CYCLIC-PARALLEL REAL HYPERSURFACES OF
A COMPLEX SPACE FORM

By
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Introduction.

In 1973 Takagi [14] classified homogeneous hypersurfaces of a complex projective space $P_nC$ by proving that all of them could be divided into six types, and he [15], [16] showed also that if a real hypersurface $M$ has two or three distinct constant principal curvatures, then $M$ is congruent to one of the homogeneous hypersurfaces of type $A_1$, $A_2$ and $B$ among these ones. This result is generalized by Kimura [6], who gives the complete classification that a real hypersurface $M$ of $P_nC$ has constant principal curvatures and $FC$ is principal if and only if $M$ is congruent to one of homogeneous examples, where $C$ denotes the unit normal and $F$ is the almost complex structure. The study of real hypersurfaces of type $A_1$, $A_2$ and $B$ of $P_nC$ was originated by Cecil and Ryan [1], Kimura [7], Kon [8], Maeda [10], Okumura [13] and so on.

Real hypersurfaces with cyclic-parallel Ricci tensor of a complex space form $M^n(c)$ have recently been classified by Kwon and Nakagawa [9] in the case where $FC$ is principal. They also gave another characterization of real hypersurfaces of type $A_1$ and $A_2$ of $P_nC$.

On the other hand, many subjects for real hypersurfaces of a complex hyperbolic space $H_nC$ were investigated from different points of view ([2], [3], [11], [12] etc.) one of which, done by Chen, Ludden and Montiel [3], asserts that a real hypersurface $M$ of $H_nC$ is of cyclic-parallel if and only if the structure tensor $J$ induced on $M$ and the shape operator $A$ derived from the unit normal commute each other, that is, $JA=AJ$. In particular, real hypersurfaces of $H_nC$, which are said to be of type $A$, similar to those of type $A_1$ and $A_2$ of $P_nC$, were treated by Montiel and Romero [12].

The purpose of the present paper is to show that a real hypersurface of a complex space form $M^n(c)$, $c\neq 0$, is of cyclic-parallel if and only if $JA=AJ$, and to give a complete classification of such hypersurfaces by using those examples constructed in [9], [12] and [15].

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1. Preliminaries.

We begin by recalling fundamental properties on real hypersurfaces of a Kaehlerian manifold. Let $N$ be a real $2n$-dimensional Kaehlerian manifold equipped with a parallel almost complex structure $F$ and a Riemannian metric tensor $G$ which is $F$-Hermitian, and covered by a system of coordinate neighborhoods $\{U : x^A\}$. Let $M$ be a real hypersurface of $N$ covered by a system of coordinate neighborhoods $\{V : y^h\}$ and immersed isometrically in $N$ by the immersion $i: M \rightarrow N$. Throughout the present paper the following convention on the range of indices are used, unless otherwise stated:

$$A, B, \ldots = 1, 2, \ldots, 2n; \quad i, j, \ldots = 1, 2, \ldots, 2n-1.$$ 

The summation convention will be used with respect to those system of indices. When the argument is local, $M$ need not be distinguished from $i(M)$. Thus, for simplicity, a point $p$ in $M$ may be identified with the point $i(p)$ and a tangent vector $X$ at $p$ may also be identified with the tangent vector $i_*(X)$ at $i(p)$ via the differential $i_*$ of $i$. We represent the immersion $i$ locally by $x^A = x^A(y^h)$ and $B_j = (B_j^A)$ are also $(2n-1)$-linearly independent local tangent vectors of $M$, where $B_j^A = \partial_j x^A$ and $\partial_j = \partial/\partial y^j$. A unit normal $C$ to $M$ may then be chosen. The induced Riemannian metric $g$ with components $g_{ji}$ on $M$ is given by $g_{ji} = G(B_j, B_i)$ because the immersion is isometric.

