Abstract

A millimeter-wave two-dimensional imaging array has been developed in order to diagnose a plug-cell plasma of the GAMMA10 tandem mirror. The system is configured as a heterodyne interferometer consisting of two separate oscillators and is also capable of applying to an electron cyclotron emission (ECE) measurement using a heterodyne receiver simultaneously. The optical system designed using the Gaussian-beam propagation theory and a ray-tracing code produces a diffraction-limited image of plasmas.

Detailed distributions of plasmas in the plug region are essential for studies of potential formation and of particle and energy transport in a tandem mirror. Time evolution of radial and axial line-densities and ECE profiles of the plug-cell plasma are successfully measured with the imaging system. The density profiles and their change due to an injection of the plug ECRH power provide information about potential confinement in GAMMA10. The heating efficiency of the ECRH and the density profiles of hot electrons up to 50-100 keV are obtained by the radial profiles of the ECE. The phase resolution of the interferometer is estimated to be less than 1/200 fringe, and the vertical and horizontal spatial resolutions of the optical system are estimated to be 21.0 and 10.6 mm, respectively.

The correlation analyses are applied to the imaging system to ascertain the informations about frequency and wave-number spectrum of the fluctuation components. Low-frequency density fluctuations such as a rotational mode and a drift-wave mode are
observed in the signals. The results of correlation analyses indicate that these fluctuations excited at a certain position of GAMMA10 rotate rigidly with the plasma. Numerical simulations are performed for detailed explanations on frequency spectrum and localization of the fluctuations from the interferometric data.

In order to apply the imaging system to large fusion devices, the novel detector for the ECE imaging diagnostics using a monolithic microwave integrated circuit technology is newly designed and fabricated. The heterodyne conversion loss of the detector is estimated to be 13.8 dB at the heterodyne intermediate frequency of 3 GHz and the local oscillator (LO) frequency of 70 GHz. It is noted that the rather flat response up to 10 GHz is obtained for the LO frequencies of 70 and 90 GHz in contrast to a hybrid detector using a beam-lead SBD, which is probably due to the method of housing semiconductor chips as well as due to the small inductance of the signal line and the performance of the SBD. These are quite attractive for the measurement of spatially resolved temperature fluctuations to understand turbulent-induced energy transport. The first ECE measurement on GAMMA10 using the monolithic detector is also performed.
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Chapter 1
Introduction

Nuclear fusion has been considered to be the ultimate energy resource instead of fossil fuels, since the fuel for the fusion reactor is obtained from sea water for an essentially infinite time. A fusion reactor is not only an inexhaustible power supply, but also an environmentally safer system in comparison with a nuclear fission reactor, since the nuclear fusion reaction stops immediately when a fuel is cut off and the reactor will be operated without discharging greenhouse gases into the atmosphere. In order to realize such an ideal fusion reactor, the study of controlled thermonuclear fusion was started about 40 years ago. Recently, tokamak plasmas have achieved the break even, which means that the fusion output power is equal to the heating power.

There are several ways to confine high energy plasmas for nuclear fusion, as shown in Fig. 1-1. The GAMMA10 tandem mirror is categorized to an open ended system in magnetic confinement. The mirror configuration is one of the most promising methods among the other candidates for fusion reactors because it is geometrically simple and can hold a high plasma pressure.

In order to improve the axial confinement, which is the main problem for open ended system, the idea of tandem mirror was proposed in 1976 [1,2]. In GAMMA10, the improvement of axial confinement has been attained by the formation of confining potentials during the injection of electron-cyclotron-resonance heating (ECRH). Although the mechanism of confining potential formation has been studied by
theoretical and experimental approaches [3,4], the distribution of the ion confining potential (plug potential) has not been clearly understood yet.

The purpose of this study is to develop millimeter-wave two-dimensional (2D) imaging array in order to obtain an information of 2D or 3D plasma profiles of electron density and electron cyclotron emission (ECE) with good spatial and temporal resolutions. The detailed distribution of a plasma around the plug region is essential for the study of the potential formation mechanism and of the particle and energy transports. The millimeter-wave 2D imaging array is also useful for the measurement of temperature fluctuations and heat transport in toroidal devices.

In this Chapter, a brief history and a review of millimeter-wave imaging diagnostics are described in Sec. 1.1 and the organization of the thesis is presented in
Sec. 1.2.

1.1 Millimeter-Wave Imaging Diagnostics

It is important to measure plasma density and temperature profiles for evaluation of plasma performance. One of the powerful non-perturbing diagnostics is the use of electromagnetic waves as a probe into a plasma or receiving electromagnetic wave radiations from a plasma. Recently, millimeter-wave imaging diagnostics have been developed for measurements of 2D/3D plasma density and/or temperature profiles, because multichannel detector array in millimeter to sub-millimeter regions is available due to the progress in millimeter-wave device technology. Millimeter-wave imaging techniques become more important in remote sensing, atmospheric radiometry, radio astronomy and radars as well as in plasma diagnostics, since they provide good spatial resolution compared to long-wavelength microwaves and are less affected by atmospheric conditions than infrared-to-visible systems.

1.1.1 Phase-Imaging Interferometry

For density profile measurement, several diagnostics such as interferometry, Thomson scattering and reflectometry are utilized [5]. Above all, multichannel interferometry is installed as a standard diagnostic method in many devices [6,7]. In order to extend multichannel interferometry, phase-imaging interferometry has been developed, in which a single set of optics with detector array is used instead of a multichannel optical path with a single detector for each chord.

The first experiment of phase-imaging interferometry was applied to a resistive arc plasma in 1982 [8]. The CO$_2$-laser through the plasma was focused onto a cooled
(77°K), one-dimensional 15-element PbSnTe detector array by a set of gold-coated parabolic mirror. In 1985, a 20-channel far-infrared (FIR) imaging interferometer system was used to obtain density profiles in the UCLA Microtor tokamak. A 36-element monolithic integrated microbolometer detector array provided diffraction-limited performance in the FIR region [9].

In millimeter-wave region, phase-imaging interferometer using an 11-element detector array with beam-lead type GaAs Schottky barrier diodes (SBDs) was developed for the study of density profiles of the GAMMA10 tandem mirror in 1991 [10]. The 2D phase imaging interferometer consisting of 16 elements of beam-lead type GaAs SBDs has been developed [11] and successfully measured time evolution of the radial and axial density profiles of the plug-cell plasma of GAMMA10 [12,13].

1.1.2 Electron Cyclotron Emission Imaging

An ECE is a radiation from a magnetized plasma at an electron cyclotron frequency, which is proportional to the magnetic field strength. If the ECE is well absorbed and the plasma is in a local thermal equilibrium, the intensity of the ECE equals to an electron temperature. The measurement of ECE becomes a common diagnostic method to obtain information about a local electron temperature in fusion experiments.

To measure an ECE signal, several methods are widely used, that is, a heterodyne radiometer, a Fourier transform spectrometer, a grating polychromator, etc. Since each method has an advantage and a disadvantage, the ECE measurement is usually performed with more than two methods. As a result of progress in computational
techniques such as computer tomography, the ECE measurement has been used to the study of MHD instability [14]. The study of temperature fluctuations and related heat transport using correlation radiometry has been progressed in TEXT-U [15].

To measure spatially resolved electron temperature fluctuations is one of the key to understanding turbulent-induced energy transport. The ECE imaging is a method whereby 2D image of turbulent temperature fluctuations may be obtained as well as 2D electron temperature profiles. The first experimental data of the ECE imaging was reported at the 7th international Toki conference in 1995 [12,16].

1.2 Organization of the Thesis

Chapter 2 presents the principles and techniques of millimeter-wave imaging diagnostics. The Chapter begins with a brief overview of electromagnetic wave propagation in a cold plasma. Then the phase-imaging interferometry and the reconstruction method using Abel inversion technique are shown. Finally, the basic principles of ECE are described.

Chapter 3 describes overview of the GAMMA10 device together with the brief descriptions of heating systems. Then, the several diagnostics related to this thesis are introduced briefly.

Chapter 4 describes details of imaging array. First, the 2D detector array and optics for the imaging array are described. Then the phase detection system for the phase-imaging interferometer and heterodyne radiometer system for ECE imaging are described. Finally, the simulation experiment of the imaging array system to confirm the ability of this system is also presented.
Chapter 5 describes novel monolithic-type GaAs Schottky diode detector having antenna and built-in amplifier on a GaAs substrate, which is newly fabricated using monolithic microwave integrated circuit (MMIC) technology. The result of actual implementation on GAMMA10 is also described.

Chapter 6 describes the experimental result obtained with the imaging array, which consists of density profiles, ECE and their fluctuation measurements. The effect of the high energy electrons on the ECE and the meaning of the fluctuation component in the line-integrated signals are also discussed.

Chapter 7 describes the density profiles and confinement in the GAMMA10 tandem mirror. The relation between the plug-cell plasma and the central-cell plasma are discussed from the standpoint of the density and/or density profiles in each cell of GAMMA10.

Chapter 8 summarized the results of the thesis research.
Chapter 2

Principles of Millimeter-Wave Imaging

It is important to measure plasma density and temperature profiles without perturbation for an evaluation of plasma performance. One of the non-perturbing diagnostics is using an electromagnetic wave as a probe into a plasma or receiving electromagnetic wave radiations from the plasma. The density profile can be provided by several diagnostic methods such as multichannel interferometry, Thomson scattering and reflectometry [5].

Interferometry utilizes the measurements for the refractive index of plasmas to determine electron density. On the other hand, radiometry utilizes measurements of an ECE from plasmas to determine electron temperature. Therefore, their imaging diagnostics have features that are able to measure distributions of electron density, temperature, and fluctuation components. They are quite important for studies of instabilities and anomalous transports caused by density and/or temperature fluctuations as well as for basic physics and machine operations.

The propagation of electromagnetic waves in a plasma is quite different in a vacuum due to the presence of magnetic field. Therefore, we will start with a brief review of the general problem of wave propagation in a magnetized plasma before describing the principle of interferometry. An Abel inversion technique for deducing the radial distribution from chordal interferometric measurements is then presented. The basic principle of ECE measurements, which has been discussed extensively in earlier
studies [17,18], is introduced in the case of ideal conditions. Finally, the basic of imaging diagnostics including optical theory is also described.

2.1 Propagation of Electromagnetic Waves in a magnetized Plasma

The way in which waves propagate in a magnetized plasma is rather more complicated than that in most other media because a magnetic field causes electrical properties to be highly anisotropic. The basic field equations (Maxwell’s equations) in an equilibrium plasma are

\[ \nabla \times E = -\frac{\partial B}{\partial t}, \]  

\[ \nabla \times B = \mu_0 j + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}, \]  

where \( B, E \) are magnetic and electric field respectively, and \( j \) is the conduction current density, and \( \varepsilon_0 \) and \( \mu_0 \) are the dielectric constant and magnetic permeability of free space respectively. We can eliminate \( B \) by using Eqs. (2.1) and (2.2) to get

\[ \nabla \times (\nabla \times E) + \frac{\partial}{\partial t} \left( \mu_0 j + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t} \right) = 0 . \]  

For present purposes it turns out that the simplest of treatments is adequate. The reason for this is that we shall be concerned with waves traveling at phase velocities close to the speed of light in plasmas whose thermal electron speed is \( v_t \ll c \). Therefore, we are able to ignore thermal particle motions and adopt what is called the cold plasma
approximation.

