REGULARITY OF ULTRAFILTERS AND FIXED POINTS OF ELEMENTARY EMBEDDINGS

By

Yoshihiro Abe and Masahiro Shioya*

1. Introduction

Let $\kappa$ be a compact cardinal, $\lambda$ a cardinal $\geq \kappa$ and $U$ a fine ultrafilter on $\mathcal{P}_\kappa \lambda$. As is well known, the canonical embedding $j_U : V \rightarrow \text{Ult}(V, U)$ is the identity below $\kappa$ and $j_U(\kappa) \geq 2^{\lambda^\kappa}$ holds. What about the action of $j_U$ on higher cardinals? Barbanel [2] determined the class of fixed cardinals when $U$ is normal. Abe [1] solved the general case:

**Theorem 1.** Let $\kappa$, $\lambda$ and $U$ be as above. Then $j_U$ moves a cardinal $\mu > 2^{\lambda^\kappa}$ exactly when $\kappa \leq \text{cf}(\mu) \leq \lambda^{<\kappa}$ or $\mu = \nu^+$ with $\kappa \leq \text{cf}(\nu) \leq \lambda^{<\kappa}$.

The main advantage of [1] over [2] lies in the assertion that the embedding moves such cardinals. In this paper we pursue this subject in somewhat greater generality, or in terms of regularity instead of fineness. In particular we give two alternative proofs of the main claim of Theorem 1, which are both much simpler than the original one. On the other hand Abe [1] noted that some of his arguments work even for a uniform ultrafilter. We also show that such extension may fail in other cases.

2. Preliminaries

In this paper we follow the terminology of Kanamori [5] closely. We let $\kappa$ denote a regular cardinal $> \omega$ and $\lambda$ a cardinal $\geq \kappa$. We use $\mu$ and $\nu$ to denote a cardinal $\geq \omega$. By inaccessibility and compactness of a cardinal we always mean the stronger notions.

---

*The authors were partially supported by Grant-in-Aid for Scientific Research (No. 09640299), Ministry of Education, Science, Sports and Culture. Received April 10, 1996 Revised June 9, 1997
As shown by Solovay [10], cardinal arithmetic above a compact cardinal is very simple:

**Theorem 2.** Let \( \kappa \) be compact and \( \nu \geq \max\{\kappa, 2^\mu\} \). Then \( \nu^\mu = \nu^+ \) when \( \text{cf}(\nu) \leq \mu \), or else \( \nu^\mu = \nu \).

We sketch a proof of the following observation essentially due to Barbanel [2].

**Lemma 1.** Let \( U \) be a \( \kappa \)-complete ultrafilter on \( I \). Assume \( \nu^{|I|} = \nu^+ \) for any \( \nu > 2^{|I|} \) with \( \kappa \leq \text{cf}(\nu) \leq |I| \). Then \( j_U \) fixes \( \mu > 2^{|I|} \) unless \( \kappa \leq \text{cf}(\mu) \leq |I| \) or \( \mu = \nu^+ \) with \( \kappa \leq \text{cf}(\nu) \leq |I| \).

**Proof.** We proceed by induction on \( \mu \). First observe \( j_U(\mu) = \sup j_U''\mu = \mu \) for \( \mu = (2^{|I|})^+ \) by the cofinality argument and the cardinality calculation. Similarly \( j_U \) fixes \( \nu^+\) when \( \nu > 2^{|I|} \) and \( \kappa \leq \text{cf}(\nu) \leq |I| \). Next note that the successor of a fixed point is also fixed. Finally we have the equalities \( j_U(\mu) = \sup j_U''\mu = \mu \) for a limit \( \mu \) by the cofinality arguments and the inductive hypothesis.

3. Regular ultrafilters

The main result of this section is as follows:

**Theorem 3.** For a \( \kappa \)-complete \((\kappa, \lambda)\)-regular ultrafilter \( U \) we have

1. \( j_U(\kappa) \geq 2^{\kappa^\lambda} \) and
2. \( j_U(\nu) \geq \nu^+ \) when \( \kappa \leq \text{cf}(\nu) \leq \lambda^{<\kappa} \).

