How universal is the $C$ function in the bulk ABL similarity approach for estimating surface sensible heat flux?

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Abstract

The $C$ function which appears in the formulation for the BAS (bulk ABL similarity; ABL - atmospheric boundary layer) approach estimating sensible heat flux ($H$) at the surface under unstable stability conditions, was examined to investigate its universality, and whether or not it is a function of atmospheric stability alone, using data sets obtained from five large-scale experiments that took place in diverse geographical locations with different surface covers and climate conditions (ranging from the tropics to the boreal zone). Coefficients in several forms of the $C$ function, first assumed as a function of the stability alone, were calibrated against these data sets to maximize the agreement between $H$ values derived from the BAS and those reference values independently determined. The agreement of $H$ values for this calibration, for all data sets except one, was excellent with an rms error on the order of 20 W m$^{-2}$ under the condition that the exact values of surface scalar roughness and atmospheric stability were known. The findings indicate that the $C$ function is a function of stability only, and universal within this accuracy level. Inclusion of another possible parameter, the ratio of the rotational height scale and the actual depth of the mixed layer, did not improve the results. A selection of different scaling for surface layer height did not produce better results, either. Practical limitations of the BAS approach were also investigated by considering one site that did not yield satisfactory results. Using an error propagation analysis, it was determined that this finding was mainly due to weak surface heating in this area under near neutral conditions and that the difference between the potential temperature at the surface and that in the mixed layer was too small to be used beyond the measurement error limit in the BAS approach. The need for a method to estimate surface scalar roughness was also identified as the largest remaining problem in the BAS approach used for actual flux estimations.
1. Introduction

The bulk atmospheric boundary layer (ABL) similarity (BAS - the bulk ABL similarity) provides a framework that enables a functional relationship between surface fluxes and corresponding values within the ABL. The shape and nature of universal functions to be determined have been the subject of study for a long time [see e.g., Brutsaert, 1982; Garratt, 1992; Brutsaert, 1999; Brutsaert, 2005]. Only recently have they come to be recognized as a practical tool for determining surface fluxes [e.g., Brutsaert and Sugita, 1991; Sugita and Brutsaert, 1992a]. The BAS approach has an advantage since remotely sensed variables such as surface temperature and possibly mixed layer quantities can be utilized. Regional scale fluxes (of $10^2$ km [Hiyama et al., 1995]) needed for many climate studies and water resource evaluations can also be determined. Functional forms of the $C$ function in the BAS equation for sensible heat fluxes $H$ and the analogous $B_w$ function for momentum flux were determined in the studies of Brutsaert and Sugita [1991] and Sugita and Brutsaert [1992a]. To show the usefulness of this approach, the forms were applied in the BAS to estimate regional $H$ and friction velocity ($u_*$) values with the data set obtained from radiosoundings during the First ISLSCP Field Experiment (FIFE) [Sellers et al., 1988]. Since these publications, in 1991 and 1992, several tests have been made regarding applicability of the BAS equations in the context of flux estimation in several settings and with different data sets. For example, Brutsaert and Parlange [1996] tested the $B_w$ functions with data obtained over the Landes forest in southwestern France. Sugita and Brutsaert [1992b], Brutsaert and Sugita [1992], and Brutsaert et al. [1993] applied the BAS equations to estimate $H$ using satellite derived surface temperatures, and Crago et al. [1995] used $u_*$ derived from the geostrophic wind with radiosonde-derived temperature and surface measurements of the skin surface temperature to estimate regional $H$. More recently, Jacobs et al. [2000] used active radar profilers with radio acoustic sounding systems to observe ABL wind and temperature values with a much better temporal sampling rate than radiosoundings.

Although the above mentioned exploratory studies have all indicated the potential of the BAS equations for estimating regional surface fluxes, one question that has only been answered partially is whether the universal functions $C$ and $B_w$ are really universal and functions of stability alone. To answer this question, a test with as many experimental data sets as possible (not only similarity considerations to verify that all relevant variables are included and there are no redundant variables involved in the formulation [see e.g., Brutsaert and Sugita, 1991]) obtained for a range of different conditions reflecting surface features, climatic conditions, atmospheric conditions, geographic location, etc. need to be carried out in a consistent manner. To some extent, for the $B_w$ function, Sugita et al. [1999] performed this task using three data sets
and their result indicates that $B_w$ can be treated as a universal function of atmospheric stability. For the $C$ function a comprehensive study of this kind has not been carried out yet. In this work, the three data sets in Sugita et al. [1999] and two additional data sets, one obtained in a dry season in the central part of Thailand and one obtained in an extensive steppe region in Mongolia, were used to study the universality of the $C$ function, especially for the purpose of flux estimation by means of the BAS. Since the new data sets were obtained under different conditions than the others, especially in terms of surface dryness and higher surface temperatures, a better and more thorough examination of the $C$ function should be possible. Therefore, it is the purpose of this paper to try to understand the behavior of the $C$ function for estimating regional surface fluxes within the context of the BAS approach.

2. Method

2-1. Experimentations and data set

As mentioned above, five data sets were analyzed in the present study. Among them, three data sets are identical to those used for the study of the $B_w$ function in Sugita et al. (1999), namely, the data sets obtained in (i) the Tsukuba Atmospheric Boundary Layer Experiment [TABLE, Sugita et al., 1993], (ii) the Northern hemisphere Climate Processes land Surface Experiment [NOPEX, Halldin et al., 1999], and (iii) the First ISLSCP Field Experiment [FIFE, Sellers, et al., 1988]. The fourth data set (iv) came from the GEWEX Asian Monsoon Experiment (GAME), where GEWEX stands for Global Energy and Water cycle Experiment. The fifth data set (v) was obtained from the Rangelands Atmosphere-Hydrosphere-Biosphere Interaction Study Experiment in Northeastern Asia [RAISE, Sugita et al., 2007]. The setting and environmental conditions in the various data sets are quite diverse. For example, surface features in the experimental areas range from sub-urban to hilly grasslands to forests. The climates within the experimental areas range from savanna to boreal, and from temperate humid to steppe to reflect both temperature and humidity differences. The main features for each experimental setting and data set are summarized in Table 1. Since data sets (iv) and (v) are new, an outline of the observations and some more details of the measurement system are given in the discussion that follows.