In the cold plasma approximation, the effect of electron collision and pressure are negligible. Therefore the equation of motion of a single electron in a static magnetic field $B_0$ in the direction of $z$ axis is

$$m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e(E + \mathbf{v}_e \times B_0), \quad (2.4)$$

where $m_e$, $-e$ and $\mathbf{v}_e$ are the electron mass, electron charge and electron velocity, respectively. The three components of Eq. (2.4) are solved in terms of $E$ as follows,

$$\mathbf{v}_x = \frac{-ie}{m_e \omega(1 - \omega_{ce}^2 / \omega^2)} \left( -E_x - \frac{i\omega_{ce}}{\omega} E_y \right),$$

$$\mathbf{v}_y = \frac{-ie}{m_e \omega(1 - \omega_{ce}^2 / \omega^2)} \left( \frac{i\omega_{ce}}{\omega} E_x - E_y \right),$$

$$\mathbf{v}_z = \frac{-ie}{m_e \omega(1 - \omega_{ce}^2 / \omega^2)} \left( -1 + \frac{\omega_{ce}^2}{\omega^2} \right) E_z,$$

where $\omega_{ce} \equiv eB_0 / m_e$ is the electron cyclotron frequency. The current density in the cold plasma approximation is written by using Eq. (2.5),

$$j = -en \mathbf{v}_e = \mathbf{\sigma} \cdot \mathbf{E}, \quad (2.6)$$

where
\[
\sigma = \frac{ie^2 n_e}{m_e \omega (1 - \omega^2 / \omega^2)} \begin{pmatrix}
-1 & -i \omega_c \omega & 0 \\
\frac{i \omega_c \omega}{\omega} & -1 & 0 \\
0 & 0 & -1 + \frac{\omega_c^2}{\omega^2}
\end{pmatrix}.
\] (2.7)

When the wave number, the direction of propagation, is in the \(y-z\) plane of a rectangular coordinate system as shown in Fig. 2-1, a plane wave with the angular frequency \(\omega\) and the wave number \(k\) is written as

\[
E(r, t) = E_0 \exp \left( i \omega t - k \cdot r \right).
\] (2.8)

Introducing Eqs. (2.6) and (2.8) into Eq. (2.3), and noting \(\varepsilon_0 \mu_0 = 1 / c^2\), we obtain

\[
k \times (k \times E) + \frac{\omega^2}{c^2} \varepsilon \cdot E = 0.
\] (2.9)

---

Figure 2-1 Coordinate system for the wave propagation in a magnetized plasma.
The dielectric tensor $\varepsilon$ is written by

$$
\varepsilon = I + \frac{\sigma}{i\omega \varepsilon_0} = 
\begin{bmatrix}
1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} & -i\frac{\omega_{ce} \omega_{pe}}{\omega(\omega^2 - \omega_{ce}^2)} & 0 \\
\frac{i\omega_{ce} \omega_{pe}}{\omega(\omega^2 - \omega_{ce}^2)} & 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} & 0 \\
0 & 0 & 1 - \frac{\omega_{pe}^2}{\omega^2}
\end{bmatrix},
$$

(2.10)

where $\omega_{pe} \equiv (n_e e^2 / \varepsilon_0 m_e)^{1/2}$ denotes the electron plasma frequency. Equation (2.9) is written with the refractive index, $N = ck/\omega$, as

$$
N \times (N \times \mathbf{E}) + \varepsilon \cdot \mathbf{E} = 0
$$

(2.11)

or

$$
N(N \cdot \mathbf{E}) - N^2 \mathbf{E} + \varepsilon \cdot \mathbf{E} = 0.
$$

(2.12)

Using $\theta$ and $k$, where $\theta$ is the angle between $k$ and $B_0$, and $k = k \cdot (0, \sin \theta, \cos \theta)$ as shown in Fig. 2-1, Eq. (2.12) can be expanded explicitly as

$$
\begin{bmatrix}
1 - N^2 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} & -i\frac{\omega_{ce} \omega_{pe}}{\omega(\omega^2 - \omega_{ce}^2)} & 0 \\
\frac{i\omega_{ce} \omega_{pe}}{\omega(\omega^2 - \omega_{ce}^2)} & 1 - N^2 \cos^2 \theta - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} & N^2 \sin \theta \cos \theta \\
0 & N^2 \sin \theta \cos \theta & 1 - N^2 \sin^2 \theta - \frac{\omega_{pe}^2}{\omega^2}
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix} = 0.
$$

(2.13)

Now Eq. (2.13) represents three homogeneous simultaneous equations for the three components of $\mathbf{E}$ ($\neq 0$). In order for these components to have a nonzero solution, the determinant of the matrix of coefficients must be zero. The solutions are written in the form
\[
N_k^2 = 1 - \frac{\omega_{pe}^2}{\omega^2} \left( \frac{1 - \omega_{pe}^2}{\omega^2} \right) \left\{ \frac{\omega_{ce}^2}{\omega^2} \sin^2 \theta \right\}^2 \pm \left\{ \left( \frac{\omega_{ce}^2}{2\omega^2} \right)^2 + \left( 1 - \frac{\omega_{pe}^2}{\omega^2} \right) ^2 \frac{\omega_{ce}^2}{\omega^2} \cos^2 \theta \right\}^{1/2}. \tag{2.14}
\]

This expression is called the Appleton-Hartree formula for the refractive index.

When the electromagnetic waves propagate perpendicular to the magnetic field, \( \theta = \pi/2 \), the solutions are

\[
N_k^2 = 1 - \frac{\omega_{pe}^2}{\omega^2} \tag{2.15}
\]

or

\[
N_k^2 = 1 - \frac{\omega_{pe}^2}{\omega^2} \cdot \frac{\omega^2 - \omega_{pe}^2}{\omega^2 - \omega_{pe}^2 - \omega_{ce}^2}, \tag{2.16}
\]

for which the characteristic polarizations of the electric field are

\[
E_x = E_y = 0 \quad (N = N_c) \tag{2.17}
\]

and

\[
\frac{E_x}{E_y} = \frac{i\omega \left( \omega^2 - \omega_{pe}^2 - \omega_{ce}^2 \right)}{\omega_{pe}^2 \omega_{ce}}, \quad E_z = 0, \quad (N = N_e) \tag{2.18}
\]

respectively. The wave which has only \( E_z \) component is called the ordinary mode (O-mode), which corresponds to the positive sign in Eq. \(2.14\). The other solution is called the extraordinary mode (X-mode). [5]
2.2 Interferometry

Measurements of the refractive index of any medium are most often made by some form of interferometry. For the measurement of a plasma density, a Michelson interferometer configuration and a Mach-Zehnder interferometer configuration are commonly used as shown schematically in Fig. 2-2.

![Figure 2-2](image_url) Configuration of (a) a Michelson interferometer and (b) a Mach-Zehnder interferometer.

An O-mode in which the effect of magnetic field can be neglected is generally used for the measurement of the refractive index. From Eq. (2.15), we can write the refractive index as

\[ N^2 = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{n_e}{n_c}, \]  

(2.19)

where \( n_c = \frac{\omega_p^2 m \epsilon_0}{e^2} \) is called the cutoff density.
Now, we consider a situation in which the refractive index is measured by the Mach-Zehnder interferometer with one beam passing through the plasma as shown in Fig. 2-3. The phase difference $\phi(x)$ between the two arms (Fig. 2-2 (b)) is

$$\phi(x) = \int_{y_1}^{y_2} (k_0 - k_p) \, dy = \frac{2\pi}{\lambda} \int_{y_1}^{y_2} (1 - N) \, dy,$$

(2.20)

where $k_0 = \frac{2\pi}{\lambda_0}$ and $k_p$ are the wave numbers of the incident beam in the vacuum and plasma, respectively, and $y_2-y_1$ is the path length inside the plasma. If $n_e \ll n_c$ is satisfied, Eq. (2.19) can be expanded to $N = 1 - \frac{1}{2}(n_e/n_c)$ with good approximation. Using this value and an assumption of cylindrical symmetry, the phase difference simplifies to

$$\phi(x) \equiv \frac{\pi}{\lambda n_c} \int_{y_1}^{y_2} n_e(r) \, dy = \frac{2\pi}{\lambda n_c} \int_{y_1}^{y_2} \frac{n_e(r)}{\sqrt{r^2 - x^2}} \, r \, dr, \quad r > x.$$

(2.21)

Figure 2-3 Schematic of the interferometer probe beam geometry, and the refractive effect of a plasma column.
We have to choose a wave length of the probe beam in considering refractive effects caused by the density gradient in the plasma cross section. Especially, when a detector array having many elements with close spacing is applied to the interferometer, the effect must be kept smaller than the distance between each element. The angle of refraction $\delta$ varies with the position of the probe beam $x$ shown in Fig. 2-3. For an axisymmetric parabolic density profile with a center density $n_e(0)$, the maximum value of $\delta_m$ is given to a good approximation [19] by

$$\delta_m \approx \sin^{-1}\left(\frac{n_e(0)}{n_c}\right) = \frac{n_e(0)}{n_c}.$$  \hspace{1cm} (2.22)

Its maximum value is for a distance from the center equal to about 0.7 times the plasma radius.

### 2.3 Abel Inversion Technique

The measured phase difference is proportional not only to the local density but to so called line-integrated density (line-density). The derivation of local electron density from chordal interferometric measurements using a scanning type interferometer, a multi-channel interferometer and a phase-imaging interferometer is performed by the Abel transform.

When we consider an electron density as a cylindrically symmetric quantity, the integral equation for $n_e(r)$ written in Eq. (2.21) can be transformed to that for $\phi(x)$ as

$$n_e(r) = -\frac{\lambda n_c}{\pi^2} \int^x d\phi(x) \frac{1}{\sqrt{x^2 - r^2}} dx, \; x > r.$$  \hspace{1cm} (2.23)
If the approximate function of the electron density profile can be determined by the fitting process of line-density along each path, the solution of Eq. (2.23) will be obtained analytically. On the other hand, the numerical inversion method with a symmetric approximation is studied for the application to the plasma, for which the fitting function is not easy to find [20]. However, when the electron density is an asymmetric quantity, Eq. (2.21) can not be solved by the analytical approach. Therefore, the asymmetric Abel transforms [20-23], which are the numerical approach, are commonly used.

In the present experiment, most plasmas in GAMMA10 can be considered to be a cylindrical plasma. We use two symmetric Abel inversion methods [see Appendix], one is numerical method and the other is analytical one, in order to confirm the accuracy of the inverted density profile.

2.4 Electron Cyclotron Emission Radiometry

In the beginning, we consider the transport of radiation emitted from plasmas. The equation of transfer through the plasma using first order of Wentzel-Kramers-Brillouin (WKB) approximation is given by

\[
\frac{dI_\omega}{ds} = \eta_\omega - \alpha_\omega I_\omega,
\]

where \( s \) is the space coordinate along the ray-path, \( I_\omega \) is the specific intensity of the radiation, \( \eta_\omega \) and \( \alpha_\omega \) are the emission and absorption coefficients respectively. When we define the optical depth \( \tau \) and the source function \( S_\omega \) by

\[
d\tau = -\alpha_\omega ds
\]

\[
(2.24)
\]

\[
(2.25)
\]
and

\[ S_\omega = \frac{n_\omega}{\alpha_\omega}, \quad (2.26) \]

Equation (2.24) becomes

\[ \frac{dI_\omega}{d\tau} = I_\omega - S_\omega \quad (2.27) \]

The solution of this equation for the intensity at some point \( A \) on the ray in terms of the intensity at another point \( B \) in Fig. 2-4 is

\[ I_\omega(A) e^{-\tau(A)} = I_\omega(B) e^{-\tau(B)} + \int_{\tau(A)}^{\tau(B)} S_\omega e^{-\tau} d\tau, \quad (2.28) \]

where \( \tau(A) \) and \( \tau(B) \) are the total optical depths from an observation point \( O \) to the point \( A \) and \( B \), respectively. Integration of definition (2.25) leads

Figure 2-4 A ray passing through the plasma. The optical depth \( \tau \) is measured from \( A \), the point of emergence of the ray.
\[ \tau = \int d\tau = -\int_0^a \alpha_\omega d\tau, \quad (2.29) \]

In order to calculate the intensity of emergent radiation, we assume the plasma boundary as shown in Fig 2-4, that is, A is the emergence point of the ray from the plasma and B is the entry point of the ray into the plasma. When diffusions and reflections can be neglected in a vacuum region, point A to O, we can denote \( \tau(A) = 0 \), \( I_\omega(A) = I_\omega(O) = I_\omega \), and \( \tau(B) = \tau_0 \), which is equal to the total optical depth of the plasma.

