NOTES ON $P_{\kappa}\lambda$ AND $[\lambda]^\kappa$

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This paper consists of notes on some combinatorial properties. §1 deals with $\lambda$-ineffability and the partition property of $P_{\kappa}\lambda$ with $\lambda$ ineffable. In §2 we combine the flipping property and a filter investigated by Di Prisco and Marek to characterize huge cardinals.

We work in ZFC and the notations are standard. $P_{\kappa}\lambda = \{x \subseteq \lambda : |x| < \kappa\}$, $[\lambda]^\kappa = \{x \subseteq \lambda : |x| = \kappa\}$, $D_{\kappa}\lambda = \{|x, y) : x, y \in P_{\kappa}\lambda \text{ and } x \not\subseteq y\}$.

§1 $P_{\kappa}\lambda$ when $\lambda$ is ineffable.

$\kappa$ is called $\lambda$-ineffable if for any function $f : P_{\kappa}\lambda \rightarrow P_{\kappa}\lambda$ such that $f(x) \subseteq x$ for all $x \in P_{\kappa}\lambda$, there is a subset $A$ of $\lambda$ such that the set $\{x \in P_{\kappa}\lambda : A \cap x = f(x)\}$ is stationary. We abbreviate the following statement to $\text{Part}^*(\kappa, \lambda)$:

"For any function $F : D_{\kappa}\lambda \rightarrow 2$, there is a stationary homogeneous set $H$ i.e. $|F''([H]^\kappa \cap D_{\kappa}\lambda)| = 1$.

If $\text{Part}^*(\kappa, \lambda)$, then $\kappa$ is $\lambda$-ineffable. We shall show the converse is true when $\lambda$ is ineffable.

Lemma 1. $X \subseteq P_{\kappa}\lambda$ is closed unbounded iff $\{\alpha < \lambda : X \cap P_{\alpha}\lambda \text{ is closed unbounded in } P_{\alpha}\lambda\}$ contains a closed unbounded subset of $\lambda$. Hence $S$ is stationary in $P_{\kappa}\lambda$ if $\{\alpha < \lambda : S \cap P_{\alpha}\lambda \text{ is stationary in } P_{\alpha}\lambda\}$ is a stationary subset of $\lambda$.

Theorem 2. Suppose that $\lambda$ is ineffable. If $\text{Part}^*(\kappa, \alpha)$ for all $\alpha < \lambda$, then $\text{Part}^*(\kappa, \lambda)$.

Proof. Let $F : D_{\kappa}\lambda \rightarrow 2$ and $F_\alpha = F|D_{\alpha}\lambda$ for every $\alpha < \lambda$. By our assumptions, there is a stationary subset $A_\alpha$ of $P_{\alpha}\lambda$ such that

$F''([A_\alpha]^{\kappa} \cap D_{\alpha}\lambda) = \{k_\alpha\}$, $k_\alpha \in [0, 1)$.

Since $\lambda$ is ineffable, we can find an $A \subseteq P_{\kappa}\lambda$ so that

$S = \{\alpha < \lambda : A_\alpha = A \cap P_{\alpha}\lambda\}$ is stationary in $\lambda$.

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A is stationary by Lemma 1.

Let \( t, u \in [A]_\alpha \cap D, \lambda \). Since \( S \) is unbounded in \( \lambda \), there is a member of \( S, \alpha \) such that both \( t \) and \( u \) are in \( [A_\alpha]_\alpha \cap D, \alpha \). Hence \( F(t) = F(u) = k_\alpha \). So, \( A \) is a stationary homogeneous set for \( F \).

**Definition.** \( \kappa \) is \( \lambda \)-almost ineffable if for any function \( f : P, \lambda \rightarrow P, \lambda \) such that \( f(x) \subset x \) for all \( x \in P, \lambda \), there is a subset \( A \) of \( \lambda \) such that the set \( \{ x \in P, \lambda : A \cap x = f(x) \} \) is unbounded.