GAME

Intensive observations took place toward the end of a dry season from February 15 through March 3, 1999. Approximately 80 radiosoundings were launched 3-7 times per day from approximately 830 TST (Thai Standard Time) to 1800 TST together with observations of surface fluxes and relevant meteorological and hydrological variables on and around a 120-m tower.
The surrounding area consists mainly of a mixture of deciduous and evergreen forests with variable heights of 5-20 m (82%), grasslands and farmland (9%) and paddy fields (8%) on a generally flat area within 20 km of the tower in the dominant wind direction of E-S-W. Photographs of the area are available in Toda et al. [2002].

The radiosounding system used for measurements (GPSonde, Atmospheric Instrumentation Research) consisted of a disposable sonde attached to a balloon with a thermometer, a relative humidity sensor, a barometer with a GPS receiver, and a ground station that received a signal from the sonde and also from GPS satellites every second. According to the manufacturer [Atmospheric Instrumentation Research, 1995], each sensor on the sonde has the specification of accuracy of $\Delta T = 0.3^\circ$C with a response time constant $\tau < 1$ s for its temperature sensor, $\Delta RH = 3\%$ and $\tau < 1$ s for the relative humidity, and $\Delta p = 1$ hPa and $\tau < 0.1$ s for the pressure sensor. At the ground station these raw data, transmitted from the sonde every second, were further processed to produce time averages. An averaging time of 5 s was selected.

Wind speeds were also calculated every 5 s from the movement of a sonde determined by consecutive differential measurements of the GPS signals at the sonde location and at the ground station. Atmospheric Instrumentation Research [1995] indicates (from a comparison of wind speeds determined by this system to those from a radar) that an accuracy of 0.5 ms$^{-1}$ for wind speeds can be obtained from this system. A typical vertical resolution for the data is around 10 to 30 m.

Other variables measured during the observation and used for the present analysis include sensible heat flux ($H$) and friction velocity ($u_*$), estimated using the eddy correlation approach with a sonic anemometer (R3A, Gill Instruments) mounted at 60 m on the tower; the surface temperature ($T_s$), obtained by means of an infrared radiation thermometer (IRT, 4000.4G, Everest Interscience) at 30 m on the tower pointing obliquely downward; air temperature ($T_a$) and relative humidity ($RH$), measured by a ventilated hygro-thermometer at 60 m; air pressure ($p$) at the surface; and the latent heat flux ($LE$), obtained through the bandpass covariance technique with $T_a$ and $RH$ measurements together with $H$ values from the eddy correlation method. Details of the $LE$ derivation at this site have been documented in Toda et al. [2002]. When $u_*$ values were not available at the time of radiosoundings, $u_*$ from the wind speed profile equation together with wind speed measurements from radiosoundings within the surface layer of ABL were used, using roughness parameters determined for this area from Toda and Sugita [2003]. When $H$ and $LE$ observations from the above methods were not available, those derived from another sonic anemometer (Kaijo DA-600 with a TR-90AH prove) mounted at 30 m on the tower and from energy balance consideration were used. In the following discussion, $H$ and $LE$ values, measured
or determined from observations of surface flux stations, are referred to as the reference sensible and latent heat flux \(H_s\) and \(LE_s\), respectively.

RAISE

A small aircraft (Antonov, AN-2) was used as a platform for measuring the temperature and humidity within the ABL above the surface of an extensive steppe region in Mongolia in the summer of 2003 [Kotani and Sugita, 2007]. An aircraft flight to the experimental area usually consisted of consecutive flight path segments for measurements for approximately half an hour at three fixed levels of, approximately, 200, 500 or 1000 m over a horizontal distance of 5-10 km. Data sets used for the analysis described here were derived from each of these 3-level flight path segments. A comparison of the temperatures from these 3-level measurements indicated a well–mixed condition for the ABL [Kotani, 2006]. Time averages of the variables measured in each flight segment were therefore treated as representative of, or equivalent to, ABL averages in the discussion that follows. A flux station was also in operation to provide reference surface fluxes of \(H_s\) and \(LE_s\) by means of the eddy correlation method. Surface temperature obtained by an IRT and other standard meteorological variables were also measured at this station [see Sugita et al., 2007 for the details of this station].

The regional friction velocity \(\left(u_*,\right)\), estimated by Kotani and Sugita [2007] by applying the Rossby number similarity, which relates the surface stresses and the geostrophic wind \(G\) [e.g., Zilitinkevich, 1975], was used in the analysis. The roughness length \(z_0\) needed in this analysis was determined as \(z_{0,NW}=0.054\) m and \(z_{0,SE}=0.430\) m for NW and SE directions, respectively, from the topographic analysis by applying the formulation of Grant and Mason [1990] to a DEM data set with a horizontal resolution of 7-12.5 m and a vertical resolution of 15 m produced as part of the ASTER 3D data set (Abrams, 2000). The northward and eastward components of \(G\), i.e., \(u_g\) and \(v_g\), were determined from the pressure gradient on a 750 hPa isobaric surface from the outputs of the regional climate model applied to the study area [TERC-RAMS, Sato and Kimura, 2005, Sato et al., 2007; Sugita et al., 2007] with a horizontal resolution of 30 km and a time interval of one hour.