If incident intensity \( I_\omega(inc) \) is not exist, Eq. (2.28) for the observed intensity at point O along the ray path s becomes

\[ I_\omega = \int_0^\tau S_\omega(\tau) e^{-\tau} d\tau. \quad (2.30) \]

In the particularly important case of thermodynamic equilibrium, the source function \( S_\omega \) of Eq. (2.26) is related by Kirchhoff’s law,

\[ I_{boe} \equiv S_\omega = \frac{\omega^3}{8\pi^3 c^2} k_B T_e, \quad (2.31) \]

where \( k_B \) is Boltzmann’s constant and \( T_e \) is the electron temperature. Equation (2.31) is well known as the Rayleigh-Jeans blackbody intensity in an anisotropic medium, which is appropriate as long as quantum effects are small, \( \hbar \omega \ll T_e \). Since the source function \( S_\omega \) is no longer a function of \( \tau \), we obtain the result of integration Eq. (2.30)

\[ I_\omega = I_{boe} \left(1 - e^{-\tau_0}\right), \quad (2.32) \]

which is the specific intensity for ECE without reflection of the vacuum vessel wall.

The observed intensity of ECE is conditioned by the value of optical thickness \( \tau \)
defined by Eq. (2.29). The optical thickness of the O-mode and X-mode harmonics for perpendicular propagation is given by [24]

\[
\tau^{(O)}_n = \frac{\pi^2 n^{2(n-1)}}{2^{n-1}(n-1)!} \left(1 - \frac{\omega_{pe}^2}{n^2 \omega_{ce}^2}\right)^{n-1} \left(\frac{\omega_{pe}}{\omega_{ce}}\right)^{2n} \left(\frac{v_{te}}{c}\right)^{2n} \frac{1}{\lambda_0} \frac{B}{dB / ds},
\]

(2.33)

\[
\tau^{(X)}_n = \frac{\pi^2 n^{2(n-1)}}{2^{n-1}(n-1)!} \left(\frac{\omega_{pe}}{\omega_{ce}}\right)^{2(n-1)} \left(\frac{v_{te}}{c}\right)^{2(n-1)} \frac{1}{\lambda_0} \frac{B}{dB / ds},
\]

(2.34)

respectively, where \(\lambda_0 \equiv 2\pi c / \omega_{ce}\) and \(n \geq 2\).

The cyclotron emission can be considered as a summation of radiation emitted from a single electron and its frequency is related to the electron cyclotron frequency \(\omega_{ce}\). The observed frequency of ECE, which takes account of the relativistic mass change and its harmonics, is given by

\[
\omega_n = \frac{n\omega_{ce}}{1 - \beta^2} \sqrt{1 - \beta \eta} \cos \theta \quad (n = 1, 2, 3, \ldots),
\]

(2.35)

where \(\beta \equiv v_{te} / c\), \(v_{te} \equiv (k_B T_e/m_e)^{1/2}\) is the electron thermal velocity, \(c\) is the velocity of light, \(\beta\) denotes the parallel component of \(\beta\) to the magnetic field, and \(\theta\) is the angle between the magnetic field and the direction of the radiation propagation. The term \((1 - \beta^2)^{1/2}\) and \(\beta \eta \cos \theta\) come from the relativistic effect and the Doppler shift of the radiation, respectively.

The line-broadening mechanisms are as follows, (1) Doppler effect, (2) relativistic effect, (3) collisions, (4) radiation damping, and (5) quantum effect. In consideration of the plasma parameters, the line-broadening width of ECE is mainly caused by (1) and
(2), which are given by

\[ \Delta \omega_n \equiv \sqrt{2\pi n} \omega_c \left( \frac{k_B T_e}{m_e c^2} \right)^{\frac{1}{2}} |\cos \theta| \]  

(2.36)

and

\[ \Delta \omega_n \equiv n^{\frac{3}{2}} \omega_c \left( \frac{k_B T_e}{m_e c^2} \right), \]  

(2.37)

respectively.

### 2.5 Millimeter-wave Imaging Diagnostics

The imaging technique for the measurement of plasmas has been stated from the visible wavelength range. The optics of the imaging system, constructed by mirrors, lenses and a detector array, is designed and evaluated by the optical theory. The brief review of the theory is described in this Section.

A lens acts as a low-pass filter having the spatial cutoff frequency \( f_0^E \) given by

\[ f_0^E = \frac{D}{2\lambda z'}, \]  

(2.38)

where \( D \) is the diameter of the objective lens, \( z' \) is the distance between the objective lens and image plane, and \( \lambda \) is the wave length in the substrate. The sampling spacing for diffraction-limited resolution is given by

\[ T_E = \frac{1}{2f_0^E} = F_{eff} \lambda, \]  

(2.39)

where \( F_{eff} = z' / D \) is the effective F number of the objective lens. The discrete data value
sampled at regular intervals $T_E$, $G_n = g(nT_E)$, can provide a band limited function using Whittaker-Shannon sampling theorem. The algorithm given by this theorem is

$$g(x) = \sum_{n=-\infty}^{\infty} G_n \frac{\sin \pi \left( x / T_E - n \right)}{\pi \left( x / T_E - n \right)}.$$ (2.40)

Two optical theories are used in order to find good alignment of the optical system within the detector array, which has many detectors at intervals $T_E$. One is the Gaussian beam propagation theory and the other is the geometric optical theory.

According to the Gaussian beam propagation theory, the spot-size $w(z)$ and curvature $R(z)$ at point $z$, which is the direction of wave propagation, are given by

$$w(z) = w_0 \sqrt{1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2}$$ (2.41)

and

$$R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right],$$ (2.42)

where $w_0$ is the minimum spot-size at $z = 0$, which point is called beam waist, and $\lambda$ is the wave length of the beam. The relation between focal length $f$ of the lens and curvature $R$ is given by

$$\frac{1}{R_z} = \frac{1}{R} - \frac{1}{f}.$$ (2.43)
Using Eqs. (2.41)-(2.43) under the condition in Fig. 2-5, the results of parameter conversion through the lens are

\[
\frac{1}{w_{02}^2} = \frac{1}{w_{01}^2} \left(1 - \frac{d_1}{f}\right)^2 + \frac{1}{f^2} \left(\frac{\pi w_{01}}{\lambda}\right)^2, \tag{2.44}
\]

\[
d_2 = f + (d_1 - f) \frac{f^2}{(d_1 - f)^2 + \left(\frac{\pi w_{01}}{\lambda}\right)^2}. \tag{2.45}
\]

On the other hand, the geometric optical theory gives the simple relation between the refractive index of the material and refractive angle of the incident ray. The relation, which is well known as the Snell’s law, is given by

\[
N_1 \sin \theta_1 = N_2 \sin \theta_2, \tag{2.46}
\]

where \(N_1, N_2\) are the refractive indices of the media, and \(\theta_1, \theta_2\) are the incident angle and refractive angle respectively.
Chapter 3
Description of the GAMMA10 Device

The GAMMA10 is a tandem mirror device with an effectively axisymmetric magnetic configuration at Plasma Research Center, University of Tsukuba. The validity of a tandem mirror with thermal barriers as to the improvement of axial confinement has been confirmed. In recent experiments, moreover, neutrons attributed to the deuterium-deuterium (D-D) fusion reaction at the temperature of above 10 keV are observed [25]. In this Chapter, the overview of the GAMMA10 device is presented including the description of heating systems and diagnostics.

3.1 Overview of the GAMMA10 Tandem Mirror

The GAMMA10 consists of four cell regions, that is, a central cell, anchor cells, plug/barrier cells, and end regions are arranged straight as shown in Fig. 3-1, which shows the coil system associated with the magnetic flux tube, magnetic field distribution and the expected axial potential distribution along with the $z$ axis. The total length of GAMMA10 is about 27 m and the total volume of the vacuum vessel is about 170 m$^3$. In order to improve the wall condition, steady ECR discharge cleaning is applied before experimental term. The base pressure of $7 \times 10^{-8}$ Torr in the central cell is maintained by the pumping system, which consists of six turbo-molecular pumps, eight helium cryopumps and four liquid helium cooled cryopanels.

The central cell consists of 12 circular coils. The magnetic field strength of the
midplane \((z = 0 \, \text{m})\) is 0.405 T in the standard operation, which can be varied from 0.3 to 0.57 T, and the mirror ratio is 5. The length and diameter of the vacuum vessel in the central cell are 6 and 1 m, respectively. A fixed limiter with a radius 0.18 m is installed at \(z = 0.33 \, \text{m}\) in the central cell.

The anchor cells are formed by a set of baseball coils creating minimum-\(B\) field for MHD stability with recircularizing coils for axisymmetrized magnetic field. This configuration is effective in cancellation of the geodesic curvature effect. The magnetic field strength is 0.61 T and mirror ratio in the minimum-\(B\) mirror region is 3.

The plug/barrier cells consist of the axisymmetric mirror coils. These cells are used to create an ion confining potential (plug potential) and thermal barrier potential.
The magnetic fields in the plug and barrier regions are 1.0 and 0.5 T respectively, which correspond to the fundamental and second-harmonic resonance points for ECRH at 28 GHz respectively.

In the end regions, the end plates are installed in front of both end walls in order to control the plasma potential. The end plates, can be segmented radially and azimuthally, are electrically grounded or floated to the machine ground. In the standard operation, the end plates, segmented radially and connected azimuthally, are floating through 1 MΩ resister to reduce non-ambipolar diffusion. Figure 3-2 shows the schematic of the plug/barrier cell and end region.

Figure 3-2  Schematic of plug/barrier cell and end region.
3.2 Heating Systems

A GAMMA10 plasma is produced and sustained by the following way in standard operations. Initial plasmas, produced by the Magneto-plasma-dynamic (MPD) plasma guns installed at both ends, are injected into the central cell along the magnetic-field line. Then Ion Cyclotron Range of Frequency (ICRF) heating systems together with fueling by the gas puffing systems sustain a target plasma. Two separate ECRH powers installed in the plug/barrier cells are applied to the RF heated plasma in order to create confining potentials. Figure 3-3 shows the arrangement of above-mentioned heating systems in GAMMA10.

ICRF (Ion Cyclotron Range of Frequency)

Two types of ICRF antennas are installed in the central cell, one is so-called Nagoya type-III antenna installed at $z = \pm 2.2 \text{ m}$ for RF1 system and the other is conventional double-half-turn antenna installed at $z = \pm 1.7 \text{ m}$ for RF2 system. The RF1
and RF2 systems have independent oscillators which generate the frequency range from 6.2 to 30 MHz and from 4.6 to 9.9 MHz respectively, and the maximum output power of 1 MW with 500 ms pulse duration. In the standard operations, the RF1 antennas excite ICRF fast waves at the frequency of about 10 MHz which propagate into the anchor cells in order to produce a high beta plasma for MHD stability [26]. Then, the fast waves are converted into slow waves and heat anchor ions [27]. The RF2 antennas excite ICRF slow waves at the frequency of 6.36 MHz that heat central cell ions.

**ECRH (Electron Cyclotron Resonance Heating)**

The plug/barrier ECRH systems consist of four gyrotrons at the frequency of 28 GHz and maximum output power of 150 kW. The plug ECRHs (fundamental resonance) are applied to form the ion confining potential by producing warm electrons, and the barrier ECRHs (second-harmonic resonance) are applied to deepen the thermal barrier potential by producing mirror-trapped hot electrons.

The additional second-harmonic ECRH system is applied to the central cell at the frequency of 28 GHz and maximum output power of 150 kW. The aim of this system is the heating of electrons in the central cell so as to suppress the ion energy loss induced by the electron drag [28].

**NBI (Neutral Beam Injection)**

Two types of NBI systems are installed in the plug/barrier cells, one is called sloshing NBI system and the other is called pumping NBI system. The sloshing NBI system injects a neutral hydrogen beam at the angle of 41 degree to the magnetic-field line. This system produces sloshing ion distribution at plug/barrier region in order to
form ion confining potential efficiently. The pumping NBI system also injects a neutral hydrogen beam at the angle of 30 degree. This system is used to sustain the confining potential by driving cold ions out of the thermal barrier by a charge exchange process.

