TSUKUBA J. MATH.
Vol. 13 No. 1 (1989). 165–173

NEAR-HOMEOMORPHISMS ON HEREDITARILY INDECOMPOSABLE CIRCLE-LIKE CONTINUA

By

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1. Introduction

A *continuum* means a compact connected metric space. A continuun is said to be *circle-like* if it is represented as an inverse limit of simple closed curves. A continuum X is said to be *hereditarily indecomposable* if each subcontinuum Y can not be represented as the union of two proper subcontinua of Y. The class of hereditarily indecomposable circle-like continua contains the pseudo-arc and the pseudo-circle.

Several authors have obtained some sufficient conditions or necessary conditions on an inverse sequence that the limit is hereditarily indecomposable (see, for example, [3], [10], [12], [15] etc.). In section 2 of this paper, we will give some equivalent conditions on inverse sequence of simple closed curves that the limit is hereditarily indecomposable. AOP (see Definition 1), one of these conditions, corresponds to "crookedness" of Bing [1] and Fearnley [4] and "Oscillating Property" of Mioduszewski [14]. AEOP (See Definetion 1), one of the other conditions, correspondes to "Everywhere Oscillation Property" of Mioduszewski [14].

In section 3, we will characterize near-homeomorphisms on a hereditarily indecomposable circle-like continuum in terms of shape theory. As a corollary, we have that any monotone map on a hereditarily indecomposable circle-like continuum is a near-homeomorphism.

The author wishes to thank to Professors K. Sakai and T. Yagasaki for their helpful advices.

2. Inverse limit representations of hereditarily indecomposable circle-like continua

First we will prepare some definitions and notations. For an interval $J=[a, b]$, $bd\ J$ denotes $\{a, b\}$. For two intervals $J_1=[a, b]$ and $J_2=[b, c]$, J_1+J_2 denotes $[a, c]$ and then the collection $\{J_1, J_2\}$ is called a *decomposition* of $[a, c]$.

Received November 19, 1987. Revised June 7, 1988

A subinterval of J always means *closed* interval contained in J. Let ε be a positive number and X and Y be continua. Two maps f and $g: X \to Y$ are said to be *ε-near*, denoted by $f \underset{\varepsilon}{=} g$, if $\sup\{d(f(x), g(x)) \mid x \in X\} < \varepsilon$, where d is a metric on Y. A map $h: X \twoheadrightarrow Y$ is called an ε-map if diam $h^{-1}(y) < \varepsilon$ for each $y \in Y$. H denotes the Hausdorff metric induced by a metric on a continuum.

Let $\underline{X} = (X_n, p_{n\,n+1})$ be an inverse sequence continua X_n and maps $p_{n\,n+1}$: $X_{n+1} \to X_n$. For each pair of integers $m > n$, p_{nm} denotes $p_{n\,n+1} \circ p_{n+1\,n+2} \circ \cdots \circ p_{m-1\,m}$. The limit of $\underline{X}$ is denoted by $\varprojlim \underline{X}$ and the projection map from $\varprojlim \underline{X}$ to X_n is denoted by p_n.

A collection of finite open sets $U = \{U_1, \cdots, U_n\}$ is called a *taut circular chain* if $clU_i \cap clU_j \neq \emptyset$ if and only if $|i-j| \leqq 1 \pmod n$. A taut circular chain $V = \{V_1, \cdots, V_m\}$ is called a *closure refinement* of U if, for each $V_i \in V$, there exists $U_j \in U$ such that $clV_i \subset U_j$. A function $f: \{1, \cdots, m\} \to \{1, \cdots, n\}$ is called a *cyclic pattern* if $|f(i) - f(i+1)| \leqq 1 \pmod n$ for each $i = 1, \cdots, m-1$. V is said to *follow f in U* if $V_k \subset U_{f(k)}$ for each $k = 1, \cdots, m$.

DEFINITION 1. Let $\underline{X} = (S_n, p_{n\,n+1})$ be an inverse sequence of simple closed curves and essential bonding maps.