We remind the reader that a filter \( F \) is \((\kappa, \lambda)\)-regular if there is a family \( \{X_\alpha : \alpha < \lambda\} \subseteq F \) such that \( \bigcap_{\alpha \in S} X_\alpha = \emptyset \) for any \( S \subseteq [\lambda]^\kappa \). Note that a fine filter on \( \mathcal{P}_\kappa \lambda \) is \((\kappa, \lambda)\)-regular, as witnessed by \( \{\{x : \alpha < x\} : \alpha < \lambda\} \). Hence together with Theorem 2 and Lemma 1, Theorem 3(2) establishes Theorem 1.

We prove Theorem 3 via two lemmas on a regular filter. For the rest of this section we fix a \( \kappa \)-complete \((\kappa, \lambda)\)-regular filter \( F \) on \( I \) witnessed by \( \{X_\alpha : \alpha < \lambda\} \) and write \( \kappa^* \) for \( \sup_{\alpha < \kappa}(2^\alpha)^+ \) for brevity. Our first lemma is a modification of a classical result of Frayne, Morel and Scott [4]:

**Lemma 2.** There is a family \( H \subseteq \mathcal{P}(\kappa^{<\kappa})^\omega \) of size \( \kappa^{<\kappa} \) such that \( \{i : f(i) \neq g(i)\} \subseteq F \) for any distinct \( f, g \in H \).
Regularity of ultrafilters and fixed

Proof. First fix a bijection \( \pi_i : \{ x : i \in \bigcap_{x \in X} X_a \} \rightarrow \{ x : i \in \bigcap_{x \in X} X_a \} \) for \( i \in I \). Note that \( \text{dom} \pi_i < \kappa^+ \). For \( s : \mathcal{P}_\kappa \lambda \rightarrow v \) and \( i \in I \) define \( h_s(i) : \text{dom} \pi_i \rightarrow v \) by \( (h_s(i))(\xi) = s(\pi_i(\xi)) \). We show that \( H = \{ h_s : s : \mathcal{P}_\kappa \lambda \rightarrow v \} \) is as desired. Fix distinct \( s, t : \mathcal{P}_\kappa \lambda \rightarrow v \). Take \( x \in P_\kappa \lambda \) with \( s(x) \neq t(x) \). Then \( \bigcap_{x \in X} X_a \subset \{ i : h_s(i) \neq h_t(i) \} \).

We remark here that the combination of Theorem 2, Lemmas 1 and 2 provides the third proof of Theorem 1 via Theorem 3(2) for a compact \( \kappa \).

Lemma 3. Let \( \kappa^+ \leq \text{cf}(v) \leq \lambda^{<\kappa} \). Then there is a family \( H \subset \mathcal{P}_\kappa \lambda \) of size \( v^+ \) such that \( \{ i : f(i) \neq g(i) \} \in F \) for any distinct \( f, g \in H \).

Proof. First we construct a family \( A \subset [v]^\kappa \) of size \( v^+ \) such that \( a \cap b \) is bounded in \( v \) for any distinct \( a, b \in A \). We may assume that \( v \) is singular. Let \( \{ \nu \xi : \xi < \text{cf}(v) \} \subset v \) be a club set of cardinals with \( v_0 = 0 \). By induction on \( \alpha < v^+ \) define \( f_\alpha : v \rightarrow v \) with \( f_\alpha''v \xi \subset v \xi \) for any \( \xi < \text{cf}(v) \) as follows: Suppose that \( \{ f_\beta : \beta < \alpha \} = \{ g_\beta^\alpha : \beta < v \} \) is defined. Take \( f_\alpha(y) = \nu \xi + 1 - \{ g_\beta^\alpha(y) : \beta < v \xi \} \) for \( y \in v_{\xi + 1} \setminus v_\xi \). By construction \( A = \{ \pi''f_\alpha : \alpha < v^+ \} \) is as desired, where \( \pi : v^2 \rightarrow v \) is the canonical bijection.

Now fix an unbounded \( \{ \delta_x : x \in \mathcal{P}_\kappa \lambda \} \subset v \). For \( a \in A \) define \( h_a : I \rightarrow v \) by \( h_a(i) = \min\{ a - \sup\{ \delta_x : i \in \bigcap_{x \in X} X_a \} \} \). We show that \( H = \{ h_a : a \in A \} \) is as desired. Fix distinct \( a, b \in A \). Take \( x \in \mathcal{P}_\kappa \lambda \) with \( a \cap b \subset \delta_x \). Then \( \bigcap_{x \in X} X_a \subset \{ i : h_a(i) \neq h_b(i) \} \).