**Theorem 3.** Suppose that \( \lambda \) is almost ineffable. Then \( \kappa \) is \( \lambda \)-almost ineffable iff \( \kappa \) is \( \alpha \)-almost ineffable for all \( \alpha < \lambda \).

**Proof.** \( \rightarrow \) is proved by the same argument as the lemma in Magidor [9] p.281.

\( \leftarrow \) Let \( f : P, \lambda \rightarrow P, \lambda \) and \( f(x) \subset x \) for all \( x \in P, \lambda \). Considering a function \( f \upharpoonright P, \alpha \) and using \( \alpha \)-ineffability, we get an \( A_\alpha \subset \alpha \) for every \( \alpha < \lambda \) such that

\[
X_\alpha = \{ x \in P, \alpha : f(x) = x \cap A_\alpha \}
\]

is unbounded in \( P, \alpha \).

Using now the almost ineffability of \( \lambda \), there is an \( A \subset \lambda \) so that

\[
S = \{ \alpha < \lambda : A_\alpha = A \cap \alpha \}
\]

is unbounded in \( \lambda \).

Let \( X = \{ x \in P, \lambda : f(x) = x \cap A \} \). If \( \alpha \in S \) and \( x \in P, \alpha \), then \( x \cap A_\alpha = x \cap A \cap \alpha = x \cap A \).

Hence \( X_\alpha \subset X \cap P, \alpha \) for every \( \alpha \in S \). This gives

\[
\{ \alpha < \lambda : X \cap P, \alpha \text{ is unbounded in } P, \alpha \}
\]

is unbounded in \( \lambda \).

Thus \( X \) is unbounded in \( P, \lambda \).

**Corollary 4.** The following are equivalent for \( \kappa < \lambda \) with \( \lambda \) ineffable.

(a) Part*(\( \kappa, \alpha \)) for all \( \alpha < \lambda \).
(b) Part*(\( \kappa, \lambda \)).
(c) \( \kappa \) is \( \lambda \)-indefinable.
(d) \( \kappa \) is \( \alpha \)-indefinable for all \( \alpha < \lambda \).
(e) \( \kappa \) is \( \alpha \)-almost indefinable for all \( \alpha < \lambda \).
(f) \( \kappa \) is \( \lambda \)-almost indefinable.
(g) \( \kappa \) is \( \alpha \)-supercompact for all \( \alpha < \lambda \).

**Proof.** (a) \( \rightarrow \) (b) is Theorem 1. (b) \( \rightarrow \) (c) is Theorem 2 in Magidor [9].

(c) \( \rightarrow \) (d) is the lemma also in [9]. (d) \( \rightarrow \) (e) is trivial. (e) \( \leftrightarrow \) (f) is Theorem 3. (e) \( \rightarrow \) (g) is by Carr's result: If \( \kappa \) is \( 2^{\kappa^+} \)-shelah, then \( \kappa \) is \( \alpha \)-supercompact. (\( \kappa \) is \( \alpha \)-shelah if \( \kappa \) is \( \alpha \)-almost indefinable.) See [3].
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On the coding of $P_\lambda$, there are works of Zwicker [14] and Shelah [12]. The author cannot answer this question.

**Question 5.** Is there a function $t: \lambda \to P_\lambda$ such that for any stationary subset $A$ of $\lambda$, $t''A$ is stationary in $P_\lambda$.

It is, of course, true if $\kappa = \lambda$. In fact let $t = id\mid \kappa$. The question is interesting when $\lambda$ is ineffable.

**Proposition 6.** If $\lambda$ is ineffable and there is a $t: \lambda \to P_\lambda$ such that $t''A$ is stationary for any stationary subset $A$ of $\lambda$, then $\kappa$ is $\lambda$-ineffable.

**Proof.** Suppose that $f: P_\lambda \to P_\lambda$ and $f(x) \subseteq x$ for all $x \in P_\lambda$. Let $A_x = \{ \beta < \alpha : \beta \in f(t(\alpha)) \}$. Since $\lambda$ is ineffable, there is a stationary subset $S$ of $\lambda$ and $A \subset \lambda$ so that $A_x = A \cap \alpha$ for all $\alpha \in S$.

