ZERO-DIMENSIONAL SUBSETS OF HYPERSPACES

By

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Abstract. Let $X$ be a metric continuum, let $2^X$ be the hyperspace of all the nonempty closed subsets of $X$ and let $C(X)$ be the hyperspace of subcontinua of $X$. In this paper we prove:

THEOREM 1. If $\mathcal{H}$ is a 0-dimensional subset of $2^X$, then $2^X - \mathcal{H}$ is connected.

THEOREM 2. If $\mathcal{H}$ is a closed 0-dimensional subset of $C(X)$ such that $C(X) - \{A\}$ is arcwise connected for each $A \in \mathcal{H}$, then $C(X) - \mathcal{H}$ is arcwise connected.

Theorem 2 answers a question by Sam B. Nadler, Jr.

Introduction

Throughout this paper $X$ denotes a nondegenerate continuum, i.e., a compact connected metric space, with metric $d$. Let $2^X$ be the hyperspace of nonempty closed subsets of $X$, with the Hausdorff metric $H$, and let $C(X)$ be the hyperspace of subcontinua of $X$.

J. Krasinkiewicz proved in [5] that if $\mathcal{H}$ is a 0-dimensional subset of $C(X)$, then $C(X) - \mathcal{H}$ is connected. In this paper we use Krasinkiewicz' result to prove the following theorem:

THEOREM 1. If $\mathcal{H}$ is a 0-dimensional subset of $2^X$, then $2^X - \mathcal{H}$ is connected.

On the other hand, in Krasinkiewicz' Theorem the word "connected" can not be replaced by "arcwise connected". Even if $X$ is the sin($1/x$)-continuum and $A$ is the limit segment, then $C(X) - \{A\}$ is not arcwise connected. In [7, Question 11.17], Nadler asked the following question: if $\mathcal{H}$ is a compact 0-dimensional
subset of $C(X)$ and if $C(X) - \{A\}$ is arcwise connected for each $A \in \mathcal{H}$, does it follow that $C(X) - \mathcal{H}$ is arcwise connected? This question has been affirmatively answered for the following particular cases:

- if $\mathcal{H}$ has two elements (Nadler and Quinn, [8, Lemma 2.4]),
- if $\mathcal{H}$ is finite (Ward, [9])
- if $\mathcal{H}$ is numerable (Illanes, [3], this result was rediscovered by Hosokawa in [1]).

Furthermore, in [3], the author showed that any two elements of $C(X) - \mathcal{H}$ can be joined by an arc which intersects $\mathcal{H}$ only a finite number of times.

In this paper we finally solve the general question by proving the following theorem.

**Theorem 2.** If $\mathcal{H}$ is a closed 0-dimensional subset of $C(X)$ such that $C(X) - \{A\}$ is arcwise connected for each $A \in \mathcal{H}$, then $C(X) - \mathcal{H}$ is arcwise connected.

**Proof of Theorem 1**

Throughout this section $\mathcal{H}$ will denote a 0-dimensional subset of $2^X$. By Krasinkiewicz' result in [5], $C(X) - \mathcal{H}$ is connected. Let $\mathcal{L}$ be the component of $2^X - \mathcal{H}$ which contains $C(X) - \mathcal{H}$.

In order to prove that $2^X - \mathcal{H}$ is connected, it is enough to prove that $\mathcal{L}$ is dense in $2^X$. Since the subset of $2^X$ which consists of all the nonempty finite subsets of $X$ is dense in $2^X$, we only need to prove the following claim:

**Claim.** For each finite subset $F = \{p_1, \ldots, p_m\}$ of $X$ and for each $\epsilon > 0$, there exists an element $L \in \mathcal{L}$ such that $H(F, L) < \epsilon$.

Let $F = \{p_1, \ldots, p_m\}$ and $\epsilon > 0$.

