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HEREDITARILY INDECOMPOSABLE COMPACTA DO NOT ADMIT EXPANSIVE HOMEOMORPHISMS

HISAO KATO AND CHRISTOPHER G. MOURON

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Abstract. A homeomorphism $h : X \longrightarrow X$ is *expansive* provided that for some fixed $c > 0$ and every $x, y \in X$ there exists an integer n, dependent only on x and y, such that $\mathrm{d}(h^n(x), h^n(y)) > c$. It is shown that if X is a hereditarily indecomposable compactum, then h cannot be expansive.

1. Introduction

A homeomorphism $h : X \longrightarrow X$ is *expansive* provided that for some fixed $c > 0$ and every $x, y \in X$ there exists an integer n, dependent only on x and y, such that $\mathrm{d}(h^n(x), h^n(y)) > c$. A *continuum* is a compact, connected metric space. A homeomorphism h is *continuum-wise expansive* provided that for some fixed $c > 0$ and every non-degenerate subcontinuum A, there exists an integer n such that $\mathrm{diam}(h^n(A)) > c$. A compactum X is *0-dimensional* provided that for every $\epsilon > 0$ there exists a finite open cover $\mathcal{V}$ with mesh less than ϵ of X such that every point of X is in exactly 1 element of $\mathcal{V}$. Similarly, a compactum X is *n-dimensional* provided that for every $\epsilon > 0$ there exists a finite open cover $\mathcal{V}$ with mesh less than ϵ of X such that every point of X is in at most $n + 1$ elements of $\mathcal{V}$ and X is not $(n - 1)$-dimensional. Every n-dimensional compactum must contain an n-dimensional continuum. For $n \geq 1$, every expansive homeomorphism of an n-dimensional compactum is continuum-wise expansive, but the converse is not so. It should be noted that h is expansive (continuum-wise expansive) if and only if h^{-1} is expansive (continuum-wise expansive).

A continuum is *decomposable* if it is the union of two of its proper subcontinua. A continuum is *indecomposable* if it is not decomposable. In order for a homeomorphism to be continuum-wise expansive, subcontinua must be continually stretched; this creates indecomposable subcontinua. There is strong evidence that continua that admit expansive homeomorphisms must contain indecomposable subcontinua (see [4] and [7]). However, there is also strong evidence that in order for a homeomorphism to be expansive, subcontinua must be stretched and wrapped around the continuum as opposed to stretched and folded. For example, tree-like continua do not admit expansive homeomorphisms [5].

A continuum is *hereditarily indecomposable* if every subcontinuum is indecomposable. Likewise, a compactum is *hereditarily indecomposable* if every non-degenerate

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subcontinuum is indecomposable. Hereditarily indecomposable continua are created by "infinite folding" of subcontinua. There are examples of 1-dimensional hereditarily indecomposable continua that admit continuum-wise expansive homeomorphisms. However, the infinite folding also raises the entropy of the homeomorphism, and it is known that expansive homeomorphisms must have finite entropy. In this paper it will be shown that continuum-wise expansive homeomorphisms on finite-dimensional hereditarily indecomposable compacta must have infinite entropy [2]. It will then be concluded that hereditarily indecomposable compacta cannot admit expansive homeomorphisms. Mañé has shown that infinite-dimensional compacta do not admit expansive homemorphisms [3].

2. Chain covers of indecomposable continua

Let $\mathcal{U}$ be a finite open cover of compactum X. The *mesh* of $\mathcal{U}$ is defined by

$$\operatorname{mesh}(\mathcal{U}) = \sup\{\operatorname{diam}(U) | U \in \mathcal{U}\}.$$

The cover $\mathcal{V}$ *refines* the cover $\mathcal{U}$ if for each $V \in \mathcal{V}$ there exist $U \in \mathcal{U}$ such that $V \subset U$. $\mathcal{V}$ *closure refines* $\mathcal{U}$ if for each $V \in \mathcal{V}$ there exist $U \in \mathcal{U}$ such that $\overline{V} \subset U$. A cover is *taut* if $\overline{U} \cap \overline{V} = \emptyset$ for all disjoint $U, V \in \mathcal{U}$. From here on out, we will assume that all covers are taut. For $U \in \mathcal{U}$, the *core* of U is defined as

$$\operatorname{core}(U) = \bigcap \{U - \overline{V} | V \in \mathcal{U} - \{U\}\}.$$