Data Selection

Among the available potential temperature profiles, only those that satisfied the following criteria were used in the analysis: (i) measurements were made either in neutral or unstable conditions on rainfree days; (ii) surface flux stations were in operation; (iii) winds came from dominant wind directions; and (iv) \(H_s>20\) W m\(^{-2}\) (for FIFE) or \(H_s>15\) W m\(^{-2}\) (for the other
data sets). Application of these criteria produced 108 profile data sets from FIFE, 23 sets from NOPEX, 39 sets from TABLE, 51 sets from GAME, and 17 flight segments from RAISE experiments.

2-2. BAS equation

The BAS formulation for $H$ can be derived by assuming an overlap region between the mixed layer and the surface layer and by joining two governing equations of these layers [e.g., Brutsaert, 1982; Brutsaert and Sugita, 1991; Brutsaert, 2005]. A recent version for the potential temperature is given by

$$H = \frac{(\theta_{s,r} - \theta_a) k u_s \rho c_p}{\ln \left( \frac{h_i - d}{z_0 h,r} \right) - C} \tag{1}$$

where $\theta_a$ is the average potential temperature in the mixed layer, $\theta_{s,r}$ is the potential surface temperature determined radiometrically by an IRT, $c_p$ is the specific heat of the air, $\rho$ is the density of the air, $z_{0h,r}$ is the radiometric scalar roughness for sensible heat, $h_i$ is the height of the ABL and $d$ is the zero-plane displacement height. Scalar roughness ($z_{0h,r}$) is called “radiometric“ because it is determined from radiometrically derived $\theta_{s,r}$ by means of an IRT. Known [e.g., Sugita and Brutsaert, 1990b; Kubota and Sugita, 1994; Brutsaert and Sugita, 1996] is that the surface temperature of vegetated and complex surfaces cannot be defined unambiguously due to the variability of surface temperature within the canopy or on complex surfaces, and that a different $\theta_{s,r}$ can be obtained for the same surface depending on how an IRT is positioned and where it is aimed, which translates into potentially different values of $z_{0h,r}$ for the same surface. $C$ is the similarity function and has been assumed to be a function of stability $(h_i - d)/L$ based on physical considerations [Brutsaert and Sugita, 1991] where $L$ stands for the Obukhov length given by

$$L = \frac{u^3}{k(g/T_a)(H+0.61T_gE)(\rho c_p)} \tag{2}$$

where $k$ (=0.4) is the von Karman's constant, $T_a$ is the air temperature in K, $g$ is the acceleration of gravity, and $E$ is the rate of evaporation. The functional form of $C$ has been studied with the FIFE data set in Brutsaert and Sugita [1991] and Sugita and Brutsaert [1992a]. Tested forms, which were either selected from common forms proposed in earlier studies or derived in their studies, can be given as,
\[ C = a \ln[-(h_i - d)/L] + b \]  
\[ (3) \]

\[ C = a \ln[1 - (h_i - d)/L] + b \]  
\[ (4) \]

\[ C = a \left[-(h_i - d)/L\right]^b \]  
\[ (5) \]

\[ C = \ln\{1 + \left[-(h_i - d)/L\right]^a/b\} \]  
\[ (6) \]

\[ C = \Psi_h [(a z_0)/L] + \ln[(h_i - d)/z_0] - b \]  
\[ (7) \]

where \( a \) and \( b \) are constants to be determined for each functional form and \( \Psi_h \) is the stability correction function for \( H \) for the surface layer equation (see below). The values of \( a \) and \( b \) of each equation are, in general, different and were determined in their studies by a trial and error method in which \( H \) values were determined from (1) and (3)-(7) first with arbitrarily selected \( a \) and \( b \) constants for each equation. Resulting fluxes were compared with reference \( H \) values and statistics such as the correlation coefficient \((r)\), the ratio of the means \((<H_i>/<H>)\), the regression coefficients \((c \text{ and } e)\) in a linear equation \( H_i = c H + e \), and the root mean square (rms) error. This process was repeated by changing the constants \( a \) and \( b \) in small steps, and \( a \)- and \( b \)-values that produced the best, on average, agreement between \( H_i \) and \( H \) were finally selected as the calibrated constants. The same method was adopted in the present analysis, except that priority was given to the selection criterion of the smallest rms error [including the total, systematic and unsystematic parts, Willmott, 1981] among the others, followed by requirements for \( c \) and \( e \) to produce the unique combination of constants for each data set. Note that the \( C \) function determined this way to optimize the best \( H \) agreement, is not necessarily the same as a function aimed at producing the best representation of the relationship between \( C \) and atmospheric stability, since the BAS approach involves non-linear operations.

An initial test has indicated that the NOPEX data set requires quite different constants \( a \) and \( b \) in the \( C \) functions than the others, and it was decided that \( C \) functions optimized for the combined four data sets excluding the NOPEX are presented first in what follows, then the NOPEX data set is treated separately with a discussion for the possible reasons for the different behavior of this data set.

Unlike the functional forms (3)-(6) which have no physical basis, the \( C \) function given by (7) was derived by assuming a certain ABL model and from (1) with the potential temperature profile equation in the surface layer of ABL:
where $\theta$ is the potential temperature in the surface layer at $z$, by taking the ratio of (8) evaluated at height $z=z_m$ and (1) with $z_m-d$ replaced by $az_0$ [Sugita and Brutsaert, 1992a]. The height of $z_m$ represents $z$ in the surface layer where $\theta(z_m)$ is equal to the mean potential temperature ($\theta_a$) in the mixed layer. If one assumes a surface layer overlain by a slightly stable mixed layer, which tends to agree with observations, $z_m$ should be somewhere between $z=d$ and the top of the surface layer. In the past, it has been assumed [e.g., Sugita and Brutsaert, 1992a] that $z_m$ can be taken as the log mean height of the lower and upper limit of the surface layer. The present analysis also followed this assumption. Also in the same manner as Sugita and Brutsaert [1992a], $b=\ln(az_0/z_0h,r)$ was treated as a constant to be determined using calibrations. From previous studies, $a=69$ derived from the height range of the surface layer $48 \leq (z-d)/z_0 \leq 101$ found for FIFE [Brutsaert and Sugita, 1991; Sugita and Brutsaert, 1992a], $a=40$ for TABLE from $18 \leq (z-d)/z_0 \leq 90$ [Hiyama et al., 1996], and $a=30$ from $16 \leq (z-d)/z_0 \leq 56$ in NOPEX [Sugita et al., 1999] were adopted in the following analysis; and the mean value of $a=112$ was obtained for the GAME data set by determining the surface layer extent from visual inspection of $\theta$ profiles. For the RAISE data set, the information is not available and an overall mean of $a=63$ for all data sets was simply adopted. Such a convention is likely acceptable since the fluxes determined from (1) with (7) are not very sensitive to the choice of the $a$ value since $\Psi_h$ is only a mild function of the stability.