For the unit normal $C$ to $M$, the following representation are obtained in each coordinate neighborhood:

$$FB_i = J_i^h B_h + P_i C, \quad FC = -P^i B_i,$$

(1.1)

where we have put $J_{ji} = G(FB_j, B_i)$ and $P_i = G(FB_i, C)$, $P^h$ being components of a vector field $P$ associated with $P_i$ and $J_{ji} = f_{ji} g_{rs}$. By the properties of the almost Hermitian structure $F$, it is clear that $J_{ji}$ is skew-symmetric. A tensor field of type $(1,1)$ with components $J_i^h$ will be denoted by $J$. By the properties of the almost complex structure $F$, the following relations are then given:

$$J_i^r J_r^h = -\delta_i^h + p_i p^h, \quad J_i^r J_r^h = 0, \quad p_i J_i^r = 0, \quad p_i \phi^i = 1,$$

that is, the aggregate $(J, g, P)$ defines an almost contact metric structure. Denoting by $\nabla_j$ the operator of van der Waerden-Bortolotti covariant differentiation formed with $g_{ji}$, equations of Gauss and Weingarten for $M$ are respectively obtained:
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(1.2) \[ \nabla_j B_i = h_{ji} C, \quad \nabla_j C = -h_{ji} B_r, \]
where \( h_{ji} \) are components of a second fundamental from \( \sigma \), \( A = (h_{ji}) \) which is related by \( h_{ji} = h_{ji} \mathcal{G}_{ri} \) being the shape operator derived form \( C \). We notice here that \( h_{ji} \) is symmetric. By means of (1.1) and (1.2) the covariant derivatives of the structure tensors are yielded:

(1.3) \[ \nabla_j J_{ih} = -h_{ji} p_h + h_{jh} p_i, \quad \nabla_j p_i = -h_{ji} J_{ir}. \]

In the sequel, the ambient Kaehlerian manifold \( N \) is assumed to be of constant holomorphic sectional curvature \( c \) and real dimension \( 2n \), which is called a complex space form and denoted by \( M^n(c) \). Then the curvature tensor \( K \) of \( M^n(c) \) takes the following form:

\[ K_{DCA} = \frac{c}{4} (G_{DA} G_{CB} - G_{DB} G_{CA} + F_{DA} F_{CB} - F_{DB} F_{CA} - 2 F_{DC} F_{BA}). \]

Thus, equations of Gauss and Codazzi for \( M \) are respectively obtained:

(1.4) \[ R_{kjih} = \frac{c}{4} (g_{kh} g_{jil} - g_{ih} g_{kli} + J_{kh} J_{jil} - J_{jih} J_{kli} - 2 J_{kij} J_{ihl}) + h_{kh} h_{jil} - h_{jih} h_{kli}, \]
(1.5) \[ \nabla_k h_{jik} - \nabla_j h_{ik} = \frac{c}{4} A_{kji}, \quad A_{kji} = p_k J_{jil} - p_j J_{kli} - 2 p_i J_{kji}, \]
where \( R_{kjih} \) are components of the Riemannian curvature tensor \( R \) of \( M \). Let \( S_{ji} \) be components of the Ricci tensor \( S \) of \( M \), then the Gauss equation implies

(1.6) \[ S_{ji} = \frac{c}{4} [(2n + 1) g_{ji} - 3 p_j p_i] + h h_{ji} - h_{ji}^2, \]
where \( h \) denotes the trace of the shape operator \( A \) and \( h_{ji}^2 = h_{ji} h_{ji}^r \).

### 2. Cyclic-parallel hypersurfaces.

Let \( M \) be a real hypersurface of a complex space form \( M^n(c) \). The hypersurface \( M \) is called cyclic-parallel if the cyclic sum of \( \nabla \sigma \) vanishes identically, namely

(2.1) \[ \nabla_k h_{jil} + \nabla_j h_{ik} + \nabla_i h_{kj} = 0. \]

It was proved in [4] that geodesic hypersurfaces of a complex space form \( M^n(c) \), \( c \neq 0 \), are cyclic-parallel and not parallel. Throughout the present paper we only consider the case where the holomorphic sectional curvature \( c \) is not zero.