(1) $\underline{X}$ is said to have *Approximate Oscillation Property* (AOP) if

for each n, for each subinterval $J \subset S_n$ and for each $\varepsilon > 0$, there exists an $m > n$ such that

for each subinterval K of $p_{nm}^{-1}(J)$ satisfying $p_{nm}(K, bd\,K) = (J, bd\,J)$, there exists a decomposition $K = K_1 + K_2 + K_3$ such that

a) $p_{nm}(bd\ K_i) = bd\ (p_{nm}(K_i))$ for $i = 1, 2, 3$.

b) $H(p_{nm}(K_i), J) < \varepsilon$ for $i = 1, 2, 3$.

(2) $\underline{X}$ is said to have *Approximate Everywhere Oscillating Property* (AEOP) if

for each n, for each $\varepsilon > 0$ and for each pair of essential maps $f_1: S_n \twoheadrightarrow S$, $f_2: C \twoheadrightarrow S$, where C and S are simple closed curves, such that $\deg f_2 \mid \deg p_{nl}$ for some $l \geqq n$, there exist an $m \geqq l$ and a map $\alpha: S_m \twoheadrightarrow C$ such that $f_2 \circ \alpha \underset{\varepsilon}{=} f_1 \circ p_{nm}$.

These two concepts are approximate versions of Mioduszewski's [14].

DEFINITION 2. Let $\underline{X} = (X_n, p_{n\,n+1})$ be an inverse sequence of continua. $\underline{X}$ is said to have *property* (*) if,

for each n, for each $\varepsilon > 0$ and for each map $f: X_l \to X_n$ which satisfies $f \simeq p_{nl}$, there exist an $m \geqq l$ and a map $\alpha: X_m \to X_l$ such that $\alpha \simeq p_{lm}$ and $f \circ \alpha \underset{\varepsilon}{=} p_{nm}$.

This concept was suggested by T. Yagasaki.

PROPOSITION 3. *Let X be an one dimensional continuum which is the limit of an inverse sequence of graphs with property* (*). *Then X is hereditarily indecomposoble.*

PROOF. Let $X=\varprojlim(X_n, p_{n\,n+1})$, where each X_n is a graph and $(X_n, p_{n\,n+1})$ has property (*). For each n, there exists an $\varepsilon_n>0$ such that

1) if $d(x, y)<\varepsilon_n$, $x, y\in X_n$, then $d(p_{in}(x), p_{in}(y))<\mathrm{diam}\, X_i/2^n$. By Lemma 1.4 of [17], there exists a map $f: X_{n+1}\to X_n$ such that $f\simeq p_{n\,n+1}$ and f is $\varepsilon_n/2$-crooked (see [3] or [17] for the definition of ε-crookedness). By property (*), there exists an $m>n+1$ and a map $\alpha: X_m\to X_{n+1}$ such that $p_{nm} \underset{\varepsilon_n/4}{=} f\circ\alpha$. Clearly, $f\circ\alpha$ is $\varepsilon_n/2$ crooked and hence p_{nm} is ε_n-crooked. So taking a subsequence, we can assume that $p_{n\,n+1}$ is ε_n-crooked for each n. By Lemma 2 of [3], we have that $X=\varprojlim X_n$ is hereditarily indecomposable.

REMARK. There exists a hereditarily indecomposable tree-like continuum X such that

1) $X=\varprojlim(T_n, p_{n\,n+1})$, where each T_n is a simple triod.

2) $(T_n, p_{n\,n+1})$ does not have property (*).

One of Ingram's examples [8] is such an example. This follows from the following proposition.

PROPOSITION 4. *Suppose that $\underline{X}=(X_n, p_{n\,n+1})$, $\underline{Y}=(Y_n, q_{n\,n+1})$ are inverse sequence of compact ANR's and both of $\underline{X}$ and $\underline{Y}$ have property* (*). *Then* $sh(\varprojlim \underline{X})=sh(\varprojlim \underline{Y})$ *if and only if $\varprojlim \underline{X}$ and $\varprojlim \underline{Y}$ are homeomorphic.*

PROOF. Using property (*), we can replace the homotopy commutative diagram which gives shape equivalence by the approximative commutative diagram as in the theorem of Mioduszewski [13]. For the detail of this argument, see also Proposition 10.