3.3 Diagnostics

In the GAMMA10 tandem mirror, several diagnostics are applied to the measurement of plasma parameters such as density, temperature, potential, etc. Table 3-1 shows the list of main diagnostics in contrast to the plasma parameter. In this Section, several diagnostics relating to the thesis are introduced briefly.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Diagnostics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Density</td>
<td>Imaging System, Interferometer, Reflectometer, Langmuir Probe</td>
</tr>
<tr>
<td>Electron Temperature</td>
<td>Soft X-ray Detector, ECE Radiometer, Imaging System</td>
</tr>
<tr>
<td>Ion Temperature</td>
<td>Charge Exchange Neutral Particle Analyzer (CX-NPA)</td>
</tr>
<tr>
<td>Potential</td>
<td>Heavy Ion Beam Probe (HIBP), Langmuir Probe</td>
</tr>
<tr>
<td>End Loss Particle</td>
<td>Slanted-Grid End-Loss Analyzer (SELA)</td>
</tr>
<tr>
<td></td>
<td>End Loss Energy Component Analyzer (ELECA)</td>
</tr>
<tr>
<td></td>
<td>Loss Electron Diagnostics (LED) System</td>
</tr>
<tr>
<td>Plasma Pressure</td>
<td>Diamagnetic Loop</td>
</tr>
<tr>
<td>Radiation</td>
<td>Spectrometer</td>
</tr>
</tbody>
</table>

Table 3-1 List of diagnostics in the GAMMA10 tandem mirror.

**Microwave Interferometer**

The principle of the interferometer is described in the previous Chapter. In
GAMMA10, the line-density is measured with seven conventional microwave Mach-Zehnder interferometers, which are installed in the central cell, east and west (E/W) anchor cells, E/W barrier cells, east plug cell, and the central mirror throat between the central cell and east anchor cell. Especially, in the central cell (z = –0.6 m), the measurement of chordal line-density profiles by a scanning-type interferometer enables us to deduce radial density profile using Abel inversion technique. The Abel inversion requires 10-20 shot with good reproducibility for the central cell plasma.

**Reflectometer** [29-31]

Reflectometry is a measurement using the reflecting wave at the cutoff layer, which corresponds to the plasma density. Recently, various types of reflectometer are applied to the measurement of density profiles and/or fluctuation distributions, because it has high time and spatial resolutions.

In GAMMA10, the broadband frequency modulation (FM) reflectometers with fast swept oscillators are applied to the central cell (z = 0 m) and west barrier cell (z = 8.6 m) for the measurement of density profiles and fluctuations. The reflectometers using yttrium-iron-garnet (YIG) oscillator-based continuous wave (CW) sources are installed in the central cell at three axial positions, z = 0, 0.6, 1.8 m, and two azimuthal positions at z = 0 m for the density and magnetic fluctuation measurements. The low-frequency (1-50 kHz) and high-frequency (5-10 MHz) fluctuations caused by instabilities are observed.

**FD (Fraunhofer-Diffraction) Method** [32,33]

The FD method is so-called far-forward scattering, and is used to measure (k, ω)
spectra of long-wavelength density fluctuations. The system consists of a set of lens and
1D detector array, and installed in the central cell of GAMMA10 \((z = -2.4 \text{ m})\). An
IMPATT oscillator with 500 mW output at 70 GHz provides a probe beam injected into
the center of a plasma through a focusing fused-quartz lens. The frequency shifted FD
signal and the unshifted transmitted wave are focused via another lens onto a detector
array, which consists of beam-lead type GaAs SBDs bonded to bow-tie antennas
fabricated on a fused-quartz substrate.

The intermediate frequency (IF) signals measured with eight mixers are analyzed
by calculating the short-time fast Fourier transform (SFFT) in one shot. The dispersion
relation of the fluctuation is obtained from the FD intensity profile at each frequency.

**SELA (Slanted-grid End-Loss Analyzer) [34]**

SELA is one of diagnostics to measure the end-loss-ion currents. The detector
consists of some biased grids in order to avoid the effect of high energy electrons and to
observe selected ion energy in the range of 0 to 3.6 keV. The SELA arrays, which are
straightly arranged 5 SELAs in each array, are installed at both ends of GAMMA10
between the end plate and vacuum vessel. The radial profiles in the direction of the
center to bottom and of the center to south are observed by four independent arrays in
one shot. The measured radial position by the arrays corresponds to \(r_c = 2.6, 5.3, 8.2,
11.2, 14.6 \text{ cm}\), where \(r_c\) denotes the radial position mapping to the central cell.

**Soft X-ray Detector [35]**

Several types of x-ray diagnostic system are installed in GAMMA10 in order to
diagnose the electron temperature and its distribution, such as x-ray pulse-height
analyzer (PHA) and x-ray absorption method using semiconductor and microchannel-plate (MCP) detectors. The PHA and absorption methods are applied to the plug/barrier and central cell plasmas for the measurement of energy spectra and temperature profiles, respectively. In GAMMA10, a few tens plasma shots are necessary in order to evaluate the energy spectrum, while the absorption method requires less than 10 plasma shots.

**HIBP (Heavy Ion Beam Probe) [36]**

The radial profiles of plasma potential in the central cell ($z = 1.2$ m) and barrier midplane ($z = -8.8$ m) are measured with HIBP using neutral Au$^0$-beam. An incident beam accelerated up to 16 kV can be swept radially across the plasma using an electrostatic deflector. Ions discharged by the plasma along with the beam orbit, Au$^+$, are detected by the electrostatic energy analyzer of a parallel plate type with micro-channel plate. The energy and time resolutions of the system are less than 50 V and 200 µs, respectively.

**CX-NPA (Charge eXchange Neutral Particle Analyzer) [37]**

CX-NPA is used for the measurement of ion temperature using charge exchange reaction with neutral hydrogen atoms. The system is installed in the central cell ($z = -0.6$ m), which is the same axial position as the scanning interferometer. The viewing angle of the system can be changed from −8.0 to +8.0 degrees, which almost covers the whole cross section of the plasma. At each angle, the energy distribution of CX-neutrals up to 30 keV is observed. The ion temperature profile is calculated by the chordal data.
obtained by radial scan together with the density profile, plasma pressure, $H_\alpha$ profile and $T_e$ profile.
Chapter 4  
Millimeter-Wave Imaging Array System

A millimeter-wave 2D imaging array system has been developed in order to diagnose the plug-cell plasma of the GAMMA10 tandem mirror. The imaging system is installed in the west plug cell ($z = 969$ cm), in which the plasma varies axially as well as radially caused by the variation of magnetic field and the formation of confining potential as shown in Fig. 4-1.

![Graph showing magnetic field strength and estimated space potential distribution](image)

**Figure 4-1** Magnetic field strength and estimated space potential distribution in the plug/barrier cell during the formation of the confining potentials. Open and closed circle means the space potential measured at the central cell and at the plug/barrier cell, respectively. The gray zone shows the axial field of view of the imaging system.
The field of view and spatial resolution of the imaging system are determined by the performance of the optics and detector array. Therefore, each component of the system is carefully designed and fabricated. In this Chapter, details of the imaging array system consisting of a 2D detector array, optics, a phase detection circuit, and a heterodyne radiometer for ECE are described. The ability of the imaging array system is also presented.

4.1 Overview of the Imaging System

The schematic of the 2D imaging system is shown in Fig. 4-2. The system is configured as a heterodyne interferometer consisting of two separate 70 GHz oscillators. An IMPATT oscillator with 500 mW output provides a probe beam injected into a plasma and a low-power reference signal. The other IMPATT oscillator with 500 mW output is used as a local oscillator (LO), which is also divided into two signals. One of the LO output and the probe beam are superimposed coaxially by a beam splitter, and focused on the detector array to produce a 150 MHz IF signal. The branches from the two sources are also combined by a directional coupler and fed to a mixer to provide another IF signal. The quadrature-type detection system provides the phase difference between two IF signals which is proportional to line-density of a plasma.

The 2D imaging system is also applied to the ECE measurement using a heterodyne receiver. The second-harmonic ECE signals in the ordinary mode are mixed with LO power on the detector array. The ECE imaging in the ordinary mode and the phase imaging can be obtained simultaneously.
4.2 Two-Dimensional Detector Array

The layout of the 2D detector array is shown in Fig. 4-3. It consists of 16 elements of beam-lead type GaAs Schottky barrier diodes [38] bonded to $4 \times 4$ bow-tie antennas which is monolithically fabricated on a fused-quartz substrate of $38.1 \times 38.1 \times 1$ mm$^3$. The bow angle of $60^\circ$ is chosen to give the resistive antenna impedance of 150 $\Omega$ for the best matching to the detector. In order to minimize direct pick-up from additional heating power, the array is installed inside an aluminum box for electrical shielding, and a 2D pyramidal horn antenna array in TE$_{10}$ mode as shown in Fig. 4-4 is attached to the input of the detector.
Figure 4-3  Layout of the detector array. (a) mask pattern of the detector, (b) magnification of one channel of the mask pattern which is located at bottom right.

Figure 4-4  Layout of the antenna array which is configured as a waveguide in TE\textsubscript{10} mode. (a) array structure on the detector side, (b) horizontal cross section of the antenna array. The cutoff frequency of the waveguide is 48.4 GHz.
The each detector is biased at bias current 200 \( \mu \text{A} \) by a constant-current biasing circuit as shown in Fig. 4-5. In order to avoid the influence of ripple noise, the biasing circuit works by using batteries. The IF signal of each detector is separated with the DC output by a bias tee, and fed to the phase detection system or ECE detection system described in the following Section.

![Image of Circuit Diagram](image)

**Figure 4-5** Circuit diagram of the bias circuit.

### 4.3 Optical System

The optical system is designed using Gaussian-beam propagation theory and ray-tracing code. The scalar feed horn produces an axisymmetric radiation pattern with low sidelobes, which is well fitted by Gaussian distribution. The probe beam is expanded by the off-axis parabolic mirror installed inside the vacuum vessel to cover upper-half of a plasma. This enables the axial viewing chord larger than the size of the vacuum window. The cross section of the probe beam is more than \( 200 \times 200 \text{ mm}^2 \) at the plasma center. On the receiver side, an ellipsoidal mirror and polyethylene lenses focus an image of a plasma onto the 2D detector array.
4.3.1 Optical System for Incident Wave

The optics is utilized to the interferometer in order to expand the probe beam. The matching between horn and optics is very important to obtain a suitable parallel beam. At first, the radiation patterns are compared between conventional conical horn and the scalar feed horn [39] as shown in Fig. 4-6. It is clear that the radiation pattern obtained from the scalar feed horn is more circular and well fitted to the theory than that obtained by the conical horn.

![Figure 4-6](image)

Radiation pattern of the scalar feed horn in comparison with that of the conical horn. The solid line shows the calculated one using the Gaussian beam propagation theory.

The focus of the off-axis parabolic mirror corresponds to the beam waist of the scalar feed horn in order to obtain the best matching. The performance of the optics is confirmed by the measurement of beam profiles as shown in Fig. 4-7. The beam width at each point seems to be almost same. The difference of the half width at half maximum (HWHM) between horizontal and vertical planes comes from the tilt of the mirrors for 30 degree.
4.3.2 Optical System for Received Wave

The receiving optics focuses plasma images, that is, the phase and ECE images, onto the detector array, and determines the performance such as field of view, magnification and spatial resolution. This optics consists of two reflecting mirrors and two polyethylene lenses installed inside and outside of the vacuum vessel, respectively. The magnification and spatial resolution can be varied by changing the outside lenses.

Figures 4-8 and 4-9 show the results of ray-tracing from a point source as an example. The object between plane mirror and first lens is vacuum window made of Teflon plate with 30 mm thick. Since most of rays from the point source will be focused onto the detector position, this optics is adequate for the imaging system.
Figure 4-8  Side view of the rays calculated by the ray-tracing code. The vertical axis of the coordinate corresponds to the radial position of the plasma. The position –70 mm corresponds to the plasma center.
Figure 4-9  Top view of the rays calculated by the ray-tracing code. The vertical axis of the coordinate corresponds to the axial plasma position. The position 0 mm corresponds to $z = 969$ cm.
In order to confirm the performance of actual optical system, the Airy pattern of the point source is measured as shown in Fig. 4-10, and is in good agreement with the optics theory as

\[
I(\rho) = \left( \frac{2J_1(\pi \rho / F_{\text{eff}} \lambda)}{\pi \rho / F_{\text{eff}} \lambda} \right)^2,
\]  

(4.1)

where \( J_1 \) is Bessel function of the first kind, \( \rho \) is the distance from the image point and \( F_{\text{eff}} \) is the effective \( F \) number defined by Eq. (2.39). The magnification of this optical system is designed to be 0.33, which is also in good agreement with the experimental result at the test stand.