From now on we suppose that \( M \) is of cyclic-parallel. Then we have from (1.5)

\[ 2 \nabla_k h_{ji} = -\nabla_i h_{kj} + \frac{c}{4} A_{kji}, \]
or equivalently $3\nabla_k h_{ji} = c/4(A_{kj} - A_{ik})$. By the second equation of (1.5), it follows that

$$\nabla_k h_{ji} = \frac{c}{4}(p_j J_{ik} + p_i J_{jk}).$$

Differentiating this covariantly along $M$ and making use of (1.3), we find

$$\nabla_m \nabla_k h_{ji} = \frac{c}{4}\{(\nabla_m p_j)J_{ik} + (\nabla_m p_i)J_{jk} - h_{mi} p_j p_k - h_{mj} p_k p_i + 2 h_{mk} p_j p_i\}.$$

Since equation (2.2) tells us that $\nabla_k h_j = 0$, the Ricci formula for $h_{ji}$ gives rise to

$$\nabla_k \nabla_i h_{ji} = S_{ji} h_{ir} - R_{kijh} h^{ik}.$$

If we substitute (1.4), (1.6) and (2.3) into the last equation and take account of (1.3), we get

$$h h_{ji} = \left\{h_{ji} - \frac{c}{2}(n+1)h_{ji} + c h_{ki} J_{ij} J_{ij}^* \right\} + \frac{c}{2}(\langle h_{jr} p^r \rangle p_i + \langle h_{ir} p^r \rangle p_j) + \frac{c}{4} h (g_{ji} - p_j p_i),$$

where $h_2 = h_{ji} h_{ij}$, which yields

$$h h_{jr} = \left(h_{2} - \frac{c}{2} n\right) h_{jr} p^r + \frac{c}{2} \alpha p_j,$$

where we have have defined $\alpha = h_{rs} p^r p^s$. Thus, it follows that

$$h \beta = \left\{h_{2} - \frac{c}{2}(n-1)\right\} \alpha, \quad \beta = h_{jr} p^r p^i.$$

On the other hand, if we substitute (1.4) and (2.3) into the Ricci formula, which is given by

$$\nabla_m \nabla_k h_{ji} - \nabla_k \nabla_m h_{ji} = - R_{mkjr} h_{ir} - R_{mkrjr} h_{jr},$$

then we have

$$h_{ik} h_{mj} - h_{im} h_{jk} + h_{jm} h_{ik} - h_{jm} h_{ik} = \frac{c}{4}\{h_{mi} (g_{kj} - p_k p_j) - h_{mj} (g_{ki} - p_k p_i) + h_{jm} (g_{ki} - p_k p_i) - h_{jk} (g_{mi} - p_m p_i) \}
+ J_{jk} (\nabla_m p_i + \nabla_i p_m) - J_{jm} (\nabla_k p_i + \nabla_i p_k) + J_{ik} (\nabla_m p_j + \nabla_j p_m)
- J_{jm} (\nabla_k p_j + \nabla_j p_k) + 2 J_{mk} (\nabla_j p_i + \nabla_i p_j),$$

where we have used the second equation of (1.3). By transvecting (2.7) with $J^r$ and $p^i p^r p^s$ respectively and making use of the fact that properties of the almost contact metric structure $(J, g, P)$, we can see that
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\[ f^{rr}(h_m, h_j, h_m^2 + h_j, h_m^2) = \frac{1}{4}(2n+1)c(\nabla_j p_m + \nabla_m p_j) - \frac{1}{4} c \left\{ (p^r \nabla_r p_j) p_m + (p^r \nabla_r p_m) p_j \right\}, \]

\[ \alpha h_m^r p^r = \beta h_m, p^r. \]

Combining (2.5) and (2.6) with (2.9), it follows that \( \alpha(h_j, p^r - \alpha p_j) = 0 \) and hence \( \alpha(\beta - \alpha^2) = 0. \)

Let \( M_i \) be a set consisting of points of \( M \) at which the function \( \beta - \alpha^2 \) does not vanish. Suppose that \( M_i \) is not empty. We then have \( \alpha = 0 \) and thus \( \beta h_m, p^r = 0 \) because of (2.9). By transvecting \( h_m^r p^s \), it follows that \( \beta^s = 0 \) and hence \( \beta \) vanishes on \( M_i \). Therefore the assumption of \( M_i \) will produce a contradiction. Accordingly we have \( \beta = \alpha^2 \) on \( M \), which means that \( P \) is the principal curvature vector corresponding to \( \alpha \), that is,

\[ h_j, p^r = \alpha p_j. \]