Values of $h_i$ were determined for GAME at the lowest height where $d\theta/dz \geq 6.0$ K/km in radiosounding profiles, as for previous studies [Brutsaert and Sugita, 1991 and Sugita and Brutsaert, 1992a for FIFE; Sugita et al., 1999 for TABLE; and Hiyama et al., 1999 for NOPEX]. To avoid possible spurious values due to small oscillations in the radiosonde profiles, the validity of the selected height was confirmed visually on a $\theta$ vs $z$ plot. For the RAISE data set, use was made of $h_i$ values estimated by Kotani and Sugita [2007] by applying the method of Liu and Ohtaki [1997] in which the peak frequency of the spectra of the horizontal wind speed data obtained at the surface flux station was analyzed to estimate $h_i$.

In the application of (1) to estimate $H$, it was necessary to use an iteration procedure since (1) was implicit. However, in the present analysis, values of $L$ were evaluated independently with $H_s$, $LE$, and $u^*$ to simplify the procedure. This procedure was acceptable since the purpose of the present analysis was to test the universality of the $C$ function, and not to give estimates of $H$. However, even for the purpose of deriving surface heat fluxes, this method of
evaluating \( L \) should not change the values markedly since \( C \) is only a mild function of \((h_r-d)/L\).

As for the surface roughness parameters \( d \) and \( z_0 \), those previously determined for each area and listed in Table 2 were adopted in the analysis. The value of \( z_{0h,r} \) must also be specified. As mentioned above, when surface temperature is determined by an IRT over a vegetated or complex surface, \( z_{0h,r} \) cannot be estimated unambiguously. In the present analysis, to avoid the propagation of error due to the uncertainty of \( z_{0h,r} \) into the final result, we decided to use a \( z_{0h,r} \) value evaluated independently from (8) for each profile, first using known \( H=H_s \) and \( u^* \) values.

The \( z_{0h,r} \) values adequate for the surface layer should be the same as those for the mixed layer, except for the minor difference of the footprint area that variables in these two layers represent. Therefore, in the analysis of \( C \) functions, it can be assumed that the influence of \( z_{0h,r} \) is minimal.

3. Results and discussion

As mentioned above, the \( C \) functions \((3)-(7)\) were calibrated to maximize the agreement between \( H \) and \( H_s \) in all of the data sets excluding NOPEX. Resulting coefficients \((a \text{ and } b)\) for each equation \((3)-(7)\) are listed in Table 4. Relevant statistics of the comparison between \( H_s \) and \( H \) estimated from (1) with \((3)-(7)\) using the calibrated constants in Table 4, are given in Table 5 for each data set and for the four combined data sets. In general, all of the functional forms of \((3)-(7)\) appear to produce the same value, more or less, for \( H \) for the combined data sets, with a slightly better outcome for \((7)\). A comparison of \( H \) values from the \( C \) function \((7)\) using \( H_s \) is shown in Fig.1. Our findings are in agreement with those of Sugita and Brutsaert [1992a] for the FIFE data set, and with those of Sugita et al. [1999] for the \( B_w \) function. Our findings indicate: 1) as long as the \( C \) function is well calibrated and the coefficients \( a \) and \( b \) are determined, the exact shape of the function is not quite relevant for the purpose of flux estimations; 2) a single function of atmospheric stability can produce consistent results for all data sets excluding NOPEX. Therefore, it is likely that the \( C \) function is universal. The first observation was not unexpected since it is known that \( C \) is a mild function of stability, as can be seen in Fig.2 where the \( C \) functions \((3)-(7)\) optimized for the combined four data sets are compared with \( C \) values calculated from (1) for each observation. Also shown in the figure is the difference \((\delta C)\) between \( C \) values from (1) and those from \((3)-(7)\). As mentioned previously, although the best agreement of \( H \) does not necessarily mean the best overall agreement of \( C \), \((7)\) appears to provide better results than the others both for \( C \) and \( H \) in this case; the rms value of \( \delta C \) for the four combined data sets is, 1.62 for \((3)\), 1.72 for \((4)\), 1.86 for \((5)\), 2.55 for \((6)\) and 1.54 for \((7)\). The difference among the equations is small except for near neutral stability ranges where a larger scatter of \( \delta C \)
can be seen, and this is likely one of the reasons for the slightly better performance of (7). For

\[- \left( \frac{h_i - d}{L} \right) \to 0 \], (7) behaves according to the assumed ABL structure and approaches

\[\ln \left( \frac{h_i - d}{z_m - d} \right)\] which is around 2.3 if one considers the fact that \(z_m\) is on the order of 1/10

of \(h_i\) [Sugita and Brutsaert, 1992; Hiyama et al., 1999]. For the others the limiting value is

arbitrary and without physical basis. For the data sets used in the analysis, the difference only

makes a minor difference in the \(H\) comparison.