Let $B = t''S$ is stationary and for any $x \in B$ there is an $\alpha_x \in S$ such that $x = t(\alpha_x)$. Hence $f(x) \cap \alpha_x = A \cap \alpha_x$.

Let $B' = \{ x \in B : f(x) \neq A \cap x \}$ and $\delta_x = \text{the least ordinal in } f(x) \delta(A \cap x)$. $\delta_x \in x$ for all $x \in B'$.

Suppose that $B'$ is stationary. There is an ordinal $\delta < \lambda$ such that $C = \{ x \in B' : \delta_x = \delta \}$ is stationary.

$$\forall x \in C (f(x) \cap (\delta + 1) = A \cap (\delta + 1)).$$

So,

$$\forall x \in C (\alpha_x < \delta).$$

Thus there is an $x \in C$ such that $\delta < \alpha_x$.

Hence $\{ x \in B : f(x) = A \cap x \}$ is stationary.

**Remark.** $t''A$ is a stationary subset which splits into $\lambda$ disjoint stationary subsets. Gitik constructed a model of $\text{ZFC}$ in which there is a stationary set that cannot be split into $\lambda$ disjoint stationary subsets in [6].

§ 2 $[\lambda]^\kappa$ when $\kappa$ is huge.

Let $j: V \to M$ be a huge embedding with critical point $\kappa$ and $j(\kappa) = \lambda$ in this section.

At first we recall a filter on $[\lambda]^\kappa$ investigated by Di Prisco and Marek in [5]. It is analogous to the closed unbounded filter on $P_\lambda$. 
Definition. For $X \subseteq P_\lambda$, define $A_x$, the basic set generated by $X$, as follows: $A_x = \{x \in [\lambda]^\kappa : x$ is the union of an increasing $\kappa$-chain of elements of $X\}$. Define $F_{*, i}$ by

$A \in F_{*, i}$ iff there is a closed unbounded $X \subseteq P_\lambda$ such that $A_x \subseteq A$.

Theorem (Di Prisco, Marek, Baumgartner)\n
$F_{*, i}$ is the least $\kappa$-complete, normal, fine filter on $[\lambda]^\kappa$. If $U$ is the normal ultrafilter on $[\lambda]^\kappa$ induced by $j$, then every set in $F_{*, i}$ is in $U$. In this case $F_{*, i}$ is not $\kappa^+$-complete.

$X \subseteq [\lambda]^\kappa$ is unbounded if $\forall x \in [\lambda]^\kappa \exists y \in X(x \subseteq y)$. $X$ is $F_{*, i}$ stationary if $X \cap Y \neq 0$ for all $Y \in F_{*, i}$.

Proposition 1. Any $X \in F_{*, i}$ is unbounded.

Proof. There is a $C \subseteq P_\lambda$ that is closed unbounded and $C_x \subseteq X$. Let $a \in [\lambda]^\kappa$ and $f : \kappa \rightarrow a$ be a bijection, $x_a = f'' \alpha$ for all $\alpha < \kappa$. We can find, using induction, $y_a \in C$ such that $y_a \supseteq x_a \cup \{y_\beta : \beta < \alpha\}$ for every $\alpha < \kappa$.

$\{y_\alpha | \alpha < \kappa\} \subseteq C$ is a $\kappa$-chain and $x = \bigcup \{x_\alpha : \alpha < \kappa\} \subseteq \bigcup \{y_\alpha : \alpha < \kappa\} = y \in C_x \subseteq X$.

Next proposition shows the situation is different from $P_\lambda$.

Proposition 2. If $\kappa$ is huge, there is a $F_{*, i}$-stationary set that is not unbounded.

Proof. $(\lambda)^\kappa = \{x \in [\lambda]^\kappa :$ the order type of $x$ is $\kappa\}$ is in $U$. Clearly $(\lambda)^\kappa$ is not unbounded.

Moreover, we shall show that there is a $F_{*, i}$-stationary set $S$ such that for any $x, y$ in $S$, $x \not\in y$. Thus, partition property may not be directly extended to $[\lambda]^\kappa$ as for $P_\lambda$.