The following two theorems are fundamental in the arguments of this paper.

THEOREM 5 [15, Theorems 1 and 2]. *Let $f, g: S\to S$ be simplicial maps between simple closed curves such that $k=\deg f>0$ and $l=\deg g>0$. Then there exist simplicial maps α and $\beta: S\to S$ such that $f\circ\alpha=g\circ\beta$ and $\deg\alpha=m/k$, $\deg\beta=m/l$, where m is the least common multiple of k and l.*

THEOREM 6 [7, Theorem 3.1]. *Let* $(f_i: S_{i+1}\to S_i)$ *be a sequence of simplicial maps between simple closed curves such that*

1) $\deg f_i\neq 0$ *for each* i.

2) *each* f_i *is a crooked pattern* (*see* [4] *for the definition of crooked pattern*).

Then for each simplicial map $f: S\to S_n$ *from a simple closed curve* S *such that* $\deg f\,|\deg f_{nl}$ *for some* $l>n$, *there exist an* $m>l$ *and a map* $r: S_m\to S$ *such that* $f\circ r=f_{nm}$.

Using the above theorems, we have

THEOREM 7. *Let* $\underline{X}=(S_n, p_{n\,n+1})$ *be an inverse sequence of simple closed curves and essential bonding maps. Then the following statements are equivalent.*

(1) $\underline{X}$ *has AOP.*

(2) $\underline{X}$ *has AEOP.*

(3) $\underline{X}$ *has property* (*).

(4) $\underline{X}=\varprojlim \underline{X}$ *is hereditarily indecomposable.*

PROOF. All ideas of the proof are already known, but we will give it for completeness. We will show implications

$$1\longrightarrow 4\longrightarrow 3\longrightarrow 1 \quad\text{and}\quad 3\longleftrightarrow 2.$$

$1\to 4$ (see [12], Theorem 5). We only have to show that each proper subcontinuum of X is indecomposable. Assume that X contains a proper subcontinuum Y which is a union of its proper subcontinua H and I. Take $x\in H-I$ and $y\in I-H$. There exists an integer n such that for each $m\geqq n$, $p_m(x)\notin p_m(I)$ and $p_m(y)\notin p_m(H)$. Let $J=p_n(Y)$ and $0<\eta<\min\{d(p_n(x), p_n(I))/4, d(p_n(y), p_n(H))/4\}$.

Applying AOP to n, J and $\eta/2$, we have an $m>n$ satisfying the condition of AOP. Let K be the subinterval of $p_m(Y)$ which is irreducible with respect to being mapped onto J under p_{nm}. Then $p_{nm}(bd\,K)=bd\,J$. Using the decomposition $K=K_1+K_2+K_3$ required in AOP, we can see that $d(p_n(y), p_n(H))<\eta$ or $d(p_n(x), p_n(I))<\eta$. This contradicts the choice of η.

$4\to 3$. Suppose that $X=\varprojlim \underline{X}$ is hereditarily indecomposable and give n, $\varepsilon>0$, and $f: S\to S_n$ as in the hypothesis of property (*). By the simplicial approximation theorem, we may assume that f is simplicial with respect to suitable subdivisions T and T_n of S and S_n respectively. Let U_0 be a taut circular chain cover of S_n such that

a) mesh $U_0<\varepsilon/4$ and each vertex of T_n is contained in the unique link

of U_0.
Set $C_0=p_n^{-1}(U_0)$ and $k_0=n$.

Using an induction, we can take a sequence $(C_n)_{n\geq 0}$ of taut circular chain covers satisfying the following conditions.

b) mesh $C_i\to 0$ as $i\to\infty$, and C_{i+1} is a closure refinement of C_i.

c) Each link of C_i contains a subchain of C_{i+1} consisting of two links.

d) There exist a subsequence (k_i) and a sequence (U_i) of taut circular chain covers of S_{k_i} such that mesh $U_i<\varepsilon/3\cdot 2^{i+1}$ and $C_i=p_{k_i}^{-1}(U_i)$ for each i.