Notice that if $\mathcal{U}$ is a taut open cover of a compact space, then $\operatorname{core}(U) \neq \emptyset$ for every $U \in \mathcal{U}$. The *star* of $\mathcal{U}$ is defined by

$$\mathcal{U}^* = \bigcup_{U \in \mathcal{U}} U.$$

A *chain* $[C_1, C_2, ..., C_n]$ is a collection of open sets such that $C_i \cap C_j \neq \emptyset$ if and only if $|i - j| \leq 1$. The elements of a chain are called *links* and C_1 and C_n are called *end-links* for $n \geq 2$. A *generalized chain* $\mathcal{C}_G = [C_1, C_2, ..., C_n]_G$ is a collection of open sets such that $C_i \cap C_j \neq \emptyset$ implies that $|i - j| \leq 1$. The maximal chains of a generalized chain $\mathcal{C}_G$ are called the *chain components* of $\mathcal{C}_G$. A subcontinuum H *runs through* a chain $[C_i, ..., C_j]$ if $\operatorname{core}(C_i) \cap H \neq \emptyset$ and $\operatorname{core}(C_j) \cap H \neq \emptyset$. A chain is *proper* if there exists a subcontinuum of X running through it.

Suppose that chain $\mathcal{C}$ has $7q$ links for some positive integer q. Then the subchains of the form $\mathcal{C}_p = [C_{7p+1}, ..., C_{7p+7}]$ where $p \in \{0, ..., q-1\}$ are called *Lucky 7 subchains*. Notice that if H intersects at least 15 elements of $\mathcal{C}$, then H must run through some (proper) Lucky 7 subchain.

If $m < n$ are positive integers, then we denote the set $\{m, m+1, ..., n\}$ by $[m, n]$. A function $f : [1, m] \longrightarrow [1, n]$ is called a *pattern* provided $|f(i+1) - f(i)| \leq 1$ for $i \in \{1, 2, ..., m-1\}$. A pattern $f : [1, km+2] \longrightarrow [1, m+2]$ is a *proper simple k-fold* provided that

$$f(i) = \begin{cases} 1 & \text{if } i = 1, \\ (i - 1 \mod 2m-2) + 1 & \text{if } i \in [p(2m-2)+2, \\ & \qquad p(2m-2)+m+1], \\ m - 1 - (i - 1 \mod 2m-2) & \text{if } i \in [p(2m-2)+m+2, \\ & \qquad (p+1)(2m-2)+1], \\ m+2 & \text{if } i = km+2, \end{cases}$$

where $p \geq 0$.

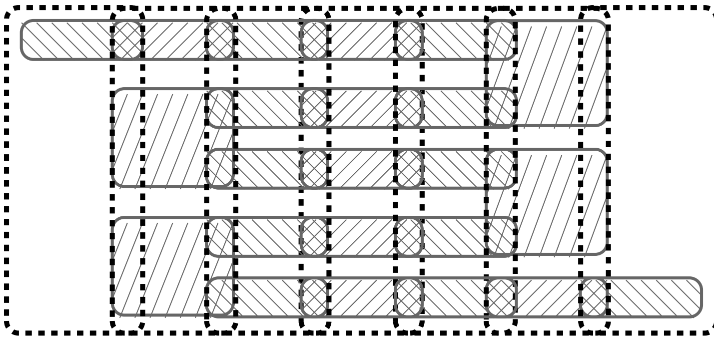

FIGURE 1. Proper 5-fold.

Let $\mathcal{V} = [V_1, ..., V_m]$ and $\mathcal{U} = [U_1, ..., U_n]$ be chain covers of a compactum X and let $f : [1, m] \longrightarrow [1, n]$ be a pattern. We say that $\mathcal{V}$ *follows pattern* f in $\mathcal{U}$ provided that $V_i \subset U_{f(i)}$ for each $i = 1, ..., m$. If f is a proper simple k-fold, then we say that $\mathcal{V}$ is a k*-fold refinement* of $\mathcal{U}$ (see Figure 1). In a k-fold refinement, the links of the form $V_{f(i)}$, where $f(i-1) = f(i+1)$, are called the *bend links* of the k-fold. Suppose that $\mathcal{U}$ is a taut finite open cover. Then define

$$\mathrm{d}(\mathcal{U}) = \min\{\mathrm{d}(\overline{U}_i, \overline{U}_j) | U_i \cap U_j = \emptyset \text{ for } U_i, U_j \in \mathcal{U}\}.$$