Take an order arc $\gamma$ from a fixed one-point set $\{p_0\}$ to $X$ (see [7, 1.2] for the definition of order arc). Since $\mathcal{H}$ is 0-dimensional, there exists an element $M \in \gamma - \mathcal{H} \subset C(X) - \mathcal{H}$ such that $H(M, X) < \epsilon/2$ and $M$ is nondegenerate. Choose points $q_1, \ldots, q_m \in M$ such that $d(p_i, q_i) < \epsilon/2$ for each $i \in \{1, \ldots, m\}$. Let $\{U_n\}_{n=1}^{\infty}$ be a sequence of proper open subsets of $M$ such that $q_1 \in U_n$ for every $n \geq 1$, $U_1 \supset \text{cl}(U_2) \supset U_2 \supset \text{cl}(U_3) \supset U_3 \supset \cdots, \text{cl}(U_n) \rightarrow \{q_1\}$ (convergence in $2^X$) and $M \neq \text{cl}(U_1) \subset \{x \in X : d(q, q_1) < \epsilon/2\}$.

Let $L_0 = \{q_1, \ldots, q_m\} \cup (\text{Bd}_{M}(U_1) \cup \text{Bd}_{M}(U_2) \cup \text{Bd}_{M}(U_3) \cup \ldots)$. Clearly, $L_0 \in 2^X$. Fix a nondegenerate subcontinuum $D$ of $U_1 - \text{cl}(U_2)$. Then the set $\{L_0 \cup \{x\} \in 2^X : x \in D\}$ is a nondegenerate subcontinuum of $2^X$. Since $\mathcal{H}$ is 0-dimensional, there exists a point $x_0 \in D$ such that $L_0 \cup \{x_0\} \notin \mathcal{H}$.
Define $L = L_0 \cup \{x_0\}$. Then $L \in 2^X - \mathcal{H}$ and $H(F, L) < \epsilon$.

We will show that $L \in \mathcal{L}$.

For each $n \geq 1$, let $A_n = M - U_n \subset M - \text{cl}(U_{n+1})$. Take an order arc $\gamma_n$ from $A_n$ to $M$. Since $M - \text{cl}(U_{n+1})$ is an open subset of $M$, there exists a (non-degenerate) subarc $\sigma_n$ of $\gamma_n$ such that each of its elements is contained in $M - \text{cl}(U_{n+1})$ and $A_n \subset \sigma_n$. Consider the set $\theta_n = \{L \cup K : K \subset \sigma_n\}$. It is easy to show that $\theta_n$ is a (non-degenerate) order arc from $L \cup A_n$ to some element in $2^X$.

Since $\mathcal{H}$ is 0-dimensional, we can choose an element $B_n = L \cup K_n \in \theta_n - \mathcal{H}$, where $K_n \subset \sigma_n$. Notice that $A_n \subset K_n \subset A_{n+1}$.

Next, we will check that every component of $B_n$ intersects $L$. Let $C$ be a component of $B_n$. Since the subarc of $\sigma_n$ which joins $L \cup A_n$ and $B_n$ is an order arc, then (see [7, 1.8]), $C \cap (L \cup A_n) \neq \emptyset$. If $C \cap L = \emptyset$, we can take an element $x \in C \cap A_n$. Let $C_1$ be the component of $A_n$ which contains $x$. Thus $C_1 \subset C$, and by ([7, 20.2]), $C \neq C_1 \cap \text{Bd}_M(U_n) \subset C \cap L$. This contradiction completes the proof that $C \cap L \neq \emptyset$.

As a consequence of the claim of the paragraph above, we obtain that every component of $B_{n+1}$ intersects $B_n$.

Let $B_0 = L$. Notice that $B_{n-1}$ is a proper subset of $B_n$ for every $n \geq 1$. By [7, 1.8], there exists a map $\alpha_n : [0, 1] \to 2^M$ such that $\alpha_n(0) = B_{n-1}$, $\alpha_n(1) = B_n$, and if $0 \leq s < t \leq 1$, then $\alpha_n(s)$ is a proper subset of $\alpha_n(t)$.

For each $n \geq 1$, let $\alpha_n : [0, 1] \to 2^X$ be a map such that $\alpha_n(0) = \text{Bd}_M(U_{n+2})$, $\alpha_n(1) = M$ and if $0 \leq s < t \leq 1$, then $\alpha_n(s)$ is a proper subset of $\alpha_n(t)$. Since $\text{Bd}_M(U_{n+2}) \subset U_{n+1} - \text{cl}(U_{n+3})$, there exists $t_n > 0$ such that $\alpha_n(t_n) \subset U_{n+1} - \text{cl}(U_{n+3})$.