In order to determine the universality of the \(C\) function, particularly with respect to

the NOPEX data set, it was necessary to analyze the difference between the data sets. As can be

seen from Fig.1 and Table 5, there is a small but systematic difference among the data sets when

(7) is used to estimate \(C\). This difference is due to an over and underestimation of \(H\) depending

on the data set. For example, for the RAISE data set, \(H\) tends to be underestimated. For the

NOPEX data set, \(H\) tends to be overestimated. The TABLE, FIFE and GAME data sets seem to

produce \(H\) values that on average agree with \(H_s\). Although there are several possible reasons for

these findings, they could result from either an overestimation or underestimation of \(H_s\). As seen

in Table 1, \(H_i\) was estimated solely from using the eddy correlation approach for GAME and

RAISE and partially for NOPEX, while the other data sets came largely from the Bowen ratio

method. The energy balance is quite often not closed when \(H_i\) and \(LE_s\) are measured

independently by an eddy correlation system. Majority reports that the sum of \(H_i\) and \(LE_s\) is

smaller than the available energy, which implies a possible relative underestimation of \(H_i\) and

\(LE_s\) using the eddy correlation approach, although other energy balance terms also need to be

considered. However, even though this could explain the overestimation in the NOPEX data set,

it does not provide a consistent and comprehensive explanation for the behavior of all data sets,

and, therefore, cannot be the only reason. Another possible explanation for the systematic

difference found for each data set is that the \(C\) function is a function of not only atmospheric

stability but also other factors even though their influences may be much smaller. Possible

factors that were dealt with in the past include baroclinicity, the ratio of the convective height

scale \((h_i)\) and the rotational height scale \((h_r)\), the diurnal heating effect, inertia, large-scale

advection, entrainment, and subsidence among others. Brutsaert and Sugita [1991] concluded

from reviews of past studies and from an additional considerations that these factors are probably

negligible as long as a layer averaged scalar variable such as \(\theta_a\) is used in the BAS. However,

as mentioned above, there has not been an extensive experimental evaluation of this issue.

Therefore, it is still possible that some of these additional parameters play a role in the shape of

the \(C\) function. Recently, Kotani and Sugita [2007] explored this idea in the formulation of the

mixed layer variance similarity. Although, in the past, there have not been many studies on this
subject, the mixed layer temperature variance $\sigma_\theta$ scaled with the convective temperature scale $T_*$, namely, $\phi_0 = \sigma_\theta / T_*$ has been treated as a function of $h_i$ and entrainment fluxes [e.g., Sugita and Kawakubo, 2003]. Kotani and Sugita [2007] tested this idea further by considering additional non-dimensional variables that may affect the mixed layer variance using large-scale data obtained by means of a regional climate model calibrated specifically for the area [Sato et al., 2007]. In their analysis, $\phi_0 = \sigma_\theta / T_*$ was treated as a multi-variable equation

$$\sigma_\theta / T_* = \phi_{\phi_0} \left( \frac{z-d}{L, \mu, \nu_0, \beta, \beta_1, L, A, \gamma_x, \gamma_y} \right)$$

(9)

in which non-dimensional variables are defined as $\nu_0 = \frac{h_i}{h_r}$, $\beta_\mu = \frac{\partial u_x}{\partial z} \left( \frac{h_i}{h_r} \right)^2 \left| f \right|$, $\beta_w = \frac{\partial \nu_r}{\partial z} \left( \frac{h_i}{h_r} \right)^2 \left| f \right|$, $A = \frac{\bar{w} \theta}{\nu_0 \bar{\theta}}$, $\gamma_x = \frac{\partial u \theta}{\partial x} \frac{h_i}{\bar{\theta} \bar{h}}$, and $\gamma_y = \frac{\partial v \theta}{\partial y} \frac{h_i}{\bar{\theta} \bar{h}}$ where $f$ is the Coriolis parameter; $\bar{w} \theta$ is the sensible heat flux with the subscripts $s$ and $t$ indicating surface and the top of ABL; and $h_i (= ku_f f^{-1})$ is the Ekman layer depth. Kotani and Sugita [2007] found that each parameter was equally important and with the inclusion of all these additional parameters, the accuracy of the sensible heat flux, as estimated from $\sigma_\theta$ measurements, increased by approximately 25%. Therefore, it is possible that these factors also have some influence on $C$ values. Unfortunately, due to data limitations, the only parameter that could be tested in the present data sets was $\nu_0$.

The $C$ function was modified to include the $\nu_0$ term; in the case of (7), it reads

$$C = \Psi_h \left[ ((az_0)/L) + \ln((h_i - d)/z_0) \right] + m_1 \nu_0 + m_2$$

(10)

The coefficients $m_1$, $m_2$, and $m_3$ were determined in the same manner to determine (3)-(7) to achieve the goal of the best agreement, on average, between $H$ from (1) and (10), and $H_s$. The coefficients of $m_1 = 0.01$, $m_2 = 0.354$, $m_3 = 4.17$ were determined and the relevant statistics were determined again for the same four data sets. However, the improvement was not substantial and the systematic difference among the data sets in Fig.1 was not reduced significantly. Perhaps this is not too surprising since Kotani and Sugita [2007] reported that each of the possible additional parameters has similar effect in reducing the error and perhaps $\nu_0$ alone is not quite sufficient. Also, the study of Kotani and Sugita [2007] deals with the second moment while the current paper utilizes the first moment; higher moments in general tend to be susceptible to many factors. Clearly, further study is needed to solve this problem.