Applying \( p^m \) to (2.8) and summing up \( m \), we obtain

\[ p^r \nabla_j p_j = 0 \]

because of the fact that \( c \neq 0 \). By means of (2.2), (2.10), (2.11) and the definition of \( \alpha \), we can easily see that \( \alpha \) is constant everywhere. Thus, differentiating (2.10) covariantly along \( M \), we find

\[ (\nabla_k h_j, p^r) + h_j, \nabla_k p^r = \alpha \nabla_k p_j, \]

which together with (1.3) and (2.2) yield

\[ \frac{c}{4} J_{jk} - h_j, h_k, f^{*s} = \alpha \nabla_k p_j. \]

If we take the symmetric part of this, then we obtain \( \nabla_k p_j + \nabla_j p_k = 0 \) provided that \( \alpha \neq 0 \). But, if \( \alpha = 0 \), then (2.12) implies \( h_j, h_k, f^{*s} = - (c/4) \nabla_k p_j \) with the aid of (1.3), which together with (2.8) and (2.11) give \( \nabla_j p_m + \nabla_m p_j = 0 \). Consequently we see in any case that \( h_j, f^{*s} = f_j, h^{*s} \). Thus we have the following fact:

**Lemma 1.** Let \( M \) be a cyclic-parallel real hypersurfaces of \( M^n(c), c \neq 0 \). Then the shape operator and the induced structure tensor commute each other, that is,

\[ AJ = JA. \]

**Remark 1.** Chen, Ludden and Montiel [3] proved this lemma for the case where \( c < 0 \). The converse assertion of Lemma 1 is well known. The proof was used the theory of Riemann fibre bundles (cf. [3], [8]). But, we introduce here the other simple proof. The method is similar to that used in the previous paper [5].
From (2.13), it is easy to see that

\[ h_{jr} p^j = \alpha p_j \]

by means of the properties of the almost contact metric structure. Differentiating (2.14) covariantly and taking account of (1.3), we obtain

\[ (\nabla_k h_{jr}) p^r - h_{jr} h_{ks} J^s = \alpha_k p_j - \alpha h_{kr} J^r, \]

where \( \alpha_k = \nabla_k \alpha \), which together with equations of Codazzi and (2.13) give

\[ \frac{c}{2} f_{jk} + 2 h_{jr} h_{ik} J^s = \alpha_k p_j - \alpha p_k + 2 \alpha h_{jr} J^r. \]

It means that \( \alpha_k = B p_k \) for some function \( B \). It is easy to see that \( \alpha \) is constant everywhere. Thus, the last equation reduces to

\[ h_{jr} = \alpha h_{ji} + \frac{c}{4} (g_{ji} - p_j p_i) \]

because of (2.13) and the properties of \((J, g, P)\). Accordingly (2.15) becomes

\[ (\nabla_k h_{jr}) p^r = \frac{c}{4} f_{jk}. \]

**Lemma 2.** Let \( M \) be a real hypersurface satisfying (2.13) of \( M^a(c), c \neq 0 \). Then \( M \) is of cyclic-parallel provided that \( \alpha^2 + c = 0 \).

**Proof.** Since we have \( \alpha^2 + c = 0 \), the relationships (2.14) and (2.17) tell us that \( M \) has at most two constant principal curvatures \( \alpha \) and \( \alpha/2 \). Their multiplicities are denoted respectively by \( r \) and \( 2n - 1 - r \). Thus, the trace of the shape operator is given by

\[ h = \frac{\alpha}{2} (2n - 1 + r) \]

and that of \( A^2 \) is given by

\[ h_s = \frac{\alpha^2}{4} (2n - 1 + 3r). \]

On the other hand, it is seen from (2.17) that \( h_z = \alpha h - (\alpha^3/2)(n - 1) \). Therefore, the last three equations imply that \( r = 1 \) because of \( \alpha^2 + c = 0 \) and \( c \neq 0 \). Accordingly (2.19) and (2.20) reduces respectively to

\[ h = n \alpha, \quad h_z = \frac{1}{2} (n + 1) \alpha^2. \]

We also have the followings:

\[ h_3 = \frac{1}{4} (n + 3) \alpha^2, \quad h_4 = \frac{1}{8} (n + 7) \alpha^4, \]

where \( h_3 \) and \( h_4 \) denote the trace of \( A^3 \) and \( A^4 \) respectively. By using (2.21)
and (2.22), it is not hard to see that

$$h_{ji} = \frac{3}{2} \alpha h_{ji} - \frac{\alpha^2}{2} g_{ji},$$

which together with (2.17) implies that $h_{ji} = (1/2)\alpha(g_{ji} + p_j p_i)$ because of $\alpha \neq 0$. Differentiating this covariantly, we find

$$\nabla_k h_{ji} = \frac{1}{2} \alpha \left\{ (\nabla_k p_j) p_i + (\nabla_k p_i) p_j \right\}.$$

Therefore, by means of (1.3) and (2.13) we can verify that $M$ is of cyclic-parallel. This completes the proof.