Let $\varphi_n : [0, 1] \times [0, 1] \to 2^M$ be given by $\varphi_n(s, t) = \alpha_n(st_n) \cup \beta_n(t)$. It is easy to check that $\varphi_n$ is continuous, one-to-one, $\varphi_n(0, 1) = B_n$ and $\varphi_n(0, 0) = B_{n-1}$. Let $\mathcal{G}_n = \varphi_n([0, 1] \times [0, 1])$. Then $\mathcal{G}_n$ is a 2-cell. By [2, Theorem 4], $\mathcal{G}_n - \mathcal{H}$ is connected and contains $B_{n-1}$ and $B_n$.

Let $\mathcal{G} = \cup \{\mathcal{G}_n : n \geq 1\}$. Then $\mathcal{G}$ is a connected subset of $2^X - \mathcal{H}$ and contains the element $B_0 = L$. On the other hand, since $A_n \to M$, and $A_n \subset B_n \subset M$ for each $n \geq 1$, we conclude that $B_n \to M$ and $M \in \text{cl}_{2^X}(\mathcal{G})$. This implies that $\mathcal{G} \subset \mathcal{L}$. Therefore, $L \in \mathcal{L}$. This completes the proof of the claim and thus the proof of Theorem 1.

**Proof of Theorem 2**

Throughout this section $\mathcal{H}$ will denote a closed 0-dimensional subset of $C(X)$ such that $C(X) - \{A\}$ is arcwise connected for each $A \in \mathcal{H}$. 


LEMMA 1. If $A, B \in C(X) - \mathcal{H}$, $A \cap B \neq \emptyset$, $A - B \neq \emptyset$ and $B - A \neq \emptyset$, then $A$ and $B$ can be joined by an arc in $C(X) - \mathcal{H}$.

PROOF. Fix a component $C$ of $A \cap B$. Then $C$ is a proper subcontinuum of both $A$ and $B$. Let $\alpha, \beta : [0, 1] \to A \cup B$ be maps such that $\alpha(0) = C = \beta(0)$, $\alpha(1) = A$, $\beta(1) = B$ and $s < t$ implies that $\alpha(s)$ (resp., $\beta(s)$) is a proper subcontinuum of $\alpha(t)$ (resp., $\beta(t)$) (see [Nd78, 1.8]). Let $\mathcal{C} = [0, 1] \times [0, 1]$. Define $\varphi : \mathcal{C} \to C(A \cup B)$ by:

$$\varphi(s, t) = \alpha(s) \cup \beta(t).$$

Clearly, $\varphi$ is continuous, $\varphi(1, 0) = A$ and $\varphi(0, 1) = B$. If $D$ is a component of $\varphi^{-1}(\mathcal{H})$, then $\varphi(D)$ is a connected subset of $\mathcal{H}$. Thus $\varphi(D)$ has exactly one element. Therefore, $D$ is a component of $\varphi^{-1}(E)$ for some $E \in \mathcal{H}$.

Since $\varphi(1, 0)$ and $\varphi(0, 1) \notin \mathcal{H}$ and $\mathcal{H}$ is compact, there exists $0 < r < 1/2$ such that $\{(1 - r, 1] \times [0, r) \cup (0, r) \times [1 - r, 1]\} \cap \varphi^{-1}(\mathcal{H}) = \emptyset$.

Let $G_1 = ([0, 1 - r] \times \{0\}) \cup ([0, 1] \times [0, 1 - r])$ and $G_2 = ([1 - r, 1] \times \{1\}) \cup ([1, 1] \times \{1\})$. Let $G = G_1 \cup G_2 \cup \varphi^{-1}(\mathcal{H})$. Then $G$ is a compact subset of $\mathcal{C}$.

We will see that no component of $\varphi^{-1}(\mathcal{H})$ intersects both $G_1$ and $G_2$. Suppose, to the contrary, that there exists a component $D$ of $\varphi^{-1}(\mathcal{H})$ such that $D \cap G_1 \neq \emptyset$ and $D \cap G_2 \neq \emptyset$. Then there exists an element $E \in \mathcal{H}$ such that $D$ is a component of $\varphi^{-1}(E)$. Let $z = (s, t) \in D \cap G_1$ and $w = (u, v) \in D \cap G_2$. Then $\alpha(s) \cup \beta(t) = \varphi(z) = \varphi(w) = \alpha(u) \cup \beta(v)$. Notice that $s = 0$ or $t = 0$. If $s = 0$, then $\varphi(z) \subset B$. This implies that $\alpha(u) \subset A \cap B$. Hence $\alpha(u) = C$. Thus $u = 0$. This is a contradiction since $w \in G_2$. A similar contradiction can be obtained assuming that $t = 0$. Therefore, no component of $\varphi^{-1}(\mathcal{H})$ intersects both $G_1$ and $G_2$.