A dichroic plate [40], made of 10 mm thick aluminum with 4.0 mm diameter hole and 4.5 mm spacing as shown in Fig. 4-11, is attached to the vacuum window of the receiver side. The circular holes act as highpass filters to prevent the mixture of ECRH.
power with frequency of 28 GHz and the size of the hole is determined by the cutoff frequency of a circular waveguide in TE\(_{11}\) mode propagation.

4.4 Phase Detection System

The IF outputs are fed to the quadrature-type phase detection system as shown in Fig. 4-12. The detector array and reference detector provide IF signals, which are \(\sin(\omega_{\text{IF}} t + \phi)\) and \(\sin(\omega_{\text{IF}} t)\) respectively, where \(\omega_{\text{IF}} / 2\pi = 150\) MHz and \(\phi\) is the phase change due to the plasma density. The one IF signal from the detector array is fed to a
power splitter with 90° phase shift between the two ports to provide \( \sin(\omega_{IF}t + \phi) \) and \( \cos(\omega_{IF}t + \phi) \). The reference IF signal is divided into 8 channels. Then IF mixer outputs are fed to the low-pass filters, and give the DC signal components of \( \sin(\phi) \) and \( \cos(\phi) \). The line-density is numerically obtained by calculating \( \arctan(\sin\phi / \cos\phi) \).

![Block diagram of the phase detection system.](image)

**4.5 ECE Heterodyne Radiometer System**

Since the magnetic field strength varies from 1.0 to 1.5 T across the field of view of the imaging system as shown in Fig. 4-1, the frequency of the second-harmonic \( (2\omega_{ce}) \) ECE also varies from 56 to 84 GHz. Especially, the frequency at the center of the optics \( (z = 969 \text{ cm}) \) is nearly 70 GHz, at which the local oscillator of the interferometer is working. Therefore, the 2D imaging system can also be applied to the ECE measurement using a heterodyne radiometer. The \( 2\omega_{ce} \) ECE signals in the O/X-mode are mixed with LO power on the detector array.
The electrical scheme of the ECE detection system is shown in Fig. 4-13. The IF signals from the detector array are amplified by 43 dB, and then fed to the band pass filters with passband of 100 ± 10 MHz. The selected IF signals are amplified by 43 dB again, then finally square-detected. The outputs of the detectors are sent to CAMAC digitizer.

The ECE imaging in the O-mode and the phase imaging can be obtained simultaneously. In case of the measurement of the ECE in the X-mode, the detector array and the LO horn have to be rotated 90 degree in order to change the polarization of electric field.

Figure 4-13 Block diagram of the ECE detection system.
4.6 Simulation Experiment of the Imaging Array System

In order to demonstrate the ability of the imaging array, a dielectric phase object is used for the test. The experimental results of imaging a dielectric phase object are compared with the theoretical curve \([9]\) as

\[
\phi(\rho) = \sqrt{\frac{2}{\pi}} \phi_0 \left\{ S_i[2\pi f_0^E (\rho + a)] - S_i[2\pi f_0^E (\rho - a)] \right\}, \tag{4.2}
\]

where \(\rho\) is the distance from the image point, \(2a\) is the width of the object, \(f_0^E\) is the spatial cutoff frequency given by Eq. (2.38), and \(\phi_0\) and \(S_i(x)\) are the sine integral and pulse height functions given by

\[
\phi_0 = \sqrt{2\pi(N - 1)} \frac{t}{\lambda_0}, \tag{4.3}
\]

\[
S_i(x) = \int_0^x \frac{\sin t}{t} dt, \tag{4.4}
\]

respectively, where \(N\) and \(t\) are the refractive index and thickness of the object respectively, and \(\lambda_0\) is the wavelength in the vacuum. This theoretical curve of the phase distribution is obtained by the result of following scheme, Fourier-transforming the spatial pulse function into the frequency domain, filtering out all frequency components above \(f_0^E\), and then transforming back to the spatial domain. Figure 4-14 shows the image of the Teflon tile target as it is moved across the object plane which corresponds to the plasma center. There is a good agreement between theory and experiment, and the image of the target is moved according to the movement across the object plane.
Figure 4-14  Image of a Teflon tile (10.1 × 10.1 × 1.5 mm³) measured in (a) horizontal and (b) vertical directions.
Chapter 5
Development of Monolithic Microwave Integrated Circuit Detector

ECE imaging has recently been applied for measurements of electron temperature profile and temperature fluctuations in a tokamak plasma. We are planning to apply the 2D imaging array system to Large Helical Device (LHD) in the National Institute for Fusion Science (NIFS), Japan [41]. The frequency of the second-harmonic ECE on LHD ranges from 70 to 100 GHz in the initial phase experiment, from 140 to 200 GHz in phase I, and from 190 to 260 GHz in phase II, respectively. In order to cover above frequency range, the novel detector using MMIC technology is newly designed and fabricated. The characteristics of the MMIC detector and the result of the first application to the GAMMA10 plasma are also described in this Chapter.

5.1 Design of the MMIC detector

For the measurement of ECE in large devices such as LHD, there are several difficulties using present HMIC (hybrid microwave integrated circuit) detector, that is, low cutoff frequency of beam-lead SBD, transmission loss of IF signal caused by the inductance of bonding wire, etc. In order to overcome these restrictions, the novel detector is designed as shown in Fig 5-1 [42]. It consists of the integration of a bow-tie antenna, down-converting mixer using a SBD, and heterojunction bipolar transistors (HBTs) on a GaAs substrate. The HBT works as an IF amplifier with 10 dB voltage gain.
and 12 GHz bandwidth. The bow angle of 60° is chosen to have the resistive antenna impedance of 150 Ω, which is the same value as the present hybrid-type detector.

The MMIC detector is fabricated using multi-chamber Molecular Beam Epitaxy (MBE) systems. The GaAs chip with size of 4.0 × 2.0 × 0.625 mm$^3$ is mounted on the case made of gold-plated brass together with alumina substrate for dc bias of the SBD and HBT amplifier. A coplanar waveguide on the other substrate is used for the connection of the IF output to the SMA connector.

Table 5-1 shows the measured SBD characteristics of the MMIC and HMIC (SBL-221) detectors [38]. The cutoff frequency is estimated by

$$f_c = \frac{1}{2\pi R_c C_f},$$

(5.1)
where $R_T$ is the series resistance and $C_T$ is the total capacitance of the SBD. It is noted that the cutoff frequency of the MMIC detector is improved three times higher than that of the HMIC detector.

<table>
<thead>
<tr>
<th></th>
<th>MMIC</th>
<th>Hybrid (SBL-221)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series Resistance ($R_T$)</td>
<td>4.0 Ω</td>
<td>5.0 Ω</td>
</tr>
<tr>
<td>Total Capacitance ($C_T$)</td>
<td>31 fF</td>
<td>70 fF</td>
</tr>
<tr>
<td>Cutoff Frequency ($f_C$)</td>
<td>1300 GHz</td>
<td>450 GHz</td>
</tr>
</tbody>
</table>

Table 5-1 Characteristics of the SBD.

5.2 Characteristics of the MMIC detector

The characteristics of the MMIC detector are evaluated in the test stand using two 70 or 90 GHz sources, that is, two klystron oscillators (68-72 GHz) and two Gunn oscillators (84-90 and 90-100 GHz) as shown in Fig 5-2. One oscillator is used as a LO signal and the other as a radio frequency (rf) signal. Two signals are combined by a directional coupler, and radiated to the detector through a waveguide antenna. The dc bias of the SBD is 0.82 V, 0.4 mA and the power supply of the HBT amplifier is 4.87 V, 30 mA. The heterodyne IF signal is fed to a spectrum analyzer.
Figure 5-3 shows the IF signal level measured with the spectrum analyzer. Note that rather flat response from 0.2 to 9 GHz is obtained in contrast to the hybrid detector using beam-lead SBDs, which is probably due to the method of housing of the semiconductor chip as well as due to the small inductance of the signal line and the performance of the SBD. The lower frequency response of the HBT amplifier is limited

![Diagram of test stand for characterization of the detector.]

Figure 5-2 Schematic of test stand for characterization of the detector.

Figure 5-3 Heterodyne characteristics of the monolithic detector as a function of IF at 70 and 90 GHz. The levels of LO and rf are kept to –6.6 and –15.6 dBm at 70 GHz, and kept to –6.9 and –11.6 dBm at 90 GHz.
by the capacitance, which is inserted in order to isolate between the IF signal and dc bias for the SBD.

Figure 5-4 shows the measurement of the conversion loss, $L_c = -10 \log \left( \frac{P_{rf}}{P_{IF}} \right)$, plotted as a function of LO power, where $P_{rf}$ is the rf power and $P_{IF}$ is the resultant IF power. The absolute value of the single sideband (SSB) conversion loss is estimated to be $13.8 \pm 3$ dB at IF = 3 GHz and LO = 70 GHz. This value can be compared with that of other monolithic detector for ECE imaging diagnostic [43].

![Conversion Loss Graph](image)

Figure 5-4 Conversion loss of the monolithic detector as a function of LO power.

### 5.3 Application of the MMIC Detector to ECE Measurement

This novel detector is applied to the ECE measurement on GAMMA10. The instrument of the ECE radiometer is modified from the interferometer as shown in Fig. 5-5. One branch from an IMPATT oscillator to a plasma, which is used as a probe beam for the interferometer, is interrupted by an attenuator (ATT), and the other branch is used as a LO source. The IF output of the MMIC detector is fed to 4-channel filter-bank.
system, which includes IF amplifiers with voltage gain of 70 dB, 4-channel power splitter, band-pass filters, and detectors, as shown in Fig. 5-6.

![Diagram of a filter-bank system for ECE measurement](image)

**Figure 5-5** Schematic of the ECE radiometer.

![Diagram of MMIC detector and bias](image)

**Figure 5-6** Filter-bank system for ECE measurement.

MMIC Detector → Diode bias 0.82 V

AMP 34dB 0.5~8GHz → Amp bias 30 mA

AMP 36dB 0.5~8GHz → 4ch PS

Filter (CF=1GHz) Detector

Filter (CF=2GHz) Detector

Filter (CF=3GHz) Detector

Filter (CF=4GHz) Detector

Band width ± 150 MHz
Figure 5-7 shows the time evolution of the ECE signal measured by the heterodyne receiver using the MMIC detector together with measured by the present imaging system using the HMIC detector. The LO frequency is fixed at 70 GHz, and IF is 3 and 4 GHz. The rf frequency corresponds to the second harmonics of the electron cyclotron frequency. It is seen that the signal level is enhanced when the ECRH power is applied. The broadband frequency spectrum up to 4 GHz is observed, probably due to the relativistic broadening of hot electrons produced by the ECRH and to the variation of the magnetic field across the observation region. The wideband characteristic of the detector is quite convenient for the application of correlation radiometry as well as the measurement of broadband spectrum with fixed LO frequency.
Figure 5-7  Time evolution of the ECE signal measured by the present imaging array system and the heterodyne receiver with the MMIC detector.
Chapter 6
Experimental Results of the Imaging Array System

The millimeter-wave imaging array system is applied to the plug-cell plasma of the GAMMA10 tandem mirror, and measures time evolution of radial and axial line-densities and ECE profiles in one plasma shot. This feature is quite attractive for studies of particle and heat transports, potential formation, and fluctuation distribution in a plasma.

In this Chapter, the experimental results of the density and ECE profile measurements using the imaging system are described. The fluctuation spectra of the signals obtained by this system are compared with that obtained by other diagnostics such as far-forward scattering (Fraunhofer Diffraction: FD) method and reflectometry.