Lemma 1. *Let $\mathcal{U} = [U_1, ..., U_n]$ be a taut open chain cover of continuum Y where $n \geq 7$ and suppose that $\mathcal{V} = [V_1, ..., V_m]$ is a k-fold refinement of $\mathcal{U}$. Then there exists k subcontinua $\{Y_i\}_{i=1}^k$ of Y such that $diam(Y_i) \geq d(\mathcal{U})$ and $d(Y_i, Y_j) \geq d(\mathcal{V})$ whenever $i \neq j$.*

Proof. Let $\{n(i)\}_{i=1}^{k-1}$ be an increasing sequence of integers such that $V_{f(n(i))}$ is a bend link of $\mathcal{V}$. Define $\mathcal{C}_1 = [V_3, ..., V_{f(n(1))-1}]$, $\mathcal{C}_i = [V_{f(n(i-1))+1}, ..., V_{f(n(i))-1}]$ for $2 \leq i \leq k-1$ and $\mathcal{C}_k = [V_{f(n(k-1))+1}, ..., V_{f(m-2)}]$. Then for each i there exists a subcontinuum Y_i contained in the chain $\mathcal{C}_i$ which intersects both end-links of $\mathcal{C}_i$. Since the end-links of $\mathcal{C}_i$ are contained in U_3 and U_{n-2}, it follows that $\mathrm{diam}(Y_i) \geq \mathrm{d}(\mathcal{U})$. Also, since $\mathrm{d}(\overline{\mathcal{C}_i^*}, \overline{\mathcal{C}_j^*}) \geq \mathrm{d}(\mathcal{V})$ for $i \neq j$, it may be concluded that $\mathrm{d}(Y_i, Y_j) \geq \mathrm{d}(\mathcal{V})$ whenever $i \neq j$. $\square$

The next theorem is due to Oversteegen and Tymchatyn:

Theorem 2 ([6])**.** *Let X be a hereditarily indecomposable compactum and let $\mathcal{U} = [U_1, ..., U_n]$ be an open taut chain cover of X such that there exists a subcontinuum $Z \subset X$ such that $Z \cap core(U_1)$ and $Z \cap core(U_n)$ are both nonempty. Let $f : [1, m] \longrightarrow [1, n]$ be a pattern on $\mathcal{U}$. Then there exists an open taut chain cover $\mathcal{V}$ of X such that $\mathcal{V}$ follows pattern f in $\mathcal{U}$.*

Corollary 3. *Suppose that X is a finite-dimensional hereditarily indecomposable compactum and $\mathcal{V}_G$ is a taut generalized chain cover for X with $7q$ links. Then for each k, there exists a taut refinement $\mathcal{V}_k$ such that each proper Lucky 7 subchain in $\mathcal{V}$ is refined with a proper k-fold. (See Figure 2.)*

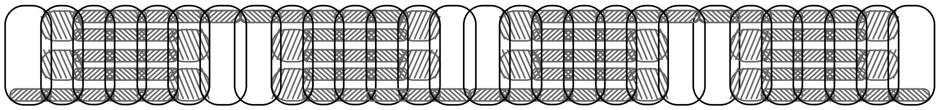

FIGURE 2. Each Lucky 7 subchain is refined by a 5-fold.

Proof. This follows from the fact that if $[C_{7p+1}, ..., C_{7p+7}]$ is a proper Lucky 7 chain, then

$$X \cap \overline{\text{core}(C_{7p+1}) \cup C_{7p+2} \cup ... \cup C_{7p+6} \cup \text{core}(C_{7p+7})}$$

is a hereditarily indecomposable compactum that satisfies the hypothesis of Theorem 2. $\square$

Next we must find ways to cover higher-dimensional compacta with chains so that "large" subcontinua must run through some Lucky 7 subchain. The following results give the prescription for this:

We define a map $g : X \longrightarrow Y$ to be *light* if $g^{-1}(y)$ is 0-dimensional for $y \in Y$.