Differentiation (2.17) covariantly and making use of (1.3), we get

$$(\nabla_k h_{ji}) h_i^r + (\nabla_k h_i^r) h_{jr} = \alpha \nabla_k h_{ji} + \frac{c}{4} \left\{ (h_{kr} J_{jr}) p_i + (h_{jr} J_{ki}) p_j \right\},$$

from which, taking the skew-symmetric part with respect to indices $k$ and $j$ and utilizing (2.13) and (2.14),

$$h_{jr} \nabla_k h_{i^r} - h_{i^r} \nabla_k h_{jr} = \frac{c}{4} \alpha (p_k J_{ji} - p_j J_{ki}) + \frac{c}{2} p_i (h_{kr} J_{jr}).$$

Thus, it follows that

$$h_{jr} \nabla_k h_{i^r} - h_{i^r} \nabla_k h_{jr} = \frac{c}{4} \left\{ p_j h_{i^r} J_{kr} - p_i h_{jr} J_{k^r} + \alpha (p_j J_{ik} - p_i J_{jk}) \right\},$$

where we have used (1.5), (2.13) and (2.14). From this and (2.23), it is seen that

$$2 h_{jr} \nabla_k h_{i^r} - \alpha \nabla_k h_{ji} = \frac{c}{4} \left\{ -2 p_i (h_{jr} J_{k^r}) + \alpha (p_j J_{ik} - p_i J_{jk}) \right\}.$$

Transforming this by $h_m^r$ and using (2.13), (2.17) and (2.18), we obtain

$$\alpha h_{jr} \nabla_k h_{i^r} + \frac{c}{2} \nabla_k h_{ji} = \frac{c}{4} \left\{ (\alpha^2 + \frac{c}{2}) J_{ik} p_j - \frac{c}{2} J_{kj} p_i - \alpha p_i (h_{jr} J_{k^r}) \right\}.$$

Combining this with (2.24), it follows that

$$(\alpha^2 + c) \left\{ (\nabla_k h_{ji} - \frac{c}{4} (p_j J_{ik} + p_i J_{jk}) \right\} = 0,$$

which shows that $M$ is of cyclic-parallel because of Lemma 2.

From this fact and Lemma 1 we have

**Theorem 3.** Let $M$ be a real hypersurface of a complex space form $M^n(c)$, $c \neq 0$. Then $M$ is of cyclic-parallel if and only if $AL = JA$.

**Remark 2.** It is obvious that if $M$ is of cyclic-parallel, then the Ricci tensor is cyclic-parallel because of (1.3), (1.6) and (2.10).
3. Homogeneous hypersurfaces.

It is known that the complete and simply connected complex space form $M^n(c)$ consists of a complex projective space $P_nC$, a complex Euclidean space $C_n$ or a complex hyperbolic space $H_nC$, according as $c>0$, $c=0$ or $c<0$. Some standard examples given by [9], [12], [14] of real hypersurfaces $M^n(c)$, $c\neq 0$ whose second fundamental form are cyclic-parallel are introduced. In a complex Euclidean space $C_{n+1}$ equipped with Hermitian form $<j>$, the Euclidean metric of $C_{n+1}$ which is identified with $R^{2n+2}$ is given by $Re0$. The unit sphere $S^{2n+1} = \{z \in C_{n+1} : (z,z)=1\}$ is denoted.

First of all, examples of real hypersurfaces $P_nC$ are considered. For any positive number $r$ a hypersurface $N_0(2n,r)$ of $S^{2n+1}$ is defined by

$N_0(2n,r) = \{(z_1, \ldots, z_{n+1}) \in S^{2n+1} \subset C_{n+1} : \sum_{j=1}^{n+1} |z_j|^2 = r |z_{n+1}|^2 \}$.