Another argument that can be applied to (7) is that the scaling variable for deriving this particular form of the $C$ function may not be applicable to all data sets. As noted, $z_m - d = az_0$.
was assumed in the derivation, and $z_m - d$ represents the height in the surface layer whose potential temperature is the same as the mixed layer average. Brutsaert [1999] reasoned that $z_m - d$ could be scaled with $h_i$ for moderately rough terrains while for rougher surface scaling using $z_o$ was more adequate. Actually other suggestions have also been made. For example, Garratt [1980] proposed using the spacing of the surface roughness elements. At any rate, if $h_i$ is to be used, (7) takes the form

$$C = \Psi_h \left( b \frac{h_i - d}{L} \right) - \ln(b)$$

(11)

and $b=0.12$ has been suggested [see Brutsaert, 2005]. The version of the $C$ function that should be applied depends on the surface features, and Brutsaert [1999] provided criteria that (11) can be used whenever $b(h_i - d)$ is larger than $az_o$. For mean values of $h_i - d$ and $z_o$, GAME and RAISE are judged to fall in the category of moderately rough terrain. Thus for these two data sets (11) was adopted in (1), while for the other data sets (7) (with the same coefficients in Table 4) was used. However, the resulting statistics for the $H$ and $H_r$ comparison are essentially the same as those in Table 5; and therefore the choice of relevant scaling in the height range of the surface layer does not appear as important as the stability. Therefore, it is not completely conclusive, whether or not the systematic difference among the data sets is the manifestation of additional factors not considered in the formulation of the $C$ function. However, it is also quite clear that the role of additional factors, if any, is likely to be small and that the $C$ function can be treated as a universal function of stability alone, for the purpose of estimating $H$ fluxes within an accuracy level of 10-20 W m$^{-2}$; provided that a properly calibrated $C$ function is used and that other errors can be assumed negligible (see below).

Another aspect one notices easily in Fig. 1 is the difference of the degree of scatter of the points for each data set. As mentioned, the scatter is larger with the NOPEX data set followed by RAISE, while it is much smaller for the other data sets, as can be confirmed in Table 5 by noticing the smaller $R^2$ and larger $rmsd_o$ values. This finding was further investigated by evaluating the probable error ($\delta H$) of estimating $H$ by means of (1) with (7). A probable error ($\delta x$) in a function $x=f(y_1, y_2, ..., y_n)$, which consists of several variables ($y_i$) with their own absolute error ($\delta y_i$) can be evaluated, in general, by

$$\delta x = \left[ \left( \frac{\partial x}{\partial y_1} \right)^2 + \left( \frac{\partial x}{\partial y_2} \right)^2 + \cdots + \left( \frac{\partial x}{\partial y_n} \right)^2 \right]^{1/2}$$

(12)

[e.g., Bevington and Robinson, 1992]. The probable error can be easily applied to the BAS...
approach using (1) and (7) by assuming that the main sources of error are $\Delta \theta$, $u_*$, and 
$z_{0h,r}$ and by denoting their errors as $\delta \Delta \theta$, $\delta u_*$, and $\delta z_{0h,r}$. To reflect the condition and nature of 
the analysis shown in Fig.1 and Table 5, $\delta z_{0h,r}=0$ was assumed. With a typical order of 
magnitude of $\delta \Delta \theta=0.5$ K, $\delta u_*=0.1$ ms$^{-1}$ and with average conditions of $\Delta \theta=1.6$ K (NOPEX), 
3.7 K (FIFE), 9.3 K (RAISE), 7.0 K (TABLE), and 10.2 (GAME), the probable error of $\delta H=57$ 
W m$^{-2}$ was derived for NOPEX, while it was found to be smaller (14-36 W m$^{-2}$) for the other four 
data sets. Clearly, the distinctive stability condition that is closer to neutral at the NOPEX site 
than the others played an important role in the scatter of the NOPEX result. The near neutral 
stability resulted for NOPEX mainly from the two factors of smaller $\Delta \theta$ and larger $u_*$; the mean 
$u_*$ for NOPEX (=1.02 m s$^{-1}$) was much larger than the others (=0.2-0.7 ms$^{-1}$) but the mean $H_i$ was 
about the same (=100 W m$^{-2}$) for all five data sets. This can be interpreted as a practical 
limitation of the BAS approach in the estimation of $H$. When and where $\Delta \theta$ is small, BAS is not 
expected to work, not because of a theoretical shortcoming, but because the magnitude of $\Delta \theta$ is 
too close to the measurement error limit.

The scatter in the $H$ comparison for the RAISE data set, which is slightly larger than 
the others except for the NOPEX result, may not be able to be explained from the above error 
propagation analysis. In fact, the error analysis indicates a smaller probable error of 15 W m$^{-2}$. 
Therefore, additional error sources not examined above must have played a role. Possible 
additional sources of error that are specific to the RAISE observation include the fact that ABL 
measurements were carried out by an aircraft at a single level, that $h_i$ was estimated indirectly 
from the surface turbulence data, and that $u_*$ values were derived from the Rossby number 
similarity. Each of these may have contributed to the additional scatter.

Finally, it is of practical interest to apply (1) in a prognostic mode to estimate $H$, once 
$C$ functions have been calibrated, since it is important to highlight other practical limitations of 
the BAS approach besides the $C$ function. Among such possible limitations, the difficulty of 
estimating $z_{0h,r}$ is probably the largest obstacle in the application of the BAS. For this purpose, 
$z_{0h,r}$ needs to be estimated independently for each profile. This was carried out by deriving an 
empirical relationship between $z_{0h,r}$ and the solar elevation $\alpha$ (FIFE, TABLE and NOPEX), or by 
taking an average value of $\ln(z_{0h,r})$ (GAME and RAISE). For the three data sets, the adopted form 
was

$$z_{0h,r} = \exp \left[ a_1 \alpha^2 + a_2 \alpha + a_3 \right]$$  \hspace{1cm} (13)$$

where the coefficients were, with $\alpha$ expressed in degrees, $a_1=0.021$, $a_2=-1.783$, and $a_3=38.2$
northerly wind case); $a_1=0.015$, $a_2=-1.801$, and $a_3=53.0$ (SW wind case) for NOPEX; for
TABLE, $a_1 =0.020$, $a_2 =-2.231$, and $a_3 =26.0$ (southerly wind case), and $a_1 =-0.011$, $a_2 =1.533$, and $a_3 =-73.4$ (easterly wind case). For FIFE they were reported in Sugita and Brutsaert [1992a]. Although these equations gave the best agreement, the coefficient of determination was generally not very high, with $R^2 =0.2-0.6$. Therefore, the exact shape of the function does not likely provide any specific physics behind the empirical equations.

