Thermodynamics of two-flavor lattice QCD with an improved Wilson quark action at non-zero temperature and density

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Abstract. We report the current status of our systematic studies of the QCD thermodynamics by lattice QCD simulations with two flavors of improved Wilson quarks. We evaluate the critical temperature of two flavor QCD in the chiral limit at zero chemical potential and show the preliminary result. Also we discuss fluctuations at none-zero temperature and density by calculating the quark number and isospin susceptibilities and their derivatives with respect to chemical potential.

1. Introduction

In order to extract unambiguous signals for the QCD phase transition from the heavy-ion collision experiments, quantitative calculations from first principle are indispensable. At present, the lattice QCD simulation is the only systematic method to do so and various interesting results have been already reported. So far, most lattice QCD studies at finite temperature (T) and chemical potential (μ_q) have been performed using staggered-type quark actions with the fourth-root trick of the quark determinant. Thus, the other actions such as the Wilson-type quark actions are necessary to control and estimate systematic errors due to lattice discretization.

Several years ago, the CP-PACS Collaboration has studied the QCD thermodynamics using the Iwasaki (RG) improved gauge action and the two-flavor clover improved Wilson quark action [1]. We revisit this action armed with recent techniques at finite chemical potential μ_q. In this report, we present on our preliminary results for the critical temperature at $\mu_q = 0$ as well as the quark number and isospin susceptibilities at finite μ_q. The latter are related to the physics of the possible critical point in the (T, μ_q) plane.

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2. Critical temperature of two-flavor QCD

The critical temperature (T_c) is one of the most fundamental quantities in the QCD thermodynamics. We renew the analysis of T_c done in [1] performing additional simulations at $\beta = 6/g^2 = 1.8$ for $N_t = 4$ and 1.95 for $N_t = 6$. We determine the pseudo-critical point β_{pc} defined from the peak of Polyakov loop susceptibility on $N_s^3 \times N_t = 16^3 \times 4$ and $16^3 \times 6$ lattices, as a function of the hopping parameter K.

It is confirmed in Ref. [1] that a subtracted chiral condensate satisfies the scaling behavior with the critical exponents and scaling function of the three-dimensional O(4) spin model. The reduced temperature t and external magnetic field h are identified to $t \sim \beta - \beta_{ct}$ and $h \sim m_q$, respectively, where β_{ct} is the critical transition point in the chiral limit. Assuming that the pseudo-critical temperature from the Polyakov loop susceptibility follows the same scaling law as the O(4) spin model, $t_{pc} \sim h^y$ with $y = 0.537(7)$, we fit the data of β_{pc} by $\beta_{pc} = \beta_{ct} + Ah^{1/y}$ with two free parameters, β_{ct} and A, in the range of $\beta = 1.8$–1.95 for $N_t = 4$ and $\beta = 1.95$–2.10 for $N_t = 6$. First, we adopt the definition $m_q a \sim 1/K - 1/K_c$ as the quark mass where K_c is the chiral point where the pion mass vanishes at $T = 0$ for each β. See Fig. 1 (left). We calculate the critical temperature T_c in the chiral limit using $T = 1/N_t a$. The lattice spacing a is estimated from the vector meson mass assuming $m_{\rm V}(T = 0) = m_\rho = 770$ MeV at β_{ct} on K_c. By this procedure, we obtain the preliminary results $T_c = 183(3)$ MeV for $N_t = 4$ and 174(5) MeV for $N_t = 6$. We also calculate β_{ct} using the relation of $m_q^{\rm AWI} \propto m_{\rm PS}^2$, where $m_{\rm PS}$ is the pseudo-scalar meson mass and $m_q^{\rm AWI}$ is the quark mass obtained from the axial vector Ward-Takahashi identity. The results of T_c are 173(3) MeV ($N_t = 4$), 167(3) MeV ($N_t = 6$) for $h = (m_{\rm PS} a)^2$ and 176(3) MeV ($N_t = 4$) for $h = m_q^{\rm AWI} a$. We note that these O(4) fits reproduce the data of β_{pc} much better than a linear fit $\beta_{pc} = \beta_{ct} + Ah$. From these analyses, we tentatively conclude that the critical temperature in the chiral limit is in the range 170–186 MeV for $N_t = 4$ and 164–179 MeV for $N_t = 6$. There is still a large uncertainty from the choice of the fit ansatz. To remove this, we are performing further simulations at lighter quark masses.