For an integer $m (2 \leq m \leq n-1)$ and a positive number $s$, a hypersurface $N(2n,m,s)$ of $S^{2n+1}$ is defined by

$N(2n,m,s) = \{(z_1, \ldots, z_{n+1}) \in S^{2n+1} \subset C_{n+1} : \sum_{j=1}^{m} |z_j|^2 = s \sum_{j=m+1}^{n+1} |z_j|^2 \}.$

Then, for the projection $\pi$ of the Hopf-fibration $S^{2n+1}$ onto $P_nC$, $M_0(2n-1,r) = \pi(N_0(2n,r))$ and $M(2n-1,m,s) = \pi(N(2n,m,s))$ ($n \geq 3$) are examples of real hypersurfaces of $P_nC$ whose shape operator and the induced structure tensor commute each other. It is known [14] that $M_0(2n-1,r)$ and $M(2n-1,m,s)$ are both compact connected real hypersurfaces of $P_nC$ with constant two or three distinct principal curvatures respectively, which are said to be of type $A_1$ and $A_2$ respectively. In [13], it is proved that $M_0(2n-1,r)$ and $M(2n-1,m,s)$ are only hypersurfaces of $P_nC$ satisfying $AJ = JA$.

In the next place, the example of real hypersurfaces of $H_nC$ defined by Montiel [11] and Montiel and Romero [12] is introduced. In $C_{n+1}$ with standard basis, a Hermitian form $\phi$ is defined by

$\phi(z, w) = -z_0 \overline{w}_0 + \sum_{k=1}^{n} z_k \overline{w}_k,$

where $z = (z_0, \ldots, z_n)$ and $w = (w_0, \ldots, w_n)$ are in $C_{n+1}$. Let $H_1^{2n+1}$ be a real hypersurface of the Minkowski space $C_1^{n+1}$ defined by

$H_1^{2n+1} = \{z \in C_1^{n+1} : \phi(z, z) = -1\},$

and let $\tilde{G}$ be a semi-Riemanian metric of $H_1^{2n+1}$ induced from the complex Lorentzian metric $Re\phi$ of $C_1^{n+1}$. Then $(H_1^{2n+1}, \tilde{G})$ is the Lorentzian manifold of constant curvature $-1$, which is called an anti-de Sitter space.
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Let \( r \) and \( s \) be integers with \( r+s=n-1 \) and \( t \in \mathbb{R} \) with \( 0 < t < 1 \). We consider a Lorentzian hypersurface \( N_{r+s}(t) \) of \( H^{2n+1}_t \) defined by the following:

\[
N_{r+s}(t) = \{(z_0, \ldots, z_n) \in H_1^{2n+1}: t(-|z_0|^2) + \sum_{j=1}^{r} |z_j|^2 = -\sum_{k=r+1}^{n} |z_k|^2 \}
\]

and a Lorentzian hypersurface of \( H_1^{2n+1} \) is given by

\[
N_n = \{(z_0, \ldots, z_n) \in H_1^{2n+1}: |z_0-z_1|=1 \}.
\]

Since it is known that \( H_1^{2n+1} \) is a principal \( S^1 \)-bundle over a complex hyperbolic space with projection \( \pi: H_1^{2n+1} \to H_n C \), and \( N_{r+s}(t) \) and \( N_n \) are \( S^1 \)-invariant, we see that \( M_{r+s}(t) = \pi(N_{r+s}(t)) \) and \( M_n = \pi(N_n) \) are real hypersurfaces of \( H_n C \), where \( \pi: N_{r+s}(t) \to M_{r+s}(t) \) and \( \pi: N_n \to M_n \) are semi-Riemannian submersions which are compatible with \( S^1 \)-fibration. It is seen that \( M_{r+s}(t) \) and \( M_n \) are complete connected real hypersurfaces of \( H_n C \) with constant two or three distinct principal curvatures, which are said to be of type A ([9]). In [12], it is proved that \( M_{r+s}(t) \) and \( M_n \) are only complete hypersurfaces of \( H_n C \) satisfying \( AJ = JA \). Thus, by combining above facts and Theorem 3, we obtain the following classifications.

**Theorem 4.** \( M_0(2n-1, r) \), \( M(2n-1, m, s) \), \( M_{r+s}(t) \) and \( M_n \) are only complete and connected cyclic-parallel real hypersurfaces of \( M^n(c) \), \( c \neq 0 \).

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