Let $f_i: C_{i+1}\to C_i$ be a pattern which C_{i+1} follows in C_i. By the same way as in [12, Theorem 1], we can assume, taking a subsequence if necessary, that

e) f_i is a crooked pattern for each i.

Each f_i determines a simplicial map $\bar{f}_i: S_{k_{i+1}}\to S_{k_i}$ such that

f) $p_{k_ik_j}$ and $\bar{f}_i\circ\cdots\circ\bar{f}_j$ are 3·mesh U_i-near.

Applying Theorem 6, there exist an integer s with $k_s>1$ and a map $\alpha: S_{k_s}\to S$ such that $f\circ\alpha=\bar{f}_1\circ\cdots\circ\bar{f}_s$. By d) and f), k_s and α have the required property.

3→1 (see [12], Theorem 4).

Give any integer $n>0$, $\varepsilon>0$ and any arc $J\subset S_n$. Define a PL map $f: S_n\to S_n$ as follows. Let $J=[p, q]$.

J is decomposed by congruent arcs $J_1=[p, s]$, $J_2=[s, t]$, $J_3=[t, q]$. $f|J: J\to J$ is defined by $f(p)=p$, $f(q)=q$, $d(f(s), q)=\varepsilon/2$, and $d(f(t), p)=\varepsilon/2$, and $f|J$ is linear on the remaining parts. Furthermore, $f|S_n-J=id_{S_n-J}$. Note $f\simeq id_{S_n}$.

Then applying property (*), there exist an integer $m>n$ and a map $\alpha: S_m\to S_n$ such that $f\circ\alpha$ and p_{nm} are $\varepsilon/2$ near. Take an arc $K\subset S_n$ which satisfies $p_{nm}(K, bd\ K)=(J, bd\ J)$. By the same way as in [12], Theorem 4, we can find a decomposition $K=K_1+K_2+K_3$ which has the required property.

2→3. This is obvious.

3→2. This is proved by Theorem 5. Notice that if $f, g: S\to S$ are maps between simple closed curves such that $\deg g|\deg f$, then there exists a map $h: S\to S$ such that $g\circ h\simeq f$.

This completes the proof of Theorem 7.

By the similar way, we can define AOP, AEOP and property (*) for inverse sequences of arcs (homotopy conditions for maps are not required). We can obtain the similar result to Theorem 7 for inverse sequences of arcs. This gives an inverse limit characterization of the pseudo-arc, which is represented as an inverse limit of simple closed curves and null-homotopic bonding maps.

The following corollary is essentially proved by Fearnley [6].

COROLLARY 8. *Let X and Y be hereditarily indecomposable circle-like continua. X and Y are homeomorphic if and only if $sh\,X = sh\,Y$.*

3. A characterization of near-homeomorphisms

Lewis [9] and Smith [16] have shown that each onto map on the pseudo-arc is a near-homeomorphism. In this section, we will characterize near-homeophisms on a hereditarily indecomposable circle-like continuum. By the characterization, we will construct an onto map on the pseudo-circle which is not a near-homeomorphism.

Let $X = \varprojlim (S_i, p_{i\,i+1})$ be an inverse limit of n-spheres S_i's and essential bonding maps ($n \geqq 1$), and let $r_i = \deg p_{i\,i+1}$. Then $\check{H}^n(X) \cong \{j/r_1 r_2 \cdots r_k \mid j \in \boldsymbol{Z}, k \in \boldsymbol{N}\}$ and $p_i^* : H^n(S_i) \to \check{H}^n(X)$ is written by $p_i^*(e_i) = 1/r_1 \cdots r_{i-1}$, where e_i is the generator of $H^n(S_i)$. In particular, p_i^* is a monomorphism.

PROPOSITION 9. *Let X be a continuum which is an inverse limit of n-sphere and essential bonding maps and $f : X \twoheadrightarrow X$ be an onto map.*

a) *f is a shape equivalence if and only if f induces an isomorphism on n-th Čech cohomology.*

b) *If f is a near-homeomorphism, then it is a shape equivalence.*

PROOF. a) In the case $n \geqq 2$, this follows from the cohomological version of Whitehead theorem by S. Mardesič (see [11], p. 155-156). The case $n=1$ follows from [7], (2.6) and the fact that each circle-like continuum has the same shape as a solenoid.