Proof of Theorem 3. First observe \( \kappa^+ = \kappa \), as \( U \) is not \( \kappa^+ \)-complete by \((\kappa, \lambda)\)-regularity and hence \( \text{crit}(j_U) = \kappa \). Then items (1) and (2) follow from Lemma 2 with \( v = \kappa \) and Lemma 3 respectively.

4. Uniform ultrafilters

In this section we construct a model in which Theorem 3(1) fails for a uniform ultrafilter on \( \mathcal{P}_\kappa \kappa^{+3} \):

Theorem 4. Let \( \kappa \) be supercompact and \( \lambda > \kappa \) huge. Then some poset forces "\( \kappa \) is supercompact and \( \mathcal{P}_\kappa \kappa^{+3} \) carries a uniform \( \kappa \)-complete ultrafilter \( U \) with \( j_U(\kappa) < \kappa^{+4} \)."

First we fix our notation. Let \( \mu \) be regular and \( \lambda > \mu \) inaccessible. By \( C(\mu, \lambda) \) we denote the Levy collapse forcing "\( \lambda = \mu^+ \)," i.e. the set of functions.
Let \( \lambda > \kappa \) be inaccessible. Then there is a \( \kappa \)-directed closed poset \( P \subseteq V_\lambda \) satisfying \( \lambda \)-c.c. and \( \lnot \varphi \) "\( \lambda = \kappa ^+ \)" such that \( Q \ast \hat{C}(\mu , \lambda )^G \leq P \) for any regular \( \mu \in \lambda - \kappa \) and \( Q \leq P \) of size \( < \mu \).

**Proof.** Inductively we build a \( \kappa \)-support iteration \( (P_\gamma : \gamma < \lambda ) \) of posets and for \( \alpha < \lambda \) a list \( \{ (\mu _{\alpha \beta } , Q_{\alpha \beta } ) : \beta < \lambda \} \) of all pairs \( (\mu , Q) \) of a regular \( \mu \in \lambda - \kappa \) and \( Q \leq P_\alpha \) of size \( < \mu \), so that \( P_{\gamma + 1} \simeq P_\gamma \ast \hat{C}(\mu _{\alpha \beta } , \lambda )^{Q_{\alpha \beta }} \) when \( \pi (\alpha , \beta ) = \gamma \), where \( \pi : \lambda ^2 \rightarrow \lambda \) is the canonical bijection. We claim that \( P = \bigcup _{\gamma } P_\gamma \) is as desired.

To see \( \kappa \)-directed closure, fix a directed \( D \in \mathcal{P}_\kappa P \). Inductively construct a common extension \( p \in P \) of \( D \) with support \( s = \bigcup _{q \in D} \text{supp}(q) \) as follows: Suppose that \( p | \gamma \) with \( \gamma = \pi (\alpha , \beta ) \in s \) is defined and \( p | \gamma \leq q | \gamma \) for any \( q \in D \). Then a reduction \( r \in Q_{\alpha \beta } \) of \( p | \gamma \) forces in \( Q_{\alpha \beta } \) \( \{ q(\gamma ) : q \in D \} \subset \hat{C}(\mu _{\alpha \beta } , \lambda )^{Q_{\alpha \beta }} \) is directed,\) as \( p | \gamma \) forces the same formula in \( P_\gamma \). Take a canonical \( Q_{\alpha \beta } \)-name \( p(\gamma ) \) with \( r \parallel Q_{\alpha \beta } \) "\( p(\gamma ) \leq q(\gamma ) \)" for any \( q \in D \). Then \( p | (\gamma + 1) \leq q | (\gamma + 1) \) for any \( q \in D \), as desired.

Next we have \( P_\gamma \subseteq V_\lambda \) inductively, as canonical representatives from \( \{ \hat{p}^Q : \lnot \varphi \ast \hat{C}(\mu , \lambda )^G \} \) belong to \( V_\lambda \) for such a pair \( (\mu , Q) \) as above.

Also we get the universality property by \( Q \ast \hat{C}(\mu , \lambda )^G \leq P_{\pi (\alpha , \beta ) + 1} \) for a pair \( (\mu , Q) = (\mu _{\alpha \beta } , Q_{\alpha \beta } ) \).