Definition. $f$ is an $\omega$-Jonsson function over a set $x$ iff $f : "x \rightarrow x$ and whenever $y \subseteq x$ and $|y| = |x|$, $f^{y''}y = x$.

Lemma 3. Let $U$ be the normal ultrafilter on $[\lambda]^\kappa$ induced by $j$ and $f$ is an $\omega$-Jonsson function over $\lambda$. Then $\{x \in [\lambda]^\kappa : f^{n}x$ is $\omega$-Jonsson over $x\} \in U$.

Proof. The same argument as a normal ultrafilter on $P_\lambda$ can be carried out. Let $e : V \rightarrow N \uplus V^{\omega} / U$ and $X \subseteq e'' \lambda$ with $|X| = |e'' \lambda| = \lambda$. Since $Y = e^{-1}(X) \subseteq \lambda$ and $|Y| = \lambda$, $f^{n''}Y = \lambda$. So,

$\forall \alpha < \lambda \exists s \in [\lambda]^\kappa Y(\alpha = f(s))$.
This implies

$$\forall \alpha \epsilon \epsilon' \exists \epsilon X(\alpha = \epsilon(f)(\epsilon(s))).$$

Since \( \epsilon(s) = \epsilon' \epsilon X \),

$$\epsilon(f)'\epsilon X = \epsilon' \epsilon.$$

Hence \( \epsilon(f)\epsilon' \epsilon \) is \( \omega \)-Jonsson over \( \epsilon'' \epsilon \).

Thus \( \{ x : f \epsilon' x \) is \( \omega \)-Jonsson over \( x \epsilon U \).

**Theorem 4.** There is an \( \epsilon U \) such that for every pair \( x, y \) in \( A, x \epsilon y \).

**Proof.** Let \( f \) be a \( \omega \)-Jonsson function over \( \epsilon \) and \( A = \{ x \epsilon [\epsilon]': f \epsilon' x \) is \( \omega \)-Jonsson over \( x \epsilon U \).

Suppose \( y \epsilon x \epsilon A \). Since \( |x| = |y|, f''y = x \). But \( f'''y \epsilon y \).

§ 3 Flipping properties and huge cardinals, partition properties of \( P, \epsilon \).

Flipping properties were first studied by Abramson, Harrington, Kleinberg and Zwicker in [1] and turned out to be another form of large cardinal property. Di Prisco and Zwicker [4] extended this line to supercompactness. More precisely, they gave a new type of flipping properties equivalent to \( \lambda \)-ineffability and \( \lambda \)-mildly ineffability. We shall introduce an analogous type properties and discuss the relationship with huge cardinals.

**Definition.** If \( t : \epsilon \epsilon P([\epsilon]') \), we call \( t' \) a flip of \( t (t \sim t) \) if \( t' : \epsilon \epsilon P([\epsilon]') \)

and for all \( \alpha < \epsilon, t'(\alpha) = t(\alpha) \) or \( t'(\alpha) = [\epsilon]'- t(\alpha) \). Flip(\( \epsilon, \epsilon \) \( \epsilon \epsilon \epsilon P([\epsilon]') \) \( \exists t' \sim \epsilon \)

such that \( \forall t'(\alpha) \) is \( F, \epsilon \)-stationary. Inef(\( \epsilon, \epsilon \) \( \epsilon \epsilon \epsilon P([\epsilon]') \) \( \forall f : [\epsilon]'' \rightarrow [\epsilon]' \) such that \( f(x) \epsilon x \) for all \( x \epsilon [\epsilon]' \), there is a subset \( A \) of \( \epsilon \) such that the set

\( \{ x \epsilon [\epsilon]' : A \cap x = f(x) \} \) is \( F, \epsilon \)-stationary.

**Theorem 1.** (i) Flip(\( \epsilon, \epsilon \) \( \epsilon \epsilon \) Inef(\( \epsilon, \epsilon \) \( \epsilon \epsilon \).