We are ready to apply the Cut Wire Theorem ([7, 20.6]) to the compact space $\varphi^{-1}(\mathcal{H})$ and the closed sets $\varphi^{-1}(\mathcal{H}) \cap G_1$ and $\varphi^{-1}(\mathcal{H}) \cap G_2$. Thus there exist two disjoint closed sets $H_1$, $H_2$ in $\mathcal{C}$ such that $\varphi^{-1}(\mathcal{H}) = H_1 \cup H_2$, $\varphi^{-1}(\mathcal{H}) \cap G_1 \subset H_1$ and $\varphi^{-1}(\mathcal{H}) \cap G_2 \subset H_2$. Define $L_1 = G_1 \cup H_1$ and $L_2 = G_2 \cup H_2$. Then $L_1$ and $L_2$ are disjoint closed subsets of $\mathcal{C}$. Thus there exist two disjoint open subsets $U_1$ and $U_2$ of $\mathcal{C}$ such that $L_1 \subset U_1$ and $L_2 \subset U_2$.

Let $U$ be the component of $U_1$ which contains $G_1$ and let $M$ be the component of $\mathcal{C} - U$ which contains $G_2$. It is easy to prove that $\mathcal{C} - M$ is connected. Since $\mathcal{C}$ is locally connected, $M$ is closed in $\mathcal{C}$ and $\text{Bd}_\mathcal{C}(M) \subset \text{Bd}_\mathcal{C}(U) \subset \text{Bd}_\mathcal{C}(U_1)$. Let $L = \text{Bd}_\mathcal{C}(M)$. Then $L \cap (L_1 \cup L_2) = \emptyset$. Since $G_1 \subset \mathcal{C} - M$, $L$ separates $G_1$ and $G_2$ in $\mathcal{C}$. Since $\mathcal{C}$ is unicoherent ([6, Thm. 2 II, §57, Ch. VIII]), $L$ is a subcontinuum of $\mathcal{C}$.

Since $[0, r] \times [1 - r, 1]$ is a connected subset of $\mathcal{C}$ that intersects both $G_1$
and \( G_2 \), we obtain this set intersects \( L \). Similarly \( L \) intersects \([1-r, 1] \times [0,r]\). Then the set \( L_0 = L \cup ([1-r, 1] \times [0,r]) \cup ([0,r] \times [1-r, 1]) \) is a subcontinuum of \( C = \varphi^{-1}(\mathcal{H}) \). Since \( C \) is locally connected, there exists an open connected (and then arcwise connected) subset \( V \) of \( C \) such that \( L_0 \subset V \subset C - \varphi^{-1}(\mathcal{H}) \). Let \( \lambda \) be an arc in \( V \) joining \((1,0)\) and \((0,1)\). Therefore, \( \varphi(\lambda) \) is a path in \( C(X) - \mathcal{H} \) joining \( A \) and \( B \).

**Lemma 2.** If \( A, B \in C(X) - \mathcal{H} \) and \( A \subset B \neq A \), then \( A \) and \( B \) can be joined by an arc in \( C(X) - \mathcal{H} \).

**Proof.** By [7, 1.8], there is an order arc from \( A \) to \( B \). That is, there is a map \( \alpha : [0,1] \to C(B) \) such that \( \alpha(0) = A \), \( \alpha(1) = B \) and if \( s < t \), then \( \alpha(s) \) is a proper subcontinuum of \( \alpha(t) \). Let \( \mathcal{G} = \alpha^{-1}(\mathcal{H}) \).

First, we will show that for any \( t \in \mathcal{G} \), there exists \( \epsilon_t > 0 \) such that \((t - \epsilon_t, t + \epsilon_t) \subset (0,1)\) and for every \( s \in (t - \epsilon_t, t) - \mathcal{G} \) and every \( r \in (t, t + \epsilon_t) - \mathcal{G} \), \( \alpha(s) \) and \( \alpha(r) \) can be joined by an arc in \( C(X) - \mathcal{H} \).