The author wishes to thank to the referee for pointing out these results.

b) Let $X = \varprojlim (S_i, p_{i\,i+1})$, where S_i is a n-sphere and $p_{i\,i+1}$ is essential, and suppose that f is a near-homeomorphism. Using an induction, we will construct a homotopy commutative diagram which implies shape equivalence of f. Take a decreasing sequence (ε_i) of positive and sufficiently small numbers which converges to 0.

Let $S_{n_1} = S_1$ and take an integer $m_1 > 1$ and a map $f_1 : S_{m_1} \to S_{n_1}$ such that $p_1 \circ f \underset{\varepsilon/4}{=} f_1 \circ p_{m_1}$. There exists a homeomorphism $h : X \to X$ such that $p_1 \circ h \underset{\varepsilon/4}{=} p_1 \circ f_1$. Take a large $l_1 > m_1$ and a map $h_1 : S_{l_1} \to S_{n_1}$ such that $p_{n_1} \circ h \underset{\varepsilon/4}{=} h_1 \circ p_{l_1}$. Since h is a homeomorphism, there exists an integer $n_2 > n_1$ and a map $k_1 : S_{n_2} \to S_{l_1}$ such that $h_1 \circ k_1 \underset{\varepsilon/4}{=} p_{n_1 n_2}$. It is easy to see that $f_1 \circ p_{m_1 l_1} \circ k_1 \underset{\varepsilon_1}{=} p_{n_1 n_2}$ and hence

$f_1\circ p_{m_1 l_1}\circ k_1 \simeq p_{n_1\, n_2} \neq 0$. Let $g_1 = p_{m_1\, l_1}\circ k_1$. Since deg $f_1 \neq 0$, we have

(1) $$p_{m_1}{}^* = (g_1\circ p_{n_2}\circ f)^*\,.$$

There exist an integer $m_2 > m_1$ and a map $f_2\colon S_{m_2} \to S_{n_2}$ such that $p_{n_2}\circ f \underset{\varepsilon_1}{=} f_2\circ p_{m_2}$. Using (1),

(2) $$p_{m_2}{}^*\circ f_2{}^*\circ g_1{}^* = p_{m_2}{}^*\circ p_{m_1\, m_2}{}^*\,.$$

Since $p_{m_2}{}^*$ is a monomorphism, we have $f_2{}^*\circ g_1{}^* = p_{m_2}{}^*$. Hence $g_1\circ f_2 \simeq p_{m_1\, m_2}$.

Repeating this process, we obtain a homotopy commutative diagram as follows.

$$\begin{tikzcd}
S_{m_1} \arrow[d, "f_1"'] \arrow[dr, phantom, "\simeq"] & S_{m_2} \arrow[l] \arrow[d, "f_2"'] \arrow[dr, phantom, "\simeq"] & & \cdots \arrow[ll] & X \arrow[l] \arrow[d, "f"] \\
S_{n_1} & S_{n_2} \arrow[l] \arrow[ul, "g_1"'] & S_{n_3} \arrow[l] \arrow[ul, "g_2"'] & \cdots \arrow[l] & X \arrow[l]
\end{tikzcd}$$

Hence f is a shape equivalence.

PROPOSITION 10. *Let X be a continuum which is the inverse limit of an ANR sequence which has property* (*). *Let $f\colon X \twoheadrightarrow X$ be an onto map. If f is a shape equivalence, then f is a near-homeomorphism.*

PROOF. Give any $\varepsilon > 0$. We will construct a homeomorphism h which is ε-near to f. Take an integer j_1 and $\delta > 0$ such that

1) for each subset $A \subset X_{j_1}$ with diam $A < \delta$, diam $p_{j_1}{}^{-1}(A) < \varepsilon/2$.

Let $\varepsilon_i = \delta/2^i$. Take an integer i_1 and a map $f_1\colon X_{i_1} \twoheadrightarrow X_{j_1}$ such that $p_{j_1}\circ f \underset{\varepsilon_1}{=} f_1\circ p_{i_1}$ and let $h_1 = f_1$.

