through the constant checking of their quality.\textsuperscript{14} In reality, what he did was not only to force US-type competition, but also to create a spirit of co-operation between the parts’ suppliers and Toyota. The latter idea was made clear in the ‘Purchase Principle’ which was established in November 1939, and which reads as follows:\textsuperscript{15}

Once a company is accepted as a Toyota sub-contractor, Toyota will regard it as a plant of Toyota and will not easily replace it by a new company, but will, instead, do its utmost to improve the sub-contractor’s performance.

With this idea in mind, Toyota held its first meeting with sub-contractors on 8 November 1939, when they talked about how to secure the parts under severe commodity control conditions, and named the meeting ‘Kyoryoku Kai’, or ‘association of co-operative suppliers’; a clear indication of the good relations between them.\textsuperscript{16}

\textsuperscript{14}See, Toyota Motors(B) (1967), p. 389
\textsuperscript{15}See, Toyota Motors(B), (1967) p. 387
\textsuperscript{16}The member firms at that time were: Myodo Tekkosho (screw), Kojima Press Kogyosho (washer), Tsunekawa Tekkosho (press process), Ito Kinzoku Hikimono Seisakusho (screw), Tsuda Tekkosho (bolt, nut), Hayashi Spring Seisakusho (spring), Sugiura Seisakusho (screw), Showa Tankosho (piston rod), Wakabayasi Kogyosho (pin), Niwa Kogyosho (push rod, handle), Yokoi Seisakusho (screw), Komai Kikai Seisakusho (screw), Hoshin Kogyosho, Nakamura Seisakusho (press process), Tomoe Seisakusho (volt), Kato Seisakusho (crutch disk), Asahi radiator, Yajima Kogyo (valve, cover). See, also Toyota Motors (1957), p. 109
When World War II broke out in September 1939, the accessibility of materials became very severe, and this instigated a movement, initiated by the parts’ suppliers, for the tightening of the relationship between them and Toyota after the two had experienced a common sharing in hardship. As a result, they replaced the name ‘Kyoryoku Kai’ by a new one, ‘Kyohou Kai’, and changed the informal character of their meetings into a more formal one in 1943. They appointed Hisayoshi Akai, Vice-president of Toyota Motors, as Chairman of the association, and Hamakichi Kojima, President of Kojima Press, and Kozo Hasegawa, President of Myodo Tekkosho, as Vice-chairmen. The following was the remark made by Kojima about the basic idea behind ‘Kyohokai’:¹⁷

The Toyota family can be compared with the human body. The head, the heart and the internal organs are Toyota Motors and we suppliers are the hand and the feet. What are very important are the head and the heart, but although the little finger may be a small part, it is still part of the body; thus it is inconvenient for the body to be without it. Toyota Company should not make fun

¹⁷See, Toyota Motors (1957), p. 110
of a tiny factory just because it happens to be the little finger. At the same time, the small factory should not depreciate itself since it is indispensable to Toyota.

The member firms should act in cooperation with each other. Toyota and the member companies should become like one human body in order to enable the establishment of the Japanese automobile industry.

Thus the unification of the Toyota family was achieved to increase the bargaining power against the government during the war conditions, and to improve the quality of car production through the exchange of information on technology and management techniques after that.

4.3.2 Toyota Group Companies

Toyoda Steel Works.

In August 1937, almost at the same time as when Toyota was established, Japanese government enacted the Law for Iron Industry which stipulated, among other things, that: (a) government permission is required for a company to enter the steel industry; and (b) a policy of restraining and rationing
steel materials will be introduced. When the government introduced a rationing policy in each industry department in June 1938, it became extremely difficult for Toyoda Auto Loom to obtain, in the market, good quality steel materials for its automobile production. Therefore, after constructing its second steel mill, it established the ‘Toyoda Steel Works Company’ (the root of Arch Steel Works) in March 1940. This was considered desirable since the share of the steel works division in Toyoda Auto Loom was too high.\textsuperscript{18}

The President and the Vice-president of this new company were, respectively, again, Risaburo Toyoda and Kiichiro Toyoda.