6.1 Density Profile Measurements

In GAMMA10, the sequence of the standard plasma operation is as follows. The ICRF powers with frequencies of 9.9 MHz and 10.3 MHz (RF1), and 6.36 MHz (RF2) are employed to buildup a plasma and heat ions following gun-produced plasma injection. In the plug/barrier cell, two separate 28 GHz gyrotrons, fundamental ECRH at the plug region (P-ECRH) and second-harmonic ECRH at the barrier region (B-ECRH), produce warm and hot electrons which are necessary for effective formation of confining (plug) and thermal barrier potentials.
Figure 6-1 shows the time evolution of the typical line-density profile measured with the imaging system in one shot together with the time sequence of the heating systems. It is noted that the density of the plug-cell plasma decreases during the injection of the ECRH power ($t = 120$-$170$ ms). When the ECRH power is applied, the plug potential is created near the position of $z = 962$ cm where the magnetic field strength equals to 1 T. At the region of $z \geq 962$ cm where the imaging system is installed, the loss particles (density) will decrease due to the formation of the plug potential.

![Figure 6-1](image)

**Figure 6-1** Time evolution of the line-density profile with the P-ECRH.

On the other hand, time evolution of the line-density profile without injection of the P-ECRH power is shown in Fig. 6-2. The difference of the sequence between Fig. 6-1 and Fig. 6-2 is only that the P-ECRH power is applied or not, that is, the plug potential exists or not. The effect of the B-ECRH on the density profile seems to be neglected.
As mentioned in the above Chapter, the Abel inversion provides radial density profile from line-density profile measured with the imaging system. Figures 6-3 and 6-4 show the calculated density profiles at several times, before \((t = 80, 110 \text{ ms})\), during \((t = 130, 150 \text{ ms})\), and after \((t = 170, 190 \text{ ms})\) injection of the P-ECRH power, in two plasma shots. The changes of the profiles due to the ECRH power are very clear at any times. Especially, a broadening of the profile coincides with a sudden decrease in the peak density immediately after the ECRH is turned on. When the ECRH is turned off, the confining potential disappears, and a short burst appears in the line-density corresponding to the axial drain of plasmas. These results are consistent with the measurement of end-loss-fluxes using the ELA array at both ends.

Figure 6-2  Time evolution of the line-density profile without the P-ECRH.
Figure 6-3  Abel-inverted density profiles measured at $t = 80, 110, 130$ ms. Left and right rows correspond to the plasma shots as shown in Figs. 6-2 and 6-1, respectively.
Figure 6-4  Abel-inverted density profiles measured at $t = 150, 170, 190$ ms. Left and right rows correspond to the plasma shots as shown in Figs. 6-2 and 6-1, respectively.
The density profiles in other plasma sequences display different behaviors. In order to mention an example, the time evolution of the plug-cell plasma with the NBI heating is shown in Fig. 6-5. In this plasma, the line-density starts to increase when the sloshing NBI (S-NBI) is injected into the RF heated plasma during $t = 130-150 \text{ ms}$. The reason why the density increases is that the particles provided by the NBI are mirror-trapped in the plug/barrier cell.

![Figure 6-5](image)

Figure 6-5  Time evolution of the line-density profile with the S-NBI.

In the present imaging system, the phase resolution of the interferometer estimated from the fringe noise of the trace is less than $1/200$ fringe, where $1$ fringe equals to the line-density of $5.2 \times 10^{13} \text{ cm}^{-2}$ at $70 \text{ GHz}$. The spatial resolution is given by the aperture size of the antenna array, $9.0 \times 4.5 \text{ mm}^2$, and the magnification of the optical system. The vertical resolution is $21.0 \text{ mm}$ and the horizontal one is $10.6 \text{ mm}$.
6.2 ECE Measurements

The imaging system is also applied to the measurement of $2\omega_\text{ce}$ ECE imaging in the X-mode and O-mode propagations. In the plug/barrier cell, warm and hot electrons are produced by P- and B-ECRHS in order to create the confining potentials. The distribution of heated electrons in the plug region will be obtained from the ECE imaging measurement.

According to Eq. (2.32) in the Chapter 2, the form of the ECE signal depends on the optical thickness given by Eqs. (2.33) and (2.34). Therefore, the optical thickness of the plug cell plasma has to be estimated first. Since the optical thickness is functions of local plasma density, temperature, and magnetic field strength, we can estimate the value of the optical thickness from a density profile measured with the imaging system assuming the ratio of electron temperature and density among bulk, warm, and hot electrons. Using Eqs. (2.33) and (2.34) under the adequate assumptions, the optical thickness of the $2\omega_\text{ce}$ ECE in each mode is estimated to be less than 1 in the present experiment. Therefore, the $2\omega_\text{ce}$ ECE intensity in each mode is given by

$$I_{2}^{(\text{X,O})} = I_{BO}^{2} \tau_{2}^{(\text{X,O})}.$$  

The intensity is not proportional only to electron temperature. In addition, the magnetic field strength in the plug cell is nearly constant in radial direction, that is, the frequency of the ECE does not change along the radial imaging chords. Therefore, the ECE signal emitted from an optically thin plasma is a line-integrated value.

Figure 6-6 shows the time evolution of the ECE signal measured at the central chord in the X-mode and O-mode propagations. It is mentioned that the ECE signal
continues to be observed after the bulk plasma disappears at \( t = 105 \text{ ms} \). This means that the observed ECE signals are mainly caused by mirror trapped hot electrons, since the life time of the hot electron plasma is much larger than that of the bulk or warm electron plasma as shown in the following equation [44],

\[
n_{eh} \tau_h [\text{cm}^{-3}, s] = 1.63 \times 10^{10} \left( \frac{m_e}{m_i} \right)^{1/2} T_{eh}^{3/2} \text{[keV]} \log_{10} R, \quad (6.2)
\]

where \( n_{eh} \) and \( T_{eh} \) are the hot electron density and temperature respectively, and \( m_i \) is the mass of an ion and \( R \) is the mirror ratio. The hot electron temperature measured by the x-ray pulse height analyzer rises up quickly and tended to saturate in a few ms in the range of 50-60 keV due to relativistic detuning of the \( 2 \omega_{ce} \) resonance for the localized ECRH power absorption [45,46]. Assuming the hot electron density of \( n_{eh} = 1 \times 10^{10} \text{ cm}^{-3} \), and using \( T_{eh} = 50 \text{ keV} \) and \( R = 6 \), we obtain \( \tau_h \sim 1 \text{ s} \) from Eq. (6.2).

\[\begin{array}{c}
\text{ECE Intensity (a.u.)} \\
\hline
0 & 100 & 200 & 300 & 400 & 500 & 600 & 700 \\
50 & 60 & 70 & 80 & 90 & 100 & 110 & 120 \\
\end{array}\]

**Figure 6-6** Time evolution of the ECE intensity in the X-mode and O-mode propagations.
However, the ECE signals in Fig. 6-6 do not last during the flat-top time of the magnetic field, \( t = 0.6 \) s, but decay abruptly at certain time. The signal reductions are observed with the occurrence of spikes as shown in Fig. 6-7, where the sampling time of the digitizer is chosen to be \( f_s = 400 \) kHz. The spike repeats until the emission decreases to low level. This phenomenon may be caused by some kinds of instability peculiar to the hot electrons.

![Image of ECE signal](image)

**Figure 6-7** Time evolution of the ECE signal obtained by using a high speed digitizer after the bulk plasma disappears.

The ECE signals emitted from an optically thin plasma are also utilized in order to determine the electron temperature from the ratio of ECE signals between the O-mode and X-mode propagations as [47]

\[
k_B T_e \equiv m_e c^2 \frac{I^{(O)}}{I^{(X)}}.
\]  

(6.3)
The electron temperature depends mostly on the emission from the hot electrons as shown in Fig. 6-8. The value of 50-100 keV is a little higher than that obtained from the x-ray measurement. The conceivable reasons of this high estimation of $T_{eh}$ are attributed to the mixing of the O-mode and X-mode emissions.

Such high temperature electrons up to 60 keV are produced by a combination of the P-ECRH and B-ECRH. Figure 6-9 shows the time evolution of the ECE profiles for various cases of ECRH application. It is noted that though the ECRH power is fixed in all cases, the ECE intensity for the simultaneous injection of the P-ECRH and B-ECRH is much larger than the sum of the intensity for the individual injection of each ECRH. This is due to the heating efficiency of the ECRH, which is shown as an average energy gain $\langle \Delta W_n \rangle$ using the $n$-th harmonic ECRH given by [48]
Figure 6-9  Time evolution of the radial ECE profiles measured with the imaging system in case of (a) B-ECRH application, (b) P-ECRH application and (c) the combined application of the P-ECRH and B-ECRH.
\[
\langle \Delta W_n \rangle = \frac{\pi e^2 L_B |E_\perp|^2}{m_e \omega_c v_i} \left( 1 - \frac{k_i v_i}{\omega} \right) J_{n+1}^2(\frac{k_i v_i}{\omega_c}),
\]

(6.4)

where \( E_\perp \) and \( L_B \) denote the electric field of right-polarized wave and the scale length of the magnetic field, respectively. Since the symbol \( J_n \) denotes the \( n \)-th Bessel function, the average energy gains due to P-ECRH \((n = 1)\) and B-ECRH \((n = 2)\) are proportional to \( J_0^2 \) and \( J_1^2 \), respectively. This indicates that P-ECRH and B-ECRH are suitable for heating of cold electrons and high energy electrons, respectively. Using the combined application of the P-ECRH and B-ECRH, the electrons are heated up to several keV by the P-ECRH and then heated up to 60 keV by the B-ECRH.

Now, we consider what is the main parameter related to the ECE intensity emitted from high energy electrons. Inserting the optical thickness given by Eqs. (2.33) and (2.34) to Eq. (6.1), we can obtain the ECE intensity in each mode as

\[
I_{2}^{(S)} \propto n_c T_e^2,
\]

(6.5)

\[
I_{2}^{(O)} \propto n_c T_e^3.
\]

(6.6)

As above mentioned, the hot electron temperature measured with the x-ray pulse height analyzer tends to saturate in a few ms in the plug/barrier cell. Therefore, the ECE intensity is proportional not to temperature but to density of hot electrons. In other words, the combined application of the P-ECRH and B-ECRH powers can produce hot electrons effectively. The dependence of the ECE intensity on the ECRH power is almost linear as shown in Fig. 6-10. This means that the hot electron density is proportional to the ECRH power. Therefore, our explanation for the ECE intensity is also supported by this result.
Figure 6-10  Power dependence of the ECE intensity.

Figure 6-11  Contour plot of the ECE intensity at $t = 150$ ms.
Figure 6-11 shows the contour plot of the ECE intensity measured with the imaging system at $t = 150$ ms. Since the LO frequency is fixed at 70 GHz, the ECE intensity is distributed around $z = 969.5$ cm where the frequency of the $2\omega_{ce}$ ECE equals to 70 GHz.

### 6.3 Fluctuation Measurements

According to Eq. (6.5), the fluctuation level of the $2\omega_{ce}$ ECE signal in the X-mode propagation is then expressed by

$$\frac{\delta I_2^{(X)}}{I_2^{(X)y}} = \frac{\delta n_e}{n_e} + 2\frac{\delta T_e}{T_e}.$$  \hspace{1cm} (6.7)

Since present imaging system can measure density and ECE profiles simultaneously, it can also be used to evaluate density and temperature fluctuations. Radial and axial distributions of the fluctuations obtained by correlation analysis are expected to give the information of wave number and/or mode number of the fluctuations [49].

First, the fluctuation spectrum of the $2\omega_{ce}$ ECE signal in the X-mode propagation is compared to that of the far-forward scattering signal which depends on the density fluctuations in the central-cell plasma as shown in Fig. 6-12. The peak frequency $\sim 7$ kHz is close to $\omega_k / 2\pi$, the rotation frequency. The wave number measured with the far-forward scattering method is estimated to be 0.1-0.2 cm$^{-1}$. The instability will be identified to be the rotational mode driven by the $E\times B$ rotation. It is noted that the power-law falloff of the spectrum $\delta I_2^{(X)} \propto \omega^{-2.3\pm0.2}$ for $\omega / 2\pi \leq 100$ kHz is almost identical with each other. This indicates that the low-frequency fluctuation in the ECE
The fluctuation components of the ECE signals measured at two axial positions are shown in Fig. 6-13. It is seen that the phase difference between two signals almost
equal to zero, that is, \( k_z \sim 0 \). Therefore, the wave number spectrum can be related to the frequency spectrum by

\[
I(\omega)d\omega = I(k)d\mathbf{k} = I(k)k_zdk_x. \tag{6.8}
\]

The fall-off of the wave number spectrum is estimated to be \(-3.3\). This value is also in agreement with the results of Ref. [50].