With the estimated $z_{0h,r}$ value together with $\theta_a$, $\theta_{s,r}$ and $h_i$ for each profile, the application of (1) with (7) produced an estimation of $H$ for each profile. Again, for the determination of $L$, $H_s$, and $LE_s$ were used to simplify the procedure and should not make much difference in the final result. The resulting $H$ values were compared with the corresponding $H_i$ values. The statistics are given in Table 6 for the result with (1) and (7) as an example. Clearly, the uncertainty of the $z_{0h,r}$ value has a significant effect on the estimation of $H$ by means of the BAS, as the average rms error became worse from 20 W/m$^2$ to 72 W/m$^2$ when $z_{0h,r}$ derived for each profile was replaced with $z_{0h,r}$ estimated from (13) or with average $z_{0h,r}$ values. This result can also be explained using the same error propagation analysis given above, but with typical $\delta z_{0h,r}$ values assigned for each data set. The order of magnitude $\delta z_{0h,r}$ was also estimated by applying (12) to (8), with the mean values of $z_{0h,r}$, $u_*$, $H_s$, $\Delta \theta$, and $(z-d)/L$, and the prescribed value $\delta H_t =20$ W m$^2$, and should represent the probable error of $z_{0h,r}$ estimated from surface layer measurements. The values for $\delta z_{0h,r} =8 \times 10^{-2}$ m (FIFE), $3 \times 10^{-2}$ m (RAISE), $3 \times 10^{-4}$ m (TABLE), $2 \times 10^{-3}$ m and $7 \times 10^{0}$ m (NOPEX) were obtained, which will immediately produce $\delta H$ estimates of 70-103 W m$^2$. Although this simple analysis does not include error due to the use of (13) or overall averages, the resulting values show that errors from the measurements tend to dominate. Thus, a method to evaluate the scalar roughness is still an important issue for the estimation of surface fluxes [see e.g., Brutsaert and Sugita, 1996; Sugita and Brutsaert, 1996; Crago, 1998; Crago and Suleiman, 2005], although it is outside the scope of the present paper.

4. Conclusions

The $C$ function that appears in the bulk ABL similarity equation for sensible heat flux was determined and analyzed using five data sets that covered a wide range of geographic locations, stability and surface conditions. The results indicate that the $C$ function can be treated as a function of atmospheric stability $(h_t-d)/L$ alone and that a single function can be used for all data sets. In other words, the $C$ function can be regarded as a universal function of $(h_t-d)/L$. The statements above can be considered valid within the accuracy that it provides sensible heat flux estimates that agree with the reference values leading to rms errors on the order of 20 W m$^2$, on average, if the scalar roughness $z_{0h,r}$ and Obukhov length $L$ can be determined without any error.
To obtain further accuracy of the fluxes, factors other than stability such as baroclinicity, large-scale advection, etc. may still have influence, but an analysis to include $v_0 = \frac{h}{h_r}$ as an additional parameter of the C function did not improve the accuracy of $H$ estimates. Since other possible factors were not tested, this finding is not conclusive and further study is needed to fully understand the issue.

Although an investigation of BAS approach using the C function has shown great potential for the estimation of surface sensible heat fluxes, it is not without limitations. One such limitation is its application under near neutral stability conditions, as was identified with the NOPEX data set for which the difference between the surface potential temperature and the mixed layer average temperature was small and the surface friction velocity was large. Thus, measurement errors dominate the error of final estimates of $H$. Similarly, error propagation due to the problem inherent to the scalar roughness estimation was investigated by comparing the results between those $H$ estimates with $z_{0k,r}$ determined for each profile (i.e., $z_{0k,r}$ values were assumed to be known) and those with $z_{0k,r}$ estimated independently either as an average for each site or as estimates from solar elevation. Apparently this is a serious problem, in general, in bulk formulations that utilize radiometrically determined surface temperatures.

Acknowledgment
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Figure 1. Comparison of the sensible heat flux. Reference values of $H_i$ were compared against $H$ values from: (1) with the similarity function (7) and the constants given in Table 4. In the comparison, $H$ values were estimated using the assumption that $z_{0h,r}$ and $L$ were exactly known for each data set.

Figure 2. The lower right panel gives the $C$ functions (3)-(7) using constants listed in Table 4, calibrated for the four combined data sets (excluding NOPEX) aimed at producing the best agreement of $H$. Also given are the $C$ values derived from measurements by inverting (1). For (7), it was not possible to draw a single line since it includes the site specific parameter of $z_0$. Therefore cross symbols are used to show $C$ values from (7) with given conditions of each ABL measurement. The other panels indicate the difference ($\delta C$) between $C$ values derived from measurements and those estimated from (3)-(7). For clarity, those points in the range of $0 \leq -(h_i - d)/L \leq 500$, in which the majority exists, are shown.
Table 1 Experimental Settings and Data Sets