Since \( \alpha(t) \in \mathcal{H} \), \( C(X) - \{\alpha(t)\} \) is arcwise connected. Then there exists a one-to-one map \( \beta : [0,1] \to C(X) - \{\alpha(t)\} \) such that \( \beta(0) = A \) and \( \beta(1) = B \). Let \( u = \max\{v \in [0,1]; \beta(v) \subset \alpha(t) \} \) for each \( w \in [0,v] \). Then \( \beta(u) \) is a proper subcontinuum of \( \alpha(t) \). Since \( \beta \) is continuous, there exists \( z \in (u,1) \) such that the continuum \( C = \bigcup \{\beta(w) : u \leq w \leq z\} \) does not contain \( \alpha(t) \). Since \( \mathcal{H} \) is 0-dimensional, we may assume that \( C \notin \mathcal{H} \). By the definition of \( u \), \( C \) is not contained in \( \alpha(t) \).

We consider two cases:

**Case 1.** \( \alpha(t) \) is indecomposable.

By [7, 1.52.1 (2)], \( \beta(u) \) is contained in the composant of \( \alpha(t) \) which contains \( A \). Then there exists a proper subcontinuum \( D \) of \( \alpha(t) \) such that \( D \cap A \neq \emptyset \neq D \cap \beta(u) \). Growing \( D \) by using an order arc from \( D \) to \( \alpha(t) \), we may assume that \( D \) is not contained in \( C \) and \( D \notin \mathcal{H} \). Let \( \epsilon_t > 0 \) be such that \((t - \epsilon_t, t + \epsilon_t) \subset (0,1)\), \( \alpha(t - \epsilon_t) \) is not contained in \( D \), \( \alpha(t - \epsilon_t) \) is not contained in \( C \) and \( \alpha(t + \epsilon_t) \) does not contain \( C \).

In order to show that \( \epsilon_t \) has the required properties, let \( s \in (t - \epsilon_t, t) - \mathcal{G} \) and \( r \in (t, t + \epsilon_t) - \mathcal{G} \). Then \( \alpha(s) \cap D \neq \emptyset \) and \( \alpha(s) - D \neq \emptyset \).

If \( D - \alpha(s) \neq \emptyset \), then we may apply Lemma 1 to the pairs \( \alpha(s) \) and \( D \); \( D \) and \( C \); \( C \) and \( \alpha(r) \), and conclude that \( \alpha(s) \) and \( \alpha(r) \) can be joined by an arc in \( C(X) - \mathcal{H} \).
If \( D \subset \alpha(s) \), then we may apply Lemma 1 to the pairs \( \alpha(s) \) and \( C; C \) and \( \alpha(r) \), and conclude that \( \alpha(s) \) and \( \alpha(r) \) can be joined by an arc in \( C(X) - \mathcal{H} \).

**Case 2.** \( \alpha(t) \) is decomposable.

In this case \( \alpha(t) = E \cup F \), where \( E \) and \( F \) are proper subcontinua of \( \alpha(t) \). We may assume that \( E, F \notin \mathcal{H} \) and \( E - C \neq \emptyset \neq F - C \).

Let \( \epsilon_i > 0 \) be such that \( (t - \epsilon_i, t + \epsilon_i) \subset (0, 1) \), \( \alpha(t - \epsilon_i) \) is not contained in any of the sets \( C, E \) and \( F \), and \( C \) is not contained in \( \alpha(t + \epsilon_i) \).

Let \( s \in (t - \epsilon_i, t) - \mathcal{G} \) and \( r \in (t, t + \epsilon_i) - \mathcal{G} \). Then \( \alpha(s) \) is not contained in any of the sets \( E, F \) and \( C \). Since \( \alpha(s) \) is a proper subcontinuum of \( \alpha(t) \), \( E - \alpha(s) \neq \emptyset \) or \( F - \alpha(s) \neq \emptyset \). Suppose, for example, that \( E \) is not contained in \( \alpha(s) \).

If \( E \cap C \neq \emptyset \), then we may apply Lemma 1 to the pairs \( \alpha(s) \) and \( E; C \) and \( C \) and \( \alpha(r) \), and conclude that \( \alpha(s) \) and \( \alpha(r) \) can be joined by an arc in \( C(X) - \mathcal{H} \).