(ii) If Flip(\( \epsilon, 2\epsilon \) \( \epsilon \epsilon \)\), then there is a huge embedding \( j \) such that \( \epsilon \) is the critical point and \( j(\epsilon) = \epsilon \).

(iii) If \( j : V \rightarrow M \) is a huge embedding with the critical point \( \epsilon \) such that \( j(\epsilon) = \epsilon \), then Flip(\( \epsilon, \epsilon \) \( \epsilon \epsilon \).

**Proof** (i) Assume that Flip(\( \epsilon, \epsilon \) \( \epsilon \epsilon \) and \( f : [\epsilon]' \rightarrow [\epsilon]' \) such that \( f(x) \epsilon x \) for all \( x \epsilon [\epsilon]' \). Define \( t : \epsilon \epsilon P([\epsilon]') \) by

\( t(\alpha) = \{ x \epsilon [\epsilon]' : \alpha \epsilon f(x) \}. \)
Let \( t' \sim t \) be such that \( \mathcal{M}'(\alpha) \) is \( F_{\alpha, r} \)-stationary.

Put \( A = \cup \{ f(x) : x \in \mathcal{M}'(\alpha) \} \). We shall show that if \( x \in \mathcal{M}'(\alpha) \) then \( x \cap A = f(x) \).

Obviously \( f(x) \subseteq x \cap A \). If \( \alpha \varepsilon x \cap A \), then there is a \( y \varepsilon \mathcal{M}'(\beta) \) so that \( \alpha \varepsilon f(y) \). Since \( \alpha \varepsilon f(y) \), \( y \varepsilon \ell(\alpha) \) and \( \alpha \varepsilon y \). Hence \( t'(\alpha) = t(\alpha) \). Now \( \alpha \varepsilon x \in \mathcal{M}'(\beta) \) and \( t'(\alpha) = t(\alpha) \). This gives \( x \varepsilon \ell(\alpha) \). Hence \( \alpha \varepsilon f(x) \).

Conversely, let \( t : \lambda \longrightarrow P([\lambda]^r) \). Define \( f : [\lambda]^r \longrightarrow [\lambda]^r \) by
\[
f(x) = \{ \alpha \varepsilon x : x \in \ell(\alpha) \}.
\]

There is a subset \( A \) of \( \lambda \) such that \( B = \{ x \in [\lambda]^r : x \cap A = f(x) \} \) is \( F_{\alpha, r} \)-stationary.

Define \( t' : \lambda \longrightarrow P([\lambda]^r) \) by \( t'(\alpha) = t(\alpha) \) if \( \alpha \varepsilon A \) and \( t'(\alpha) = [\lambda]^r - t(\alpha) \) if \( \alpha \notin A \).

Suppose \( x \varepsilon S \) and \( \alpha \varepsilon x \). If \( \alpha \varepsilon A \), then \( \alpha \varepsilon f(x) \) hence \( x \varepsilon \ell(\alpha) = t'(\alpha) \). If \( \alpha \notin A \), then \( \alpha \notin f(x) \) hence \( x \notin \ell(\alpha) \). So \( x \notin t'(\alpha) \). Now we have shown \( S \subseteq \mathcal{M}'(\alpha) \), which must be \( F_{\alpha, r} \)-stationary.

(ii) Let \( \gamma = 2^n \) and \( \{ A_a : \alpha < \gamma \} \) be an enumeration of \( P([\lambda]^r) \). Define \( t : \gamma \longrightarrow P([\lambda]^r) \) by \( t(\alpha) = \{ x \in [\gamma]^r : x \cap A_\alpha \} \). Let \( t' \sim t \) be such that \( \mathcal{M}'(\alpha) \) is \( F_{\alpha, r} \)-stationary.

A filter \( U \) on \( [\lambda]^r \) is defined by \( A_a \in U \) if \( t'(\alpha) = t(\alpha) \). We shall show in fact \( U \) is a normal ultrafilter. The fact that for any \( \alpha \varepsilon P_{\gamma} \) the set \( \{ x \in [\gamma]^r : \alpha \subseteq x \} \) is a member of \( F_{\alpha, r} \) is often used.