To see \( \lambda \)-c.c., fix \( X \in [P]^\lambda \). Take \( Y \in [X]^\lambda \) and \( s \in \mathcal{P}_\kappa \lambda \) with \( \gamma = \sup s + 1 \) such that \( \text{supp}(p) \cap \gamma = s \) for any \( p \in Y \) and \( \{ \text{supp}(p) - \gamma : p \in Y \} \) is pairwise disjoint. Set \( \mu = \sup \{ \mu _{\alpha \beta } : \pi (\alpha , \beta ) \in s \} < \lambda \). For \( p \in Y \) we have \( d_p \in \mathcal{P}_{\mu ^* \lambda } \) with \( \lnot Q_{\alpha \beta } \) "\( \text{dom} \; p(\pi (\alpha , \beta )) \leq d_p \times \mu ^* \)" for any \( \pi (\alpha , \beta ) \in s \); Take \( d_{\pi (\alpha , \beta )} \in \mathcal{P}_{\mu _{\alpha \beta } \lambda } \) with \( \lnot \varphi_{\alpha \beta } \) "\( \text{dom} \; p(\pi (\alpha , \beta )) \leq d_{\pi (\alpha , \beta )} \times \mu _{\alpha \beta } \)" for any \( \pi (\alpha , \beta ) \in s \) and set \( d_p = \bigcup \{ d_{\pi (\alpha , \beta )} : \pi (\alpha , \beta ) \in s \} \). Take \( Z \in [Y]^\lambda \) such that \( \{ d_p : p \in Z \} \) forms a \( \Delta \)-system.
with root $d$. The standard argument yields distinct $p, q \in Z$ such that $\|\models p(\pi(\alpha, \beta)) (d \times \mu) = q(\pi(\alpha, \beta)) (d \times \mu)^{\cdot}$ for any $\pi(\alpha, \beta) \in s$. Now $p$ and $q$ are the desired compatible elements in $X$ by the compatibility of $p|\gamma$ and $q|\gamma$ in $P_\gamma$.

Finally $\|\models p^{\cdot \lambda = \kappa^+}$ follows from $C(\kappa, \lambda) \leq P$ and $\lambda$-c.c. of $P$.  

**Proof of Theorem 4.** We may assume that $\kappa$ remains supercompact in any $\kappa$-directed closed extension of the universe $V$, for Laver's poset $[7]$ forcing this property is small enough to preserve hugeness of $\lambda$. We may also assume $2^\lambda = \lambda^+$ in $V$, as we show next.

Let $j : V \to M$ witness hugeness of $\lambda$. Construct a forcing iteration $(P_\alpha : \alpha \leq j(\lambda) + 1)$ taking direct limits at inaccessible steps and inverse limits otherwise so that $P_{\alpha + 1} \simeq P_\alpha * A(x)^{P_\alpha}$ when $\alpha \geq \kappa$ is inaccessible, and $P_{\alpha + 1} \simeq P_\alpha$ otherwise.

Fix a $V$-generic $G_2 \in P_\lambda$ and $V[G_2]$-generic $G_{\lambda + 1} \in A(\lambda)^{V[G_2]}$. In $V[G_{\lambda + 1}] = V[G_2 * G_{\lambda + 1}^{\cdot}]$, set $P_{\lambda + 1} = P_{\lambda}/G_{\lambda + 1}$. Fix a $V[G_{\lambda + 1}]$-generic $G_{\lambda + 1}^{\cdot} \in P_{\lambda + 1}$ and work in $V[G_{\lambda + 1}] = V[G_{\lambda + 1} * G_{\lambda + 1}^{\cdot}]$. By abuse of notation $j : V \to M$ lifts to $j : V[G_2] \to M[G_{\lambda + 1}]$ by $j^* G_2 = G_2$ and $j(P_\lambda) = P_{\lambda}$, and $j(\lambda) * M[G_{\lambda + 1}] \subseteq M[G_{\lambda + 1}]$ by $j(\lambda)$-M $\subseteq M$ and $j(\lambda)$-c.c. of $P_{\lambda}$ in $V$. Hence $p = \bigcup j^* G_{\lambda + 1} \in j(A(\lambda)^{V[G_2]})$, as $j^* G_{\lambda + 1} \subseteq j(A(\lambda)^{V[G_2]})$ is directed and of size $\leq \lambda^+$. Fix a $V[G_{\lambda + 1}]$-generic $p \in G_{\lambda + 1}^{\cdot} \subseteq j(A(\lambda)^{V[G_2]})$. We claim that $V[G_{\lambda + 1}] = V[G_{\lambda + 1} * G_{\lambda + 1}^{\cdot}]$ is the desired model.