Next, we compare our results with those of the staggered quark action. We plot the results of the pseudo-critical temperature (T_{pc}) in unit of Sommer scale (r_0) as a function of $m_{\rm PS} r_0$ in Fig. 1 (right) together with those by the RBC-Bielefeld Collaboration using 2+1 flavor p4-improved staggered quark action [2]. As seen in this figure, results of T_{pc} obtained by different quark actions seem to approach the same function of $m_{\rm PS} r_0$ as N_t increases.

A care is in order when we convert $T_c r_0$ ($T_{pc} r_0$) to T_c (T_{pc}) in MeV using a physical value of r_0. Because the phenomenological estimate of r_0 has large theoretical uncertainties, it looks convenient to adopt a lattice result. Unfortunately, lattice results for r_0 suffer from sizable ambiguities yet. The RBC-Bielefeld Collaboration adopted the value $r_0 = 0.469(7)$ fm, which was obtained by Gray *et al.* [3] from a bottomonium mass splitting using the AsqTad-improved staggered quark action. On the other hand, the CP-PACS+JLQCD Collaboration found $r_0 = 0.516(21)$ fm from a study of light hadron spectrum using the clover-improved Wilson quark action [4]. This leads to about 10 % difference in the value of T_c and makes it difficult to naively compare results from different groups.

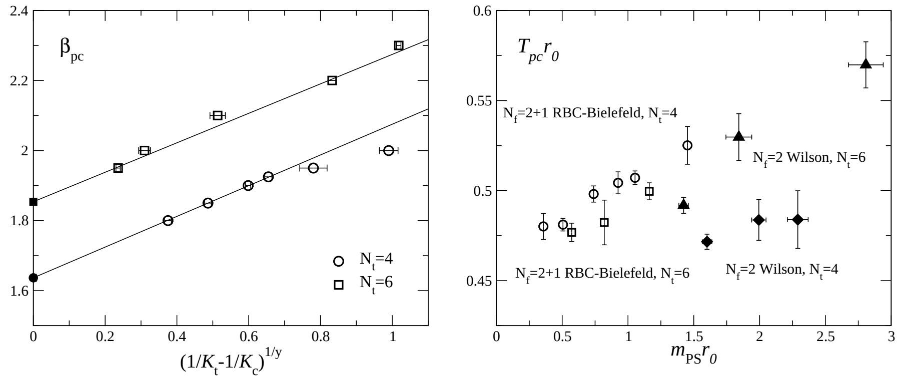

Figure 1. Left: The pseudo-critical point β_{pc} as a function of $m_q \sim 1/K - 1/K_c$ for $N_t = 4$ (circle) and $N_t = 6$ (square). Right: Comparison of T_{pc} scaled by r_0 between the staggered quark action (open symbol) and the Wilson quark action (filled symbol) for $N_t = 4$ and 6.

3. Hadronic fluctuations at finite μ_q

Hadronic fluctuations at finite density are observables closely related to the critical point in the (T, μ_q) plane and may be experimentally detected by an event-by-event analysis of heavy ion collisions. The fluctuations can also be studied by numerical simulations of lattice QCD calculating the quark number and isospin susceptibilities, χ_q and χ_I. They correspond to the second derivatives of the pressure with respect to μ_q and μ_I, where μ_I is the isospin chemical potential. From a phenomenological argument in the sigma model, χ_q is singular at the critical point, whereas χ_I shows no singularity there.

We perform simulations at $m_{\rm PS}/m_{\rm V} = 0.65$ and 0.80 on a $16^3 \times 4$ lattice. We calculate the χ_q and χ_I and their second derivatives with respect to μ_q and μ_I at $\mu_q = \mu_I = 0$. (Note that the odd derivatives are zero at $\mu_q = 0$.) The details of the calculations are reported in Ref. [5].