As stated above, the fluctuation spectrum measured at the plug cell is almost identical with that measured at the central cell. Therefore, the axial distributions of the low-frequency fluctuation measured with the imaging system such as a rotational mode (flute-like mode) and/or a drift-wave mode are compared with other diagnostics.

Figure 6-13 shows the time evolution of the far-forward scattering and reflectometer signals and their frequency spectra at the central and west-barrier cells respectively. Figure 6-15, on the other hand, shows the similar data of the line-density

![Graph showing ECE intensity over time for two axial positions](image-url)
signal measured with the imaging system.

Figure 6-14 Frequency spectra measured with the FD-method (top) and reflectometer (bottom) installed in the central cell and barrier cell, respectively.
Figure 6-15 Frequency spectra of the line-density signal measured with the imaging system at two radial positions. Channel (CH) 2 corresponds to the chord passing through the core plasma region, and CH 4 corresponds to the edge plasma region.
At all axial positions from the central cell to the plug cell, the strong fluctuation with frequency of 2.5 kHz and its higher modes are observed during the injection of the P-ECRH power \((t = 110-170 \text{ ms})\). According to the results of the Langmuir probes installed in the central cell \((z = 0.3 \text{ m})\), the fluctuations are identified as the rotational mode driven by the \(E \times B\) rotation with azimuthal mode number \(m = 2\).

Although the frequency is same in all cells, the radial electric field and magnetic field strength are, of course, different in each cell. This indicates that the fluctuations excited at a certain position of GAMMA10 rotate rigidly with the plasma, since the axial wave number \(k_z\) of a rotational mode is almost 0. In order to confirm the axial wave number, the correlation analyses are applied between two axial and radial channels as shown in Fig. 6-16.

Figure 6-16  Arrangement of the detector channels for correlation analyses. CH 2-4 and CH 1 measure the central chord \((x = 0 \text{ cm})\) along with \(z\)-axis and the chord at \(x = 6 \text{ cm}\), respectively.
The result of the correlation analysis using the line-density signals measured with the imaging system is shown in Fig. 6-17. The density fluctuations with frequency of 2.2 kHz and its harmonics are observed in both figures. These fluctuations seem to be the

![Graph showing fluctuation components, power spectral densities, coherence, and phase spectrum between two signals.](image)

**Figure 6-17** Fluctuation components of the line-density signals of CH2 and CH4 (upper). Power spectral densities of each signal (middle), coherence and phase spectrum between two signals (lower).
rotational mode, since the peak frequency is close to the rotation frequency, $\omega_r / 2\pi$, and varies when the potential distribution is changed in the central cell. The axial phase spectrum between two signals is almost 0, that is, $k_z \sim 0$, while the result of correlation

Figure 6-18 Fluctuation components of the line-density signal of CH 1 and CH 2 (upper). Power spectral densities of each signal (middle), coherence and phase spectrum between two signals (lower).
analysis between two radial positions shows the finite phase difference as shown in Fig. 6-18. This means that the fluctuation is rotating in the azimuthal direction due to the $E \times B$ rotation.

In the GAMMA10 tandem mirror, we observe not only a rotational mode but also a drift-wave mode due to the existence of density gradient. In order to increase the density in the central cell, the additional gas puffing is applied to the RF heated plasma. As a result of the gas puffing, the line-density in the central cell gradually increases up to $6.6 \times 10^{13}$ cm$^{-2}$, which is the increase of 50% of normal operation. After the injection of gas puffing, the strong drift-wave mode with frequency of 3.9 kHz is observed as shown in Fig. 6-19 (top). On the other hand, in the case of the P-ECRH application (bottom), the frequency of the fluctuation is shifted to 2.3 kHz during the injection of the ECRH power because of the Doppler shift due to the $E \times B$ rotation, i.e.,

$$v_p = \omega / k_\perp = |v_d - v_r|,$$  \hfill (6.9)

where $v_r$ is the rotational velocity and $v_d$ is the electron diamagnetic drift velocity, given by

$$v_d \equiv \frac{k_B T_e}{e B L_n},$$  \hfill (6.10)

for $(k_\perp \rho_i)^2 \ll 1$. Here, $L_n = -n_e / (\partial n_e / \partial r)$ is the density scale length and $\rho_i = (m k_B T_e / e^2 B^2)^{1/2}$. The frequency is returned to 3.9 kHz after the ECRH power is turned off.
Figure 6-19 Frequency spectra of the line-density signal measured with the imaging system using CH 2 detector. The additional gas puffing is applied to the central cell plasma during $t = 80-190$ ms. The P-ECRH power is also applied during $t = 130-180$ ms (bottom).
6.4 Discussion

(1) ECE measurement

In Section 2.4, the electron velocity distribution is assumed to be Maxwellian for the calculation of optical thickness, however, it is highly anisotropic in the plug-cell plasma. Moreover, we have to include the relativistic effects for such high energy electrons up to 60 keV. We will consider these effects in this Section.

According to the form of the ECE signal in Section 2.4, the anisotropic effect mainly appears in the emission and absorption coefficients $\eta_\omega$ and $\alpha_\omega$. For the ECE measurement using the imaging array system, the direction of the ECE propagation can be regarded to be nearly perpendicular to the magnetic field, since the imaging optics focuses the ECE source onto the detector array as shown in Fig. 4-2. We can calculate $\eta_\omega$ and $\alpha_\omega$ considering only the perpendicular component, because the parallel and perpendicular components of these coefficients are independent each other. Therefore, we can use the expression written in Section 2.4.

The relativistic effects due to the relativistic mass change are considered in deriving Eqs. (2.35) and (2.37). In order to determine the electron temperature using Eq. (6.3), the Maxwell energy distribution has to be assumed. We now estimate the discrepancy between two values with and without considering the relativistic effect for ECE of lower-harmonics ($n = 2$) and 50 keV temperature.

The relativistic effect causes contamination of lower-harmonics ECE due to the higher harmonics. Therefore, it is possible to account for the effect by multiplying the lowest-significant-order results by an exponential correction factor. In this way, the resulting expression is identical with the absorption coefficient obtained by expanding
the Bessel functions for quasi-perpendicular propagation by the permutation of [18]

\[ \alpha_n^{(O,X)} \rightarrow \alpha_n^{(O,X)} \exp \left[ \gamma_n^{(O,X)} \left( \frac{v_{te}}{c} \right)^2 z_n \right], \quad (6.11) \]

where \( \alpha_n \) is the absorption coefficient of the \( n \)-th harmonics and \( z_n \) is defined by

\[ z_n \equiv \left( \frac{c}{v_{te}} \right)^2 \frac{\omega_n \omega_{ce}}{\omega}. \quad (6.12) \]

Here, \( \gamma_n^{(O,X)} \) is obtained by

\[ \gamma_n^{(X)} = \frac{3}{4} (2n - 3) - \frac{2a_n}{1 + a_n} + \frac{2n^2}{2n + 3} \left( 1 + \frac{2}{n(1 + a_n)} \right) (N_{\perp}')^2, \quad (6.13) \]

\[ \gamma_n^{(O)} = \frac{3}{4} (2n - 1) + \frac{2n^2}{2n + 5} (N_{\perp}')^2, \quad (6.14) \]

for the X- and O-modes respectively, where \( N_{\perp}' \) is the real part of the refractive index and \( a_n \) is defined by

\[ a_n \equiv \frac{\left[ \omega_{pe} \omega_{ce} \right]^2}{n^2 - 1 - \left( \omega_{pe} / \omega_{ce} \right)^2}. \quad (6.15) \]

Using Eqs. (6.11)-(6.15) for the \( 2\omega_{ce} \) ECE of the plug-cell plasma, the final result of the relativistic effect on Eq. (6.3) is written by

\[ k_B T_e \simeq 0.956 \times m_e c^2 \frac{I_n^{(O)}}{I_n^{(X)}}. \quad (6.16) \]

The discrepancy between Eqs. (6.3) and (6.16) is a factor of 0.956 for the \( 2\omega_{ce} \) ECE of
the plug-cell plasma. As a matter of fact, the estimated temperature shown in Fig. 6-8 is calculated using Eq. (6.16).

(2) Fluctuation measurement

Interferometry is often utilized to fluctuation measurement as well as line-integrated density measurement. It is quite obvious to identify the existence of fluctuations from interferometric data, although we realize that it does not give the information about wave number at least. However, no detailed explanation has been made yet on frequency spectrum and localization of the fluctuations. In this Section, we discuss the simulation of fluctuation measurements using present imaging array system.

When we assume the fluctuations to have an azimuthal mode number $m = 1$ and a frequency $f$, the interferometric data along the central chord give the frequency of $2f$. It is because the line-integrated signal can not distinguish between the near-side and far-side of the detector as far as it observes along the central chord. The experimental results indicate the existence of a rotational mode with $m = 1$ as shown in Figs. 6-17 and 6-18. Therefore, we evaluate the fluctuation component in the line-integrated signal using a certain model.

The density profile is defined by the summation of mean density $n_0$ and fluctuation component $n_1$ as

$$n = n_0 + n_1,$$

while

$$n_0 = a_1 + a_2 r^2 + a_3 r^4 + a_4 r^6,$$
\[ n_1 = \frac{\hat{n}}{n} \exp \left( - \left( \frac{r \sin \omega t - y}{r \sin \theta} \right)^2 \right) \exp \left( - \left( \frac{r \cos \omega t - x}{r \sin \theta} \right)^2 \right). \] (6.19)

Here \( \hat{n}/n \) is the normalized fluctuation amplitude, the position is \( r = (x^2 + y^2)^{1/2} \), \( \omega/2\pi \) is the fluctuation frequency, and \( \theta \) means the existent area of the fluctuation as shown in Fig. 20(a). The parameters in Eqs. (6.17)-(6.19) such as \( a_1-a_4, \omega, \theta \) and \( \hat{n}/n \) are determined experimentally from Fig. 6-18. We can calculate the line-density including fluctuation component by using the model as shown in Fig. 6-20(b). Although, strictly speaking, this model is not a rotational mode with an azimuthal mode number \( m = 1 \), the line integrated value behaves as \( m = 1 \) fluctuation in this simulation. The correlation analysis is then applied to these signals in order to compare with the experimental data shown in Fig. 6-18.

![Fluctuation model of a rotational mode with an azimuthal mode number \( m = 1 \). (a) The gray zone means the existent area of the fluctuation. (b) The contour plot of the density profile calculated using Eqs. (6.17)-(6.19) at \( \omega t = \pi/6 \).](image-url)

Figure 6-20
A result of the simulation is shown in Fig. 6-21 using the values, $\tilde{n}/n = 0.6$, $\theta = \pi/6$, $\omega/2\pi = 2.2$ kHz, and including a random noise with amplitude of 0.001% of line-density. It is noted that the behaviors of the fluctuation components in the

![Figure 6-21](image_url)  
Figure 6-21  Fluctuation component in the line-density signal of CH 1 and CH 2 (upper). Power spectral densities of each signal (middle) and coherence and phase spectrum between two signals (lower).
line-integrated signals ($\tilde{n}l$) are similar to those observed in Fig. 6-18. The difference in the shape between two channels is caused by the integration of rotational data at different radial positions. This result is in good agreement with experiments.

According to the simulation, the frequency spectrum of the fluctuation with an azimuthal mode number $m = 1$ tends to lead some false peak like higher modes due to numerical error when we use the line-integrated data. Therefore, the evaluation of higher mode of the fluctuation must be compared carefully with that obtained by other diagnostics. On the other hand, in the case of $m = 2$, since the integration effect is canceled, the frequency spectrum seems to be more accurate. In the present configuration of the imaging array system, since the central chord observes a finite size of the plasma, the rotational mode of $m = 1$ can be observed to have the frequency of $\omega_E / 2\pi$.