Since f is a shape equivalence, there exist an integer $k > j_1$ and a map $u_1\colon X_k \twoheadrightarrow X_{i_1}$ such that $f_1\circ u_1 \simeq p_{j_1 k}$. Applying property (*), there exist an integer $j_2 > k$ and a map $v_1\colon X_{j_2} \twoheadrightarrow X_k$ which is homotopic to $p_{k\, j_2}$ such that $f_1\circ u_1\circ v_1 \underset{\varepsilon_1}{=} p_{j_1\, j_2}$. Let $g_1 = u_1\circ v_1$.

Take an integer $l > i_1$ and a map $f_2\colon X_l \twoheadrightarrow X_{j_2}$ such that $p_{j_1\, j_2}\circ p_{j_2}\circ f \underset{\varepsilon_2}{=} p_{j_1\, j_2}\circ f_2\circ p_l$. Since $v_1 \simeq p_{k\, j_2}$, we may assume that $u_1\circ v_1\circ f_2 \simeq p_{i_1\, l}$. Applying property (*) again, there exist an integer $i_2 > l$ and a map $w_2\colon X_{i_2} \twoheadrightarrow X_l$ such that $g_1\circ f_2\circ w_2 \underset{\varepsilon_2}{=} p_{i_1\, i_2}$. Let $h_2 = f_2\circ w_2$.

Repeating these processes, we obtain an approximative commutative diagram as follows.

$$\begin{tikzcd}
X_{i_1} \arrow[d, "f_1=h_1"'] & X_{i_2} \arrow[l] \arrow[d, "h_2"] & {} \arrow[l] & \cdots & X \arrow[l] \arrow[d, "h"] \\
X_{j_1} & X_{j_2} \arrow[l] \arrow[ul, "g_1"] & X_{j_3} \arrow[l] \arrow[ul, "g_2"] & \cdots \arrow[l] & X \arrow[l]
\end{tikzcd}$$

By [13], the sequence (h_i) induces a homeomorphism h. By the choice of j_1 and δ, we have $h \underset{\varepsilon}{=} f$. This completes the proof.

Combining Propositions 9, 10 and Theorem 7, we have

THEOREM 11. *Let X be a hereditarily indecomposable circle-like continuum and $f: X \twoheadrightarrow X$ be an onto map. Then the following statements are equivalent.*

1) *f is a near-homeomorphism.*
2) *f is a shape equivalence.*
3) *f induces an isomorphism on the first Čech cohomology.*

COROLLARY 12. *Each monotone map on a hereditarily indecomposable circle-like continuum is a near-homeomorphism.*

Because, each monotone map on a circle-like continuum is a cell-like map.

EXAMPLE 13. There is an onto map on the pseudo-circle which is not a near-homeomorphism.

The pseudo-circle Q is represented as the inverse limit of an inverse sequence $(S_i, p_{i\, i+1})$ of simple closed curves S_i's where $p_{i\, i+1}$ has degree 1. We may assume that each $p_{i\, i+1}$ is simplicial.

Take a map $f_1: S_1 \to S_1$ with $\deg f_1 = 2$. Applying Theorem 5 to f_1 and $p_{1\, 2}$, there exist simplicial maps $a_1: C_1 \to S_1$ and $b_1: C_1 \to S_2$ from a simple closed curve C_1 such that $p_{1\, 2} \circ b_1 = f_1 \circ a_1$ and $\deg a_1 = 1$, $\deg b_1 = 2$. Applying Theorem 6 to a_1, there exist an $n_2 > 1 = n_1$ and a map $c_1: S_{n_2} \to C_1$ such that $a_1 \circ c_1 = p_{1\, n_2}$. Let $f_2 = b_1 \circ c_1$. Then $\deg f_2 = 2$.

Repeating this step, we obtain a commutative sequence $(f_i: S_{n_i} \to S_i)$ of maps such that $\deg f_i = 2$ for each i. (f_i) induces an onto map $f: Q \twoheadrightarrow Q$ which is not a shape equivalence, hence not a near-homeomorphism.

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