(1) \( A_s \in U \wedge A_s \subseteq A_s \longrightarrow A_s \in U \).

There is a \( x \in \mathcal{M}'(\xi) \) such that \( \{ \alpha, \beta \} \subseteq x \). Since \( x \in t'(\alpha) = t(\alpha) \), \( x \cap \lambda \varepsilon A_s \subseteq A_s \). Thus \( x \varepsilon \ell(\beta) \). Hence \( t'(\beta) = t(\beta) \).

(2) \( U \) is \( \kappa \)-complete.

Suppose \( \{ B_\alpha : \alpha < \delta \} \subseteq U (\delta < \kappa) \) and \( f : \delta \longrightarrow \gamma \) such that \( B_\alpha = A_{f(\alpha)} \) for all \( \alpha < \delta \). Let \( A_s = \bigcup_{\alpha < \delta} B_\alpha \).

There is a \( x \in \mathcal{M}'(\xi) \) such that \( \{ \gamma \} \cup f'' \delta \subseteq x \). For all \( \alpha < \delta \), \( x \varepsilon t'(f(\alpha)) = t(f(\alpha)) \), so \( x \cap \lambda \varepsilon A_s \). Hence \( x \varepsilon \ell(\gamma) \) and \( t'(\gamma) = t(\gamma) \).

(3) For any \( \alpha < \lambda \), \( \{ x \in [\lambda]^r : \alpha \subseteq x \} \in U \).

Let \( A_s = \{ x \in [\lambda]^r : \alpha \subseteq x \} \). \( t(\beta) = \{ x \in [\gamma]^r : \alpha \subseteq x \cap \lambda \} = \{ x \in [\gamma]^r : \alpha \subseteq x \} \in F_{\alpha, r} \). There is a \( x \in \mathcal{M}'(\xi) \) such that \( x \varepsilon \ell(\beta) \) and \( \beta \subseteq x \). Hence \( x \varepsilon t'(\beta) \) and \( t'(\beta) = t(\beta) \).

(4) \( U \) is an ultrafilter. Obviously \( \phi \notin U \). So we have to show only that if \( A_s \notin U \), then \( [\lambda]^r - A_s \notin U \). Suppose that \( A_s \notin U \). \( t'(\alpha) = [\lambda]^r - t(\alpha) \). Let \( [\lambda]^r - A_s = A_s \). There is a \( x \in \mathcal{M}'(\xi) \) such that \( \{ \alpha, \beta \} \subseteq x \). Since \( x \varepsilon t'(\alpha) = [\lambda]^r - t(\alpha) \), \( x \cap \lambda \notin A_s \). Hence \( x \cap \lambda \varepsilon A_s \) and \( x \notin \ell(\beta) \). Thus \( t'(\beta) = t(\beta) \).

(5) \( U \) is normal.

Suppose that \( \{ B_\alpha : \alpha < \lambda \} \subseteq U \). Let \( f : \lambda \longrightarrow \gamma \) be such that \( B_\alpha = A_{f(\alpha)} \) for all \( \alpha < \lambda \),
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and $A_B= A_\beta$.

Note that $X = \{ x \in P_\lambda : \forall \alpha \in \langle \lambda \rangle \cap \lambda (f(\alpha) \in x) \}$ is a closed unbounded subset of $P_\lambda$.

Let $C = A_X = \{ y \in [\lambda]^\omega : \exists D \subseteq X (D$ is $\lambda$-chain, $y = \bigcup D) \}$. Then $C \subseteq F_{\omega_1}$.

If $y \in C$ and $a \in y \cap \lambda$, there is an $x \in D$ such that $a \in x \cap \lambda$ and $x \subset y$. Hence $f(\alpha) \in x \subset y$. Now we have got that for any $y \in C$, if $a \in y \cap \lambda$ then $f(\alpha) \in y$.

There is a $y \in \delta(\xi)$ such that $y \in C$ and $\beta \in y$. For all $a \in y \cap \lambda$, $f(\alpha) \in y$ and $y \in t'_P(f(\alpha)) = t(f(\alpha))$, hence $y \cap \lambda \in A_{f(\alpha)}$. Contradiction. Hence $t'(\beta) \neq t(\beta)$.