The phase difference between two channels is more significant in the case of edge measurement using both sides of plasmas as shown in Fig. 6-22. Since channels 1 and 2 correspond to the positions $y = 10$ and $-10$ cm respectively, the phase difference between two channels is equivalent to $\pi$. Otherwise, when the amplitude of the fluctuation has a time dependence, the contribution of the fluctuation to the phase shift can be separated from near and from far side of the plasma. Figure 6-23 shows an example of this situation, that is, the amplitude $\tilde{n}/n$ changes from 0.6 to 0.12 in half rotation. The phase difference corresponds to the azimuthal angle between CH 1 and CH 2, $\pi/3$.

The imaging system covers upper half of the plasma using four channels and the magnitude of the observed fluctuation is continuously strong as shown in Figs. 6-17 and
6-18. This is why it is seldom that such a clear phase difference as shown in Figs. 6-22 and 6-23 can be measured in the experiments.

Figure 6-22 Fluctuation component of the line-density signal of CH 1 and CH 2, which corresponds to $y = 10$ and $-10$ cm respectively. Phase difference between two signals corresponds to the azimuthal angle of $\pi$. 

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Figure 6-23  Fluctuation component of the line-density signal of CH 1 and CH 2, which corresponds to $y = 0.01$ and 6 cm respectively. Phase difference between two signals corresponds to the azimuthal angle of $\pi/3$. 
Chapter 7
Density Profiles and Confinement in GAMMA10

Since GAMMA10 is configured as a tandem mirror, the distributions of plasma density in each cell are not independent but strongly affected with each other. In this Chapter, the relation among the density profiles at the plug cell and other cells will be discussed.

In recent experiments, an increase in the central-cell density and diamagnetic signal has been observed during the formation of a confining potential by the P-ECRH. This is probably due to the reduction of radial losses by the axisymmetrization of heating profiles [51]. The relation between the density profile at the plug cell and the radiation pattern of the P-ECRH is described in this Chapter.

7.1 Axial Density Profiles of a Tandem Mirror Plasma

Now, we consider detailed discharge process along with the time sequence of the standard operation. When the initial plasma produced by the MPD plasma guns is injected into the central cell along the magnetic-field line at \( t = 50.5-51.5 \) ms, the line-density in the plug/barrier cell responds as shown in Fig. 7-1. The density profile of the gun-produced plasma is successfully measured with the imaging system in one plasma shot as shown in Fig. 7-2. This improves inconvenience in the previous measurement using a probe and a single channel interferometer [52]. After the injection of the plasma guns, most plasma particles pass through the plug/barrier cell, and only a little amount
of plasma remains in the plug/barrier cell due to pitch-angle scattering.

Figure 7-1  Time evolution of the line-density observed at the central chord. The initial plasmas from the plasma guns are injected at \( t = 50.5-51.5 \) ms.

Figure 7-2  Abel-inverted density profile at \( t = 52.1 \) ms measured with the imaging system in one plasma shot.
When the RF1 heating power is applied to the central cell in order to sustain the plasma, the plasma is mainly confined between the east and west inner mirror throat located at $z = \pm 7.6$ m (see Figs. 3-1 and 4-1) by a simple mirror confinement. In this situation, if there is no heating mechanism in the plug/barrier region, the plasma through the inner mirror throat will be lost to the end wall without any trapping. In fact, the line-density at the plug cell is proportional to that at the central cell as shown in Fig. 7-3, and shows similar behavior to the end-loss ion current measured with the SELA as shown in Fig. 7-4.

Figure 7-3 Dependence of the line-density at the plug cell on that at the central cell.
In order to create the confining potentials, ECRH powers are applied to the plug/barrier cells. The line-density at the plug cell and the end-loss-ion current decrease after the formation of the confining potential and then gradually increase, as shown in Fig. 7-4. On the other hand, the line-densities measured at the central cell and barrier
midplane which are located at inner region of the confining potentials increase as a result of the improvement of axial particle confinement as shown in Fig. 7-5. The effect of the confining potential on plasma density profile will be discussed in the next Section.

![Figure 7-5](image)

Figure 7-5 Time evolution of the line-density measured at (a) the central cell and (b) the barrier cell.
When the ECRH power is turned off, the confining potential disappears quickly. The axial drain of confined plasma is then observed. Figure 7-6 shows the time evolution of line-density at the central cell. The lower traces show the magnification of decay phase of the line density at the central cell and plug cell. Since the decay time of the line-density at each cell is almost same, the axial particle confinement time $\tau_{p//}$ is considered to be much smaller than the radial one. The line-density at the plug cell is still strongly connected with that at the central cell.

The plasma density $n$ at the decay phase in Fig. 7-6 (a) can be written by

$$n = n_B + n_P \exp\left(-\frac{t}{\tau_{p//}}\right), \quad (7.1)$$

where $n_B$ and $n_P$ are the densities at $t \sim 205$ ms and 180 ms, respectively. In this plasma, the axial particle confinement time $\tau_{p//}$ is estimated to be 7-10 ms. On the other hand, the theoretical estimation of the particle confinement time using Fokker-Planck cord is given by the following equation [53],

$$\tau_{p//} = \frac{10^{11} C E_{i}^{3/2} \log R_{\text{eff}}}{n \ln \Lambda}, \quad (7.2)$$

where $C$ is the constant having 2-3.5, $E_i$ is the ion energy in keV, $\ln \Lambda$ is the Coulomb logarithm, and $R_{\text{eff}}$ is the effective mirror ratio defined by $R_{\text{eff}} = R / \{(1-\beta)^{1/2} (1 + \phi / E_i)\}$. Using experimental values of $n = 2.2 \times 10^{12}$ cm$^{-3}$, $E_i = 2.9$ keV, and $R_{\text{eff}} = 6$, the axial particle confinement time is estimated to be 16.8 ms. This value is in good agreement with the experiment.
Figure 7-6 (a) Time evolution of the line-density at the central cell. (b) and (c) show the magnification of the line-density during $t = 180-205$ ms in Fig. (a) at central cell and plug cell, respectively.
7.2 Confining Potential Effect on Plasma Density Profile

In the previous Section, the improvement of the axial particle confinement due to the formation of confining potential is described. When the ECRH power is applied to the plug/barrier cell, the density profile is considerably changed as shown in Fig. 7-7. In the plug cell, the density decreases corresponding to the formation of the confining potential in most cases, but the density profile during the injection of the ECRH ($t = 120$ ms) is quite different from that shown in Fig. 6-3. The response at the central cell in each case is also different. The density increase due to the injection of the ECRH power is not observed clearly in the previous experiments.

The main difference between these shots is a way to inject the P-ECRH power. The radiation pattern of the P-ECRH wave becomes more axisymmetric by using a reflecting mirror attached to the Vlasov-Nakajima antenna. Figure 7-8 shows the radiation pattern of the P-ECRH together with the density profile at the plug cell. It is noted that the density profile in the core region ($r \leq 45$ mm) decreases during the injection of the ECRH power. This region corresponds to that of the effective confining potential which coincides to the radial profile of the P-ECRH power deposition. These results indicate that the formation of the effective confining potential needs a suitable ECRH power distribution, and the plug density and/or density profile as a target plasma of the ECRH is also important to create the confining potential.
Figure 7-7  Abel-inverted density profiles in (a) the plug cell and (b) the central cell measured at before, during, and after the injection of the ECRH.
The radial distribution of potentially confined plasmas can be evaluated from the discrepancy between two profiles, with and without the ECRH power. Figure 7-9 shows the subtracted values between two profiles measured at \( t = 160 \) and \( 175 \) ms. The peak position of the axial drain of potentially confined plasmas is located at the center just after the P-ECRH power is applied, however, it gradually shift to the periphery at the later time. Since the radial distribution of the plug potential measured with the SELA

\[ \text{Figure 7-8 Radiation pattern of the P-ECRH (bottom) after optimization together with the Abel-inverted density profiles at the plug cell (top). Each contour line shows the 10\% reduction from the maximum ECRH power.} \]
array peaks at the central chord, it seems that there are some kinds of diffusion mechanism in the central cell during the formation of confining potential.

![Graph showing the difference in density profiles](image)

**Figure 7-9** Difference of the density profiles measured at $t = 160$ (during) and 175 ms (after). The radial position in the plug cell is mapping into the central cell.

The axial and radial distributions of the line-density profile measured with the 2D imaging system will give more interesting information about the potential formation mechanism. Figure 7-10 shows the 2D line-density profiles obtained before and after the ECRH application, respectively. The variation of the profile in the $z$ direction is caused by the change of the magnetic field as shown in Fig. 4-1. This is the first result of the 2D phase-imaging interferometry.
Figure 7-10 Contour plot of the line-density profiles before and after the ECRH power application. The labels on each contour line show the value of line-density ($\times 10^{12}$ cm$^{-2}$).
7.3 Discussion

The relations of plasma densities between the central cell and the plug cell are discussed in this Chapter. In the GAMMA10 tandem mirror, the movable limiters (MLO) are installed at \( z = \pm 7.06 \text{ m} \) in order to restrict the plasma radius within suitable size. Therefore, we have to take into account of the effect of the MLO to the plug density. Figure 7-11 shows the dependence of the line-density at the plug cell on that at the central cell for three radial positions of the MLO. The line-density at the plug cell depends on the position of the MLO clearly, since the axial flowing plasma is restricted by the MLO.

![Figure 7-11](image_url)

**Figure 7-11** The plug-cell line-density versus the central cell one. Limiter positions of \( r = 86, 96, 101 \text{ mm} \) correspond to \( r_{cc} = 13.8, 11.6, 10.6 \text{ cm} \) respectively mapping into the central cell.
Admittedly, the MLO reduces flow plasma, but the plug cell plasma is influenced by the edge plasma in the central cell masked with the MLO as in the example given below. In the central cell, the $2\omega_k$ ECRH (C-ECRH) system is applied to RF heated plasma in order to suppress the ion energy loss induced by the electron drag [28]. The Abel-inverted density profile obtained by the imaging system at the plug cell is shown in Fig. 7-12 together with that obtained by the scanning interferometer at the central cell. It is observed that the density profile broadens when the C-ECRH is applied at $t = 115$ ms. This broadening is also observed in the central-cell plasma as shown in Fig. 7-12(b). The application of the C-ECRH power contributes to the ionization of the edge region as well as the electron heating of the plasma. The density profiles at the plug cell and central cell agree with each other assuming that the profiles are conserved along the magnetic flux tube. For the formation of the effective confining potential using the P-ECRH, the detailed information of the plug cell plasma as a target seems to be more important.
Figure 7-12 Abel-inverted density profile at (a) the plug cell and (b) the central cell. The C-ECRH power is injected at $t = 115-145$ ms.
Chapter 8
Conclusions

In summary, we have developed a millimeter-wave 2D imaging array system for measurements of line-densities and ECE profiles in the GAMMA10 tandem mirror. The optical system consisting of parabolic and ellipsoidal mirrors installed inside the vacuum vessel is effective to ease the restriction of the view by the vacuum windows. The diffraction-limited image of a dielectric target is confirmed by the simulation experiment.

The phase-imaging interferometer successfully measures the time evolution of the 3D line-density profiles of the plug-cell plasma. It gives the information of potential confinement as well as density profiles in the GAMMA10 tandem mirror, such as axial density profiles and change of density profiles due to the formation of confining potential by the P-ECRH application.

The imaging system also measures the second-harmonic ECE profiles simultaneously. The hot electron temperature is estimated from the ratio of ECE signals between the O-mode and X-mode propagations. The radial profiles of the ECE indicate the information of heating efficiency of ECRH and hot electron density.

The fluctuation spectra of the signals obtained by the imaging array system are compared with those obtained by other diagnostics such as far-forward scattering method and reflectometry. The low-frequency density fluctuations such as a rotational mode and a drift-wave mode are observed in both signals. The wave numbers of the
fluctuations are estimated by the correlation analyses. We also establish the detailed
explanation on frequency