\textbf{Toyoda Rikagaku Institute.}

Because of World War II, Toyota could no longer continue to rely on the import of technological information from the US and Europe. Moreover, since Sakichi's success was based on constant R&D on new technology, they were able to establish a research institute, ‘Toyoda Rikagaku Institute’, in February 1940. Again, the Managing Director and Vice-managing Director were, respectively, Kiichiro Toyoda and Risaburo Toyoda. However, this Institute was not confined to research on only fields relating to automobile

\textsuperscript{18}See, Toyota Motors (1977), p. 107

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production, its remit covered the pursuit of basic research.\(^{19}\)

**Toyoda Machine Works.**

Because heavy industry was relatively underdeveloped in Japan in those days, machine tools for automobile production were not available. So Toyota Motors had to establish a division for making them in the Koromo plant to enable it to expand production, as mentioned earlier. As the production in the Koromo plant increased, so did the role of the machine tools division. The demand for machine tools did not just arise from within Toyota, but also from outside; it came from the aircraft company and military plants. Again, Toyota decided in 1939 to transform this division into an independent company, and got it established as ‘Toyoda Machine Works.’ in 1941. The reason for the new company is related to government control policy. Two years after the introduction of the *Law of Automobile Production Enterprise*, the government enacted the *Law of Machine Tools Production Enterprise* (in 1938) to support the production of machine tools. This company had to be established since Toyota Motors was not allowed to take advantage

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\(^{19}\)See, Toyota Motors (1977), p. 105
of both laws simultaneously.\textsuperscript{20} The President and the Vice-president were, respectively, Risaburo Toyoda and Kiichiro Toyoda.

\textbf{Toyoda Boshoku and Toyoda Auto Loom.}

Although Toyoda Steel Works and Toyoda Machine Works were able to get established through expanded production supported by the government, Toyoda Boshoku and Toyoda Auto Loom were suffering from the depression; due to war-time controls, individual consumption had to decline. At the same time, both the import of materials and export of the final product were almost impossible. Thus, they had excess capacity and this forced them to reorganize the industry. Consequently, Toyoda Boshoku, Toyoda Oshikiri Boshoku, Chuo Boshoku, Utsumi Boshoku and Kyowa Boseki were merged in February 1942 to become the new ‘Chuo Boseki Company’.\textsuperscript{21} However, this did not help the declining company to survive; the government announced in June 1943 the idea of building up a war potential through industrial readjustment, which in effect led to the liquidation of the non-military related operations by trying to transfer both physical and human resources entirely

\footnotesize{\textsuperscript{20}See, Shioji (1991), p. 75  
\textsuperscript{21}See, Toyota Motors (1987), p. 155}
to military industry. Following this policy, Toyota Motors absorbed the Chuo Boseki Company in June 1943, the plants of which were all transformed into the aircraft division of Toyota Motors. All workers there were absorbed by Toyota Motors.

After the War, the textile industry was revived. As a result, the spinning and weaving division of Toyota Motors was again made an independent company in 1950, and was called ‘Minsei Boseki Company’. This company name was back to the original company name ‘Toyoda Boshoku’ in 1967.

Toyoda Auto Loom was also forced into abandoning its original business field to specialize in the production of automobile parts. The Toyota family company then totally changed from the light to the heavy industry.22

**Toyota Tsusho.**

The car sales department was also under the control. The government forced the establishment of the Japan Automobile Rationing Company in 1942 by integrating all Japanese car dealers (Toyota Motors, Nissan and Isuzu) in each prefecture. All cars produced were under the control of the military, hence it was impossible to sell to individual consumers. This in turn meant

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22See, Toyota Motors (1987), p. 167
that there was no longer any justification the existence of Toyota Finance Company since its main occupation was to deal with the monthly installments plan. Consequently, in 1942, the company changed the name to ‘Toyota Industrial Company’ and became a holding company of Toyota Group. However, after the War, this was designated as a holding company when the Zaibatsu were being dissolved, and was therefore liquidated. Then the trading division of the company was turned into ‘Nissin Tsusho Company’ in 1948, and was renamed ‘Toyota Tsusho’ in 1956.\(^{23}\)

**Aisin Seiki.**

After the retreat of the Japanese Navy during the *Battle of Midway* in 1942, the Japanese military demanded the expansion of aircraft production. As a result, Toyota Motors established, in 1943, ‘Tokai Aircraft Company’ to manufacture aircraft engines. The President was Risaburo Toyoda. However, the war ended before any engine could be produced. After the war, Toyota’s policy had to change dramatically in accordance with GHQ directions: it abandoned aircraft production and sales confined to the military since it became free of Zaibatsu designation. Accordingly, Tokai Aircraft Company

\(^{23}\)See, Toyota Motors (1987), p. 160
withdrew from aircraft production, and was renamed ‘Aichi Industrial Company’ in 1945.\textsuperscript{24} In 1965, it was merged with Sinkawa Industrial Company to become ‘Aisin Seiki’.