If \( F \cap C \neq \emptyset \), then we may apply Lemma 1 to the pairs \( \alpha(s) \) and \( E; E \) and \( F \); \( F \) and \( C \) and \( \alpha(r) \), and conclude that \( \alpha(s) \) and \( \alpha(r) \) can be joined by an arc in \( c(X) - \mathcal{H} \).

This completes the proof of the existence of \( \epsilon_i \).

Now we are ready to prove Lemma 2.

Let \( t \in \mathcal{G} \) and let \( \epsilon_i > 0 \) be as before. We claim that if \( s, r \in (t - \epsilon_i, t + \epsilon_i) - \mathcal{G} \), then \( \alpha(s) \) and \( \alpha(r) \) can be joined by an arc in \( C(X) - \mathcal{H} \). Indeed, if \( t \) is between \( s \) and \( r \), this claim follows from the choice of \( \epsilon_i \), and if, for example, \( s, r < t \), then fix \( r_1 \in (t, t + \epsilon_i) - \mathcal{G} \). By the choice of \( \epsilon_i \), both pairs \( \alpha(s), \alpha(r_1) \) and \( \alpha(r), \alpha(r_1) \) can be joined by an arc in \( C(X) - \mathcal{H} \). Thus, \( \alpha(r), \alpha(s) \) can be joined by an arc in \( C(X) - \mathcal{H} \).

Given a number \( t \in [0, 1] - \mathcal{G} \), there exists \( \epsilon_i > 0 \) such that \( (t - \epsilon_i, t + \epsilon_i) \cap \mathcal{G} = \emptyset \). In this case, if \( s, r \in (t - \epsilon_i, t + \epsilon_i) \cap [0, 1] \), then \( \alpha(s) \) and \( \alpha(r) \) can be joined by an arc in \( C(X) - \mathcal{H} \).

For the open cover \( \{(t - \epsilon_i, t + \epsilon_i) : t \in [0, 1]\} \), there exists \( \delta > 0 \) such that if \( s, r \in [0, 1] \) and \( |s - r| < \delta \), then \( s, r \in (t - \epsilon_i, t + \epsilon_i) \) for some \( t \in [0, 1] \).

Choose a partition \( 0 = t_0 < t_1 < \cdots < t_m = 1 \) such that \( t_i - t_{i-1} < \delta \) and \( t_i \notin \mathcal{G} \) for each \( i = 1, 2, \ldots, m \).

Thus, for each \( i = 1, 2, \ldots, m \), \( \alpha(t_{i-1}) \) and \( \alpha(t_i) \) can be joined by an arc in \( C(X) - \mathcal{H} \). Therefore, \( A \) and \( B \) can be joined by an arc in \( C(X) - \mathcal{H} \).

**Proof of Theorem 2.** We consider two cases:
CASE 1. $X$ is indecomposable.

In this case $C(X) - \{X\}$ is not arcwise connected (see [7, 1.51]). Then $X \notin \mathcal{H}$. Given an element $A \in C(X) - (\mathcal{H} \cup \{X\})$, by Lemma 2, $A$ and $X$ can be connected by an arc in $C(X) - \mathcal{H}$.

CASE 2. $X$ is decomposable.

Let $X = E \cup F$, where $E$ and $F$ are proper subcontinua of $X$. Since $\mathcal{H}$ is 0-dimensional, we may assume that $E$, $F \notin \mathcal{H}$. Given an element $A \in C(X) - (\mathcal{H} \cup \{X\})$, taking an order arc from $A$ to $X$, we can find an element $B \in C(X) - \mathcal{H}$, such that $A$ is a proper subcontinuum of $B$, $B \neq X$, $B - E \neq \emptyset$ and $B - F \neq \emptyset$. Notice that $E - B \neq \emptyset$ or $F - B \neq \emptyset$. Suppose, for example, that $E - B \neq \emptyset$. By Lemma 1, the pairs $E$, $B$ and $E$, $F$ can be joined by an arc in $C(X) - \mathcal{H}$, and by Lemma 2, $A$ and $B$ can be joined by an arc in $C(X) - \mathcal{H}$. Then $A$ can be joined to both $E$ and $F$ in $C(X) - \mathcal{H}$. In the case that $X \notin \mathcal{H}$, by Lemma 2, $X$ can be joined to both $E$ and $F$ in $C(X) - \mathcal{H}$. This completes the proof that $C(X) - \mathcal{H}$ is arcwise connected.

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References


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