Theorem 4 ([1])**.** *Let X be a compact metric space. Then $dim(X) \leq m$ if and only if there exists a light map $g : X \longrightarrow I^m$.*

Lemma 5. *Let $g : X \longrightarrow Y$ be a light map and X, Y be compact spaces. For each $\delta > 0$, there exists a finite open cover $\mathcal{U}_\delta$ such that if $U \in \mathcal{U}_\delta$, then every component of $g^{-1}(U)$ has diameter less than δ.*

Proof. Suppose on the contrary that there exists a sequence of finite open covers, $\{\mathcal{U}_i\}_{i=1}^{\infty}$, with the following properties:

(1) $\text{mesh}(\mathcal{U}_i) \to 0$ as $i \to \infty$.
(2) $\mathcal{U}_i$ closure refines $\mathcal{U}_{i-1}$.
(3) There exists $U_i \in \mathcal{U}_i$ such that some component, C_i, of $g^{-1}(U_i)$ has diameter greater than or equal to δ.

If component C_i of $g^{-1}(U_i)$ is such that $\text{diam}(C_i) \geq \delta$ and $U_{i-1} \in \mathcal{U}_{i-1}$ is such that $\overline{U_i} \subset U_{i-1}$, then there exists a component C_{i-1} of $g^{-1}(U_{i-1})$ such that $C_i \subset C_{i-1}$. Hence by considering partial orderings, it is possible to find sequences $\{U_i\}_{i=1}^{\infty}$ and $\{C_i\}_{i=1}^{\infty}$ with the following properties:

(1) $U_i \in \mathcal{U}_i$.
(2) $\overline{U_i} \subset U_{i-1}$.
(3) C_i is a component of $g^{-1}(U_i)$.
(4) $C_i \subset C_{i-1}$.
(5) $\text{diam}(C_i) \geq \delta$.

Let $\{y\} = \bigcap_{i=1}^{\infty} \overline{U_i}$ and $C = \bigcap_{i=1}^{\infty} \overline{C_i}$. Then C is connected, contained in each $g^{-1}(\overline{U_i})$, and $\text{diam}(C) \geq \delta$. However, then

$$C \subset g^{-1}(\bigcap_{i=1}^{\infty} \overline{U_i}) = g^{-1}(y),$$

which contradicts the fact that g is light. $\square$

If $g : X \longrightarrow I^m$, then define $g_i = \pi_i \circ g$ where $\pi_i : I^m \longrightarrow I$ is the ith coordinate map.

Theorem 6. *Let $g : X \longrightarrow I^m$ be a light map and $\delta > 0$. Then there exists a chain cover $\mathcal{C}$ of I such that if H is a subcontinuum of X with diam$(H) \geq \delta$, then there exists $i \in \{1, ..., m\}$ such that H runs through some (proper) Lucky 7 chain of $g_i^{-1}(\mathcal{C})$.*

Proof. By Lemma 5, there exists a finite cover $\mathcal{U}$ of I^m such that every component of $g^{-1}(U)$ has diameter less than δ for each $U \in \mathcal{U}$. Choose $\mathcal{C}_0$ to be a chain cover of I with mesh sufficiently small so that $\bigcap_{i=1}^{m} \pi_i^{-1}(\mathcal{C}_0)$ refines $\mathcal{U}$. Then choose $\mathcal{C}$ to be a chain cover of I with $7q$ links and with mesh sufficiently small so that every subchain of 14 or fewer links is contained in some element of $\mathcal{C}_0$. If for each i there exists a subchain $\mathcal{V}_i$ of $\mathcal{C}$ with 14 or fewer links such that $g_i(H) \subset \mathcal{V}_i^*$, then $g(H)$ would be contained in some $U \in \mathcal{U}$ which would violate the fact that every component of $g^{-1}(U)$ has diameter less than δ. Thus there exists some i such that no subchain of $\mathcal{C}$ can completely contain $g_i(H)$. Thus, H must run through some proper Lucky 7 subchain of $g_i^{-1}(\mathcal{C})$. Note that in general, $g_i^{-1}(\mathcal{C})$ may be a generalized chain if X is not connected, but then $g_i^{-1}(\mathcal{C})$ is a disjoint union of finite chains for each $i = 1, 2, ..., m$. If necessary, we consider such chains. $\square$

3. Entropy and expansive homeomorphisms

Entropy is a measure of how fast points move apart. Continuum-wise expansive homeomorphisms have positive entropy. The following definition of entropy is due to Bowen [8].