<table>
<thead>
<tr>
<th>Location</th>
<th>FIFE</th>
<th>TABLE</th>
<th>NOPEX</th>
<th>GAME</th>
<th>RAISE</th>
</tr>
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<td>75 km N of Uppsala, Sweden,</td>
<td>Central part, Thailand</td>
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<td>temperate humid</td>
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<td>sub-urban features with forests/woods, agricultural sites, grass fields, and rice paddies</td>
<td>dense boreal forests with clearings mainly used for agriculture.</td>
<td>mixture of deciduous and evergreen forest, paddy fields, farm land and grassland</td>
<td>extensive steppe</td>
</tr>
<tr>
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<td>flat</td>
<td>flat</td>
<td>generally flat</td>
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<tr>
<td>ABL profiles</td>
<td>radiosoundings</td>
<td>radiosoundings</td>
<td>radiosoundings</td>
<td>radiosoundings</td>
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</tr>
<tr>
<td>Surface heat fluxes</td>
<td>Bowen ratio stations, evaluated as averages of six stations for the 1987 data set or surface layer profile equation (8) for the 1989 data set.</td>
<td>Bowen ratio and eddy correlation stations, evaluated as the weighted averages of 5 stations with the fractional areas of 5 surface types as weighing factors</td>
<td>Bowen ratio and eddy correlation stations, evaluated as weighted averages of 2 stations with the fractional areas of forest and farmlands as weighing factors</td>
<td>eddy correlation station</td>
<td>eddy correlation station</td>
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<tr>
<td>Surface temperature</td>
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<td>Weighted mean of IRT measurements at 5 major surface types</td>
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Table 3 Roughness length and displacement height of each data set

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<th>$d$ (m)</th>
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<td>Hiyama et al.</td>
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<td>10.8</td>
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<tr>
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<td>1.05</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td>GAME</td>
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<td></td>
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<td>[2003]</td>
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Table 4 Coefficients $a$ and $b$ determined by calibration to optimize the agreement of $H$ for the four combined data sets.

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Table 5 Statistics in the comparison of $H_s$ and $H$ estimated by the BAS approach with the $C$ function (3)-(7) calibrated with four data sets excluding NOPEX. Statistics for each data set were calculated for $H$ estimated with $C$ functions using coefficients calibrated for the combined data set given in Table 4, under the assumption that $z_{0h,c}$ and $L$ were exactly known for each data set.

<table>
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<th>equation number</th>
<th>$R^2$</th>
<th>$c$</th>
<th>$e$</th>
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<th>rms error</th>
<th>rmsd$_s$</th>
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<td>0.684</td>
<td>52.1</td>
<td>1.08</td>
<td>26.5</td>
<td>19.4</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>0.764</td>
<td>0.652</td>
<td>56.0</td>
<td>1.08</td>
<td>27.2</td>
<td>18.3</td>
</tr>
<tr>
<td>combined data set</td>
<td>(3)</td>
<td>0.964</td>
<td>0.996</td>
<td>6.76</td>
<td>1.05</td>
<td>15.0</td>
<td>13.6</td>
</tr>
<tr>
<td>excluding NOPEX</td>
<td>(4)</td>
<td>0.958</td>
<td>0.947</td>
<td>6.06</td>
<td>0.995</td>
<td>15.2</td>
<td>14.7</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>0.958</td>
<td>1.07</td>
<td>4.63</td>
<td>1.11</td>
<td>19.7</td>
<td>14.9</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
<td>0.968</td>
<td>0.981</td>
<td>5.99</td>
<td>1.03</td>
<td>13.4</td>
<td>12.8</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>0.987</td>
<td>0.961</td>
<td>6.53</td>
<td>1.01</td>
<td>12.0</td>
<td>11.6</td>
</tr>
</tbody>
</table>

$R^2$ is the coefficient of determination, $c$ and $e$ are the slope and intercept in the regression equation $H_s=cH+e$; $[H_s]/[H]$ is the ratio of the means; rmsd$_s$ is the systematic root mean square error, equal to

$$\frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - x_i)^2$$

and rmsd$_u$ is the unsystematic root mean square error, equal to

$$\left[ \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - x_i)^2 \right]^{1/2}$$.
\[ 760 \left[ \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 \right]^{1/2} \] [Willmott, 1981].
Table 6  Statistics for the comparison of $H_s$ and $H$ estimated by means of the BAS approach with the $C_7$ function (7) and coefficients given in Table 4, and $z_{0h,r}$ estimated independently using the solar elevation (FIFE, TABLE and NOPEX) or given as averages (RAISE and GAME). For all data sets, the estimation was made under the assumption that the value of $L$ was exactly known for each data set.

<table>
<thead>
<tr>
<th>equation number</th>
<th>$R^2$</th>
<th>$c$</th>
<th>$e$</th>
<th>$[H_s]/[H]$</th>
<th>rms error</th>
<th>$rmsd_s$</th>
<th>$rmsd_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIFE</td>
<td>0.880</td>
<td>0.827</td>
<td>19.8</td>
<td>0.971</td>
<td>31.2</td>
<td>27.0</td>
<td>15.6</td>
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<tr>
<td>TABLE (7)</td>
<td>0.219</td>
<td>0.429</td>
<td>37.3</td>
<td>1.03</td>
<td>26.5</td>
<td>21.6</td>
<td>15.4</td>
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<tr>
<td>NOPEX</td>
<td>0.131</td>
<td>0.280</td>
<td>87.6</td>
<td>0.727</td>
<td>77.3</td>
<td>39.5</td>
<td>66.5</td>
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<tr>
<td>GAME</td>
<td>0.564</td>
<td>0.832</td>
<td>26.5</td>
<td>1.02</td>
<td>49.0</td>
<td>47.7</td>
<td>11.5</td>
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<tr>
<td>RAISE</td>
<td>0.912</td>
<td>0.262</td>
<td>61.3</td>
<td>0.461</td>
<td>194.8</td>
<td>11.3</td>
<td>194.4</td>
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<tr>
<td>combined data set excluding NOPEX</td>
<td>0.561</td>
<td>0.514</td>
<td>50.6</td>
<td>0.868</td>
<td>72.5</td>
<td>47.8</td>
<td>54.5</td>
</tr>
</tbody>
</table>

$R^2$ is the coefficient of determination; $c$ and $e$ are the slope and intercept of the regression equation $H_s = cH + e$; $[H_s]/[H]$ is the ratio of the means; $rmsd_s$ is the systematic root mean square error, and $rmsd_u$ the unsystematic root mean square error.