**Toyoda Gosei.**

The introduction of the military industrial readjustment plan in 1943 forced the division for rubber parts in the Kariya plant to be absorbed by ‘Kokka Kogyo’ because it did not reach the minimum size specified in the plan. The plant was moved to Nagoya, was named ‘Kokka Industrial Nagoya Plant’, later renamed ‘Nagoya Rubber Company’ before it became ‘Toyoda Gosei’\textsuperscript{25}.

**Toyota Auto Body.**

After the establishment of Toyoda Steel Works and Toyoda Machine Works, Kiichiro wanted to create an independent company out of the body division. In 1940, he applied to the Ministry of Commerce and Industry for approval to establish the Toyota Body Company. However, the application was rejected because it was deemed to diminish the power of Toyota. After splitting the

\begin{footnotesize}
\begin{itemize}
  \item \textsuperscript{24}See, Toyota Motors (1987), p. 193-194
  \item \textsuperscript{25}See, Toyota Motors (1977), p. 121
\end{itemize}
\end{footnotesize}

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rubber division and moving the electric devices section to an aircraft factory, the Kariya plant became specialized in building of car bodies. They applied for approval again in 1943 because they felt that independence was so necessary if high productivity and the introduction of new capital were to become a reality. Again, there was a rejection, and for the same reason. However, the application was finally accepted in July 1945, and the division was named ‘Toyota Body Industrial Company’. The President was Kiichiro Toyoda. Because the war ended soon after that, the company was renamed ‘Kariya Body Company’ in December 1945.\textsuperscript{26} After the start of the occupation, Toyota Motors dropped ‘Toyota’ (or ‘Toyoda’) from its company names and replaced it with local area names so as dispel any \textit{Zaibatsu} impression attributable to so many companies coming under the same umbrella name. Later on, in 1953, when the occupied forces eased the conditions, the company was renamed ‘Toyota Auto Body’.

\textbf{Nippondenso.}

As already mentioned, after the occupation, GHQ issued the order for the Dissolution of the \textit{Zaibatsu}, and this applied to four: Mitsui, Mitsubishi,

Sumitomo and Yasuda. Soon after, in 1947, GHQ declared the *Law for the Elimination of Excessive Concentration of Economic Power* with the object of promoting competition in each industry. Toyota Motors was considered ‘excessive’ in 1948. This, together with the *Law for Restructuring and Reorganizing Firms* introduced in 1936, was responsible for Toyota Motors issuing a plan of the reorganization of the company in 1948. However, when the US government policy changed to one of regarding Japan as its Asian factory, the designation of Toyota as being excessively powerful was waived in 1949. However, Toyota still wanted to reorganize the company, but with only minor modifications of its original plan: the basic idea was to separate automobile production from the divisions indirectly connected with it, such as electric devices, spinning and weaving, and enamel. Thus Toyota Motors decided to establish Nippondenso, Minsei Boseki and Aichi Horo companies in 1949. ‘Nippondenso’ was also established at the end the same year. The President was Torao Hayashi.\(^{27}\)

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\(^{27}\)See, Toyota Motors (1987), p. 208-209
Kanto Auto Works.

Amongst the Toyota group of firms, the Kanto Auto Works. is quite different from the others in terms of the background their establishment. While the others, as we have seen, were derived from a division of the Toyota Motors, this company had relation to that division. The company was originally called ‘Kanto Electric Automobile Production’ when it was established in 1941. It had a good technology for car-body production. Suffering from insufficient production equipment due to the Dodge Line, Toyota, in 1943, asked this company to manufacture and develop a body for a passenger car. Although kiichiro had dreamt of producing a passenger car, the dream could not come true before the end of the war, because it was only then, in 1944, that GHQ permitted increased production of passenger cars from 300 to 5000 as well as allowed the production of 5000 taxi cabs (in December). This made it possible to establish the company. It was renamed ‘Kanto Auto Works in 1945 and it became a member of the Group.28

28After the war, the other Toyota group companies were only Towa Real Estate and Toyota Central & Development Laboratories, established in 1953 and 1960 respectively. In addition to these, the Toyota group contains business tie-up firms such as Hino Motors (1966) and Daihatsu Motors (1967).
4.4 Concluding remarks

As we have seen, the Toyota Keiretsu was founded before the beginning of the Korean War, and that most of the Toyota group of companies had been derived from Toyota Motors and Toyota Auto Loom and its associations of parts' makers. Its creation happened in response to pre-war government policy and to postwar GHQ directives. However, given that Nissan Motors, which went through the same process, but did not form a Keiretsu system, it follows that it is far from clear why this division of Toyota Motors had developed so differently from Nissan. In order to investigate this question, one needs to study the Nissan's case. This, together with a comparison of the two cases, shall be my next task.

4.5 References


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