If $h : X \longrightarrow X$ is a map and n a non-negative integer, define

$$d_n^+(x, y) = \max_{0 \leq i < n} d(h^i(x), h^i(y)).$$

Let K be a compact subset of X and n be a positive integer. A finite subset E_n of K is said to be (n, ϵ)-*separated* with respect to map h provided that if x and y are distinct elements of E_n, then $d_n^+(x, y) > \epsilon$. Let $s_n(\epsilon, K, h)$ denote the largest cardinality of any (n, ϵ)-separated subset of K with respect to h. Then

$$s(\epsilon, K, h) = \limsup_{n \to \infty} \frac{\log s_n(\epsilon, K, h)}{n}.$$

The entropy of h on X is then defined as

$$\mathrm{Ent}(h, X) = \sup\{\lim_{\epsilon \to 0} s(\epsilon, K, h) | K \text{ is a compact subset of } X\}.$$

It can be shown that if h is a homeomorphism, then $\mathrm{Ent}(h^{-1}, X) = \mathrm{Ent}(h, X)$.

The proofs of the following theorems are exactly the same as the proofs for Corollary 2.4 and Proposition 2.5 in [2]. However the following theorems are stated for compacta, whereas Corollary 2.4 and Proposition 2.5 are stated for continua.

Theorem 7. *Let $h : X \longrightarrow X$ be a continuum-wise expansive homeomorphism on compactum X. Then there exists a $\delta > 0$ such that for every $\gamma > 0$ there exists $N_\gamma > 0$ such that if A is a subcontinuum of X with diam$(A) > \gamma$, then diam$(h^n(A)) > \delta$ for all $n \geq N_\gamma$ or diam$(h^{-n}(A)) > \delta$ for all $n \geq N_\gamma$.*

Theorem 8. *Let $h : X \longrightarrow X$ be a continuum-wise expansive homeomorphism on compactum X. Then there exists a non-degenerate subcontinuum A such that either $\lim_{n \longrightarrow -\infty}$ diam$(h^n(A)) = 0$ or $\lim_{n \longrightarrow \infty}$ diam$(h^n(A)) = 0$.*

Corollary 9. *Let $h : X \longrightarrow X$ be a continuum-wise expansive homeomorphism on compactum X. Suppose that A is a non-degenerate subcontinuum such that $\lim_{n\longrightarrow -\infty} diam(h^n(A)) = 0$. Then there exists $\delta > 0$ such that for every integer m, subcontinuum $B \subset h^m(A)$ and $\gamma > 0$ there exists $N_\gamma > 0$ such that if $diam(B) > \gamma$, then $diam(h^n(B)) > \delta$ for every $n \geq N_\gamma$.*

The next theorem is the main theorem of this section.

Theorem 10. *Continuum-wise expansive homeomorphisms on hereditarily indecomposable compacta have infinite entropy.*

Proof. Let h be a continuum-wise expansive homeomorphism on X. Without loss of generality we may assume that there exists a subcontinuum A such that $\lim_{n\longrightarrow -\infty} \mathrm{diam}(h^n(A)) = 0$ (otherwise, consider $\widehat{h} = h^{-1}$, which is also a continuum-wise expansive homeomorphism on X with the same entropy). Let δ be defined from Corollary 9. By Theorem 4, there exists a light map $g : X \longrightarrow I^m$ where $m = \dim(X)$. Let $\mathcal{C}$ be a chain cover of I that satisfies conclusion of Theorem 6. Let $\gamma = \min\{\mathrm{d}(g_\alpha^{-1}(\mathcal{C})) \mid \alpha \in \{1, ..., m\}\}$ and k be any positive odd integer greater than 1. By Corollary 3, there exists a taut refinement $\mathcal{V}_k^\alpha$ of $g_\alpha^{-1}(\mathcal{C})$ such that every proper Lucky 7 subchain of $g_\alpha^{-1}(\mathcal{C})$ is refined by a proper mk-fold. Let $\epsilon_k = \min\{\mathrm{d}(\mathcal{V}_k^\alpha) \mid \alpha \in \{1, ..., m\}\}$. Also, there exists an integer n such that $\mathrm{diam}(h^n(A)) > \delta$. Let $H = h^n(A)$. Then by Theorem 6, there exists a Lucky 7 subchain $\mathcal{C}_p^\alpha = [C_{7p+1}, C_{7p+2}, ..., C_{7p+7}]$ of $g_\alpha^{-1}(\mathcal{C})$ such that H runs through $\mathcal{C}_p^\alpha$. Then by Lemma 1, there exists mk subcontinua $\{H(i)\}_{i=1}^{mk}$ of H such that

(1) $\mathrm{diam}(H(i)) > \gamma$.
(2) $\mathrm{d}(H(i), H(j)) > \epsilon_k$ for $i \neq j$.