First observe $2^\lambda = \lambda^+$, as $2^\lambda = \lambda^+$ in $V[G_{j(\lambda + 1)}]$ by $2^\lambda = \lambda^+$ and $\lambda^+$-closure of $P_{\lambda + 1}$ in $V[G_{\lambda + 1}]$. Next we construct a normal ultrafilter $U$ on $[j(\lambda)]^X$.

Again by abuse of notation $j : V[G_2] \to M[G_{\lambda + 1}]$ lifts to $j : V[G_{\lambda + 1}] \to M[G_{\lambda + 1}]$ by $j^* G_{\lambda + 1} \subseteq G_{\lambda + 1}^{\cdot}$, and $j(\lambda) * M[G_{\lambda + 1}] \subseteq M[G_{\lambda + 1}]$ by $j(\lambda)$-M $\subseteq M$ and $j(\lambda)$-$\lambda^+$-closure of $j(A(\lambda)^{V[G_2]})$ in $V[G_{\lambda + 1}]$. Hence we have a common extension $q \in j(P_{\lambda + 1})$ of a directed $j^* G_{\lambda + 1} \subseteq j(P_{\lambda + 1})$ of size $\leq \lambda(j)$ by $j(\lambda)^+$-$\lambda^+$-directed closure of $j(P_{\lambda + 1})$. We claim that $q \models \langle \text{there is a } V[G_{j(\lambda + 1)}]-\text{normal ultrafilter } U \text{ on } [j(\lambda)]^X \rangle^{V[G_{j(\lambda + 1)}]}$.

Fix a $V[G_{j(\lambda + 1)}]$-generic $q \in G_{j(\lambda + 1)}^{\cdot} \subseteq j(P_{\lambda + 1})$. Then in $V[G_{\lambda + 1}] = V[G_{j(\lambda + 1)} * G_{j(\lambda + 1)}^{\cdot}]$, $j : V[G_{\lambda + 1}] \to M[G_{\lambda + 1}]$ lifts to $j : V[G_{j(\lambda + 1)}] \to M[G_{\lambda + 1}]$ (once again by abuse of notation) by $j^* G_{\lambda + 1} \subseteq G_{\lambda + 1}^{\cdot}$, and $U = \{X \in [j(\lambda)]^X : j^* j(f(\lambda)) \in j(X)\}$ is as desired.

Now in $V[G_{j(\lambda + 1)}]$, we have $2^{j(\lambda)} = j(\lambda)^+$, as $2^{\leftrightarrow j(\lambda)} = j(\lambda)$ and $2^{j(\lambda)} = j(\lambda)^+$ in $M[G_{j(\lambda + 1)}]$ by $2^{\leftrightarrow j(\lambda)} = \lambda$ and $2^{j(\lambda)} = \lambda^+$ in $V[G_{\lambda + 1}]$, and $j(\lambda) * M[G_{\lambda + 1}] \subseteq M[G_{\lambda + 1}]$. Let $\{X_\alpha : \alpha < j(\lambda)^+\}$ list the set $\mathcal{P}(j(\lambda)^+)^{V[G_{j(\lambda + 1)}]}$ and a $j(P_{\lambda + 1})$-name $U$ witness the above claim. By $j(\lambda)^+$-closure of $j(P_{\lambda + 1})$ build a descending $\{q_\alpha : \alpha < j(\lambda)^+\} \subseteq j(P_{\lambda + 1})$ so that $q_\alpha \leq q$ decides "$X_\alpha \in U"$ for any $\alpha < j(\lambda)^+$. Then $U = \{X_\alpha : q_\alpha \models \langle X_\alpha \in U \rangle\}$ is as desired.
Now we restart from the universe $V$ as above. Let $j : V \to M$ witness hugeness of $\lambda$ and $P$ be as in Lemma 4. We show that $P * \hat{C}(\lambda^+, j(\lambda))^P$ is the desired poset. First note in $V$, $P * \hat{C}(\lambda^+, j(\lambda))^P \leq j(P)$, as $j(P)$ has the universality property by $j(\lambda) M \subset M$, and $P \leq j(P)$ by $\lambda$-c.c. of $P$ and $P \subset V_\lambda$.

Fix a $V$-generic $G_P \subset P$ and a $V[G_P]$-generic $G_C \subset (C(\lambda^+, j(\lambda))^V[G_P]).$ We show that in $V[G_P * G_C]$, $R = j(P)/G_P * G_C$ forces “there is a uniform $V[G_P * G_C]$-$\lambda$-complete $V[G_P * G_C]$-ultrafilter $\mathcal{U}$ on $j(\lambda)$.”

Fix a $V[G_P * G_C]$-generic $R \subset R$ and work in $V[G_{j(P)}] = V[G_P * G_C * G_R]$. By abuse of notation $j : V \to M$ lifts to $j : V[G_P] \to M[G_{j(P)}]$ by $j'' G_P = G_P$, and $j(\lambda) M[G_{j(P)}] = M[G_{j(P)}]$, by $j(\lambda) M \subset M$ and $j(\lambda)$-c.c. of $j(P)$ in $V$. Hence $r = \bigcup j'' G_C \in j(C(\lambda^+, j(\lambda))^V[G_P])$, as $j'' G_C \in j(C(\lambda^+, j(\lambda))^V[G_P])$ is directed and of size $\leq j(\lambda)$. We claim that $r \models \langle \forall \mathcal{U} \rangle$ “there is a uniform $V[G_P * G_C]$-$\lambda$-complete $V[G_P * G_C]$-ultrafilter $\mathcal{U}$ on $j(\lambda)$.”

Fix a $V[G_{j(P)}]$-generic $r \in G_{j(C)} \subset j(C(\lambda^+, j(\lambda))^V[G_P]).$ Then in $V[G_{j(P)} * G_{j(C)}]$, $j : V[G_P] \to M[G_{j(P)}]$ lifts to $j : V[G_P * G_C] \to M[G_{j(P)} * G_{j(C)}]$ (again by abuse of notation) by $j'' G_C \subset G_{j(C)}$, and $\mathcal{U} = \{ X \subset j(\lambda) : \sup j'' \in j(X) \}$ is as desired.

Work again in $V[G_{j(P)}]$. We have $2^{\lambda^+} = j(\lambda)^+$, as $2^{\lambda^+} = j(\lambda)^+$ in $M[G_{j(P)}]$ by $2^\lambda = \lambda^+$ in $V[G_P]$, and $j(\lambda) M[G_{j(P)}] \subset M[G_{j(P)}]$. Let $\mathcal{X}_x : x < j(\lambda)^+$ list the set $\mathcal{P}(j(\lambda))^V[G_P * G_C]$ and a $j(C(\lambda^+, j(\lambda))^V[G_P])$-name $\mathcal{U}$ witness the above claim. By $j(\lambda)^+$ closure of $j(C(\lambda^+, j(\lambda))^V[G_P])$ build a descending $\{ r_x : x < j(\lambda)^+ \} \subset j(C(\lambda^+, j(\lambda))^V[G_P])$ so that $r_x \leq r$ decides “$X_x \in \mathcal{U}$” for any $x < j(\lambda)^+$. Then $\mathcal{U} = \{ X_x : r_x \models \langle \forall \mathcal{U} \rangle \} \models \mathcal{U}$ as desired.

Now work in $V[G_P * G_C]$. Let an $R$-name $\dot{\mathcal{U}}$ witness the claim. Then the filter $F = \{ X \subset j(\lambda) : \models \langle \forall X \in \mathcal{U} \rangle \}$ is $\lambda$-complete and contains all final segments. Also $|\eta(\lambda)^{\mathcal{U}'}| \leq |\text{ro}(R)^{\mathcal{U}'}| \leq j(\lambda)$, as the map $[\eta] \mapsto (\| j^{-1}(\alpha) \in \mathcal{U} \| : \alpha < \kappa)$ is injective and $|\text{ro}(R)| \leq j(\lambda)^{\mathcal{U}} = j(\lambda)$. By compactness of $\kappa$ we have a uniform $\kappa$-complete ultrafilter $\mathcal{U} \supset F$ on $j(\lambda) = \kappa^3$ with $j_U(\kappa) < \kappa^4$, and hence on $\mathcal{P}_\kappa \kappa^{+3}$ through a bijection. \hspace{1cm} \Box

In conclusion we ask whether the integers 3 and 4 in Theorem 4 may be replaced by smaller ones.

References

Regularity of ultrafilters and fixed


Yoshihiro Abe
Department of Mathematics
Kanagawa University
Yokohama, 221
Japan

Masahiro Shioya
Institute of Mathematics
University of Tsukuba
Tsukuba, 305
Japan