The left panel of Fig. 2 shows χ_q/T^2 (circle) and χ_I/T^2 (square) at $m_{\rm PS}/m_{\rm V} = 0.8$ and $\mu_q = \mu_I = 0$ as functions of T/T_{pc}. We find that χ_q/T^2 and χ_I/T^2 increase sharply at T_{pc}, in accordance with the expectation that the fluctuations in the QGP phase are much larger than those in the hadron phase. Their second derivatives $\partial^2(\chi_q/T^2)/\partial(\mu_q/T)^2$ and $\partial^2(\chi_I/T^2)/\partial(\mu_q/T)^2$ are shown in Fig. 2 (right). We find that basic features are quite similar to those found previously with the p4-improved staggered fermions [6]. $\partial^2(\chi_I/T^2)/\partial(\mu_q/T)^2$ remains small around T_{pc}, suggesting that there are no singularities in χ_I at non-zero density. On the other hand, we expect a large enhancement in the quark number fluctuations near T_{pc} as approaching the critical point in the (T, μ_q) plane. The dashed line in Fig. 2 (right) is a prediction from the hadron resonance gas model, $\partial^2\chi_q/\partial\mu_q^2 \approx 9\chi_q/T^2$. Although current statistical errors in Fig. 2 (right) are still large, we find that $\partial^2(\chi_q/T^2)/\partial(\mu_q/T)^2$ near T_{pc} is much larger than that at high temperatures. At the right end of the figure, values of free quark-gluon gas (Stefan-Boltzmann gas) for $N_t = 4$ and for $N_t = \infty$ limit are shown. Since the lattice discretization error in the equation of state is known to be large at $N_t = 4$ with our quark action, we need to extend our study to larger N_t for the continuum extrapolation.

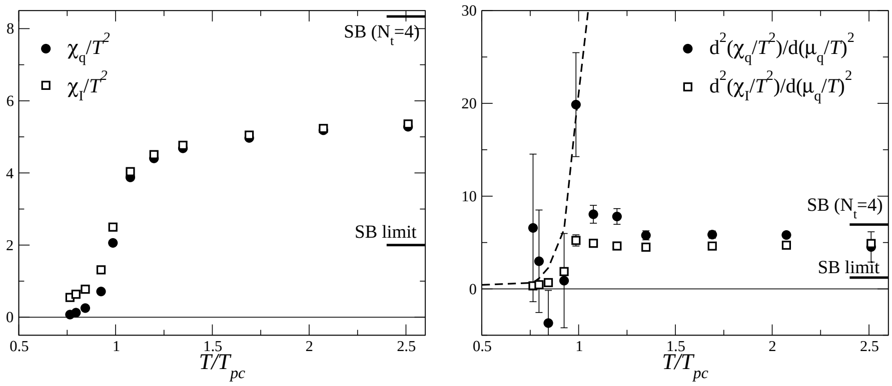

Figure 2. Left: Quark number (circle) and isospin (square) susceptibilities at $\mu_q = \mu_I = 0$. Right: The second derivatives of these susceptibilities.

4. Conclusion

We reported the current status of our study of QCD thermodynamics using the two-flavor improved Wilson quark action. The critical temperature is estimated from the Polyakov loop susceptibilities on $16^3 \times 4$ and $16^3 \times 6$ lattices. We discussed uncertainties from the chiral extrapolation and the scale parameter. Our preliminary result of T_c in the chiral limit is in the range of T_c = 170–186 MeV for $N_t = 4$ and T_c = 164–179 MeV for $N_t = 6$.

The fluctuations of quark number and isospin densities were also discussed. Although the statistical errors are still large, we find that χ_q seems to increase rapidly near T_{pc} as μ_q increases, whereas the increase of χ_I is not large near T_{pc}. These behaviors qualitatively agree with the previous results obtained with the p4-improved staggered fermions.

We also studied the static quark free energies, Debye screening masses and spatial string tension at finite temperature with the same action, which are reported in detail in Ref. [7].

Acknowledgments

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