It follows from Corollary 9 that

$$\mathrm{diam}(h^{N_\gamma}(H(i))) > \delta \text{ for each } i \in \{1, ..., mk\}.$$

Thus by the pigeonhole principle there exists an $\alpha \in \{1, ..., m\}$ such that at least k of $\{H(i)\}_{i=1}^{mk}$, say $\{\widehat{H}(i)\}_{i=1}^{k}$, have the property that each $h^{N_\gamma}(\widehat{H}(i))$ runs through some Lucky 7 subchain of $g_\alpha^{-1}(\mathcal{C})$.

Continuing inductively, suppose that $\{\widehat{H}(i_1, i_2, ..., i_n)\}_{i_j \in \{1,...,k\}}$ have been found such that

$$\mathrm{diam}(h^{N_\gamma}(\widehat{H}(i_1, i_2, ..., i_n))) > \delta \text{ for each } i_j \in \{1, ..., k\}.$$

Then by Lemma 1, there exists mk subcontinua, $\{H(i_1, i_2, ..., i_n, i_{n+1})\}_{i_{n+1}=1}^{mk}$ of $\widehat{H}(i_1, i_2, ..., i_n)$, such that

(1) $\mathrm{diam}(H(i_1, i_2, ..., i_n, i_{n+1})) > \gamma$.
(2) $\mathrm{d}(H(i_1, i_2, ..., i_n, i_{n+1}), H(i_1, i_2, ..., i_n, j_{n+1})) > \epsilon_k$ for $i_{n+1} \neq j_{n+1}$.

Again, by the pigeonhole principle there exists an $\alpha \in \{1, ..., m\}$ such that at least k of $\{H(i_1, i_2, ..., i_n, i_{n+1})\}_{i_{n+1}=1}^{mk}$, say $\{\widehat{H}(i_1, i_2, ..., i_n, i_{n+1})\}_{i_{n+1}=1}^{k}$, have the property that each $h^{N_\gamma}(\widehat{H}(i_1, i_2, ..., i_n, i_{n+1}))$ runs through some Lucky 7 subchain of $g_\alpha^{-1}(\mathcal{C})$.

Choose a point $x(i_1, ..., i_n) \in h^{-(n-1)N_\gamma}(\widehat{H}(i_1, ..., i_n))$ and let E_n be the collection of such points. Notice that $h^{(j-1)N_\gamma}(x(i_1, ..., i_n)) \in \widehat{H}(i_1, ..., i_j)$ for each $j \in \{1, ..., n\}$. Hence, E_n is an (n, ϵ_k)-separated set. Thus

$$\begin{aligned}
\mathrm{Ent}(\epsilon_k, H, h) &= \limsup_{n\to\infty} \frac{\log s_n(\epsilon_k, H, h)}{n} \\
&\geq \limsup_{n\to\infty} \frac{\log s_{nN_\gamma}(\epsilon_k, H, h)}{nN_\gamma} \\
&\geq \limsup_{n\to\infty} \frac{\log |E_n|}{nN_\gamma} \\
&= \limsup_{n\to\infty} \frac{\log k^n}{nN_\gamma} \\
&= \frac{\log k}{N_\gamma}.
\end{aligned}$$

Hence,

$$\mathrm{Ent}(h, X) \geq \sup_{k\to\infty} \mathrm{Ent}(\epsilon_k, H, h) = \infty.$$

□

The following theorem is found in Walters [8].

Theorem 11. *If h is an expansive homeomorphism on a compact space, then Ent(h) is finite.*

Now we may conclude our main result:

Theorem 12. *Hereditarily indecomposable compacta do not admit expansive homeomorphisms.*

Proof. Suppose that $h : X \longrightarrow X$ is an expansive homeomorphism on hereditarily indecomposable compactum X. Then h is continuum-wise expansive. So by Theorem 10, $\mathrm{Ent}(h) = \infty$. However, this contradicts Theorem 11. □

Corollary 13. *Hereditarily indecomposable continua do not admit expansive homeomorphisms.*

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Institute of Mathematics, University of Tsukuba, Ibaraki, 305 Japan
E-mail address: hisakato@sakura.cc.tsukuba.ac.jp

Department of Mathematics, The University of Alabama at Birmingham, Birmingham, Alabama 39254
Current address: Department of Mathematics and Computer Science, Rhodes College, Memphis, Tennessee 38112
E-mail address: mouronc@rhodes.edu