(iii) Let $U$ be the normal ultrafilter on $[\lambda]^\omega$ induced by $j$, $t: \lambda \rightarrow P([\lambda]^\omega)$. Define $t': \lambda \rightarrow P([\lambda]^\omega)$ as follows. $t'(\alpha) = t(\alpha)$ if $t(\alpha) \in U$, and $t'(\alpha) = [\lambda]^\omega - t(\alpha)$ if $t(\alpha) \notin U$.

Then $t' \sim t$ and for all $a < \lambda$, $t'(a) \in U$. Hence $\delta'(a) \in U$. Every member of $U$ is $F_{\omega_1}$-stationary.

Next the author tried to express the partition property of $P_\lambda$ in the form of a flipping property. (Though it does not seem successful.)

**Proposition 2.** The followings are equivalent.

(a) Part* $(\kappa, \lambda)$.

(b) For any $t: P_\lambda \rightarrow P(\Omega)$, there are $t' \sim t$ and a stationary set $X$ such that if $x, y \in D, \lambda \cap [\lambda]^\omega$ then $y \in t'(x)$.

**Proof.** (a) $\rightarrow$ (b). Define $F: D, \lambda \rightarrow 2$ by $F(x, y) = 0$ if $y \in t(x)$ and $F(x, y) = 1$ otherwise. Let $X$ be a stationary homogeneous set for $F$. When $F'(\langle X \rangle^\omega \cap D, \lambda) = \{ 0 \}$, $t' = t$. If $F'(\langle X \rangle^\omega \cap D, \lambda) = \{ 1 \}$, let $t'(x) = t(\lambda) \in t(x)$ for all $x \in X$.

(b) $\rightarrow$ (a). Put $t(x) = \{ y : F(x, y) = 0 \}$. There are $t' \sim t$ and a stationary set $X$ such that if $x \notin y \in X$ then $x \in t'(y)$.

Let $X_1 = \{ x \in X : t'(x) = t(x) \}$ and $X_2 = \{ x \in X : t'(x) = t(\lambda) \setminus t(x) \}$. Either $X_1$ or $X_2$ is stationary and both of them are homogeneous set for $F$.

We add easy obervations at the end of this paper.

**Definition.** A stationary coding set for $P_\lambda$ (an "SC") consists of a stationary set $A \subseteq P_\lambda$ together with a 1:1 function $c : A \rightarrow \lambda$ (called the coding function) satisfying that for each $x, y \in A$

$$x \leftrightarrow y \leftrightarrow c(x) \in y.$$  

**Proposition 3.** If Part* $(\kappa, \lambda)$, then an SC exists. (This is also seen in Zwicker [14]. The author considered this property without a word an "SC").

**Proof.** Let $F(x, y) = 0$ if $c(x) \in y$ and $F(x, y) = 1$ otherwise, for any 1:1 func-
Definition.  $X \subseteq P, \lambda$ is prestationary iff for any choice function on $X$ is constant on some unbounded set $S \subseteq X$.

This definition makes sense. In fact,

Lemma 4. (Menas in [10]) There is a prestationary set that is not stationary.

Lemma 5. If $X$ is prestationary, then $\{x \in X: a \subset x\}$ is also prestationary for all $a \in P, \lambda$.

Definition. $w\text{Part}^*(\kappa, \lambda)$ iff any partition of $P, \lambda$ has a prestationary homogeneous set.

Theorem 6. If $w\text{Part}^*(\kappa, \lambda)$, then $\kappa$ is almost $\lambda$-ineffable.

Proof. Magidor's proof of Theorem 2 in [9] can be carried out. What we really need is a homogeneous set $H$ such that for any choice function $f$ there is an unbounded subset $T$ of $H$ so that

$$\forall x \in T \exists y \in T (x \not\leq y \text{ and } f(x) \geq f(y)).$$

References

Notes on $P, \lambda$ and $[\lambda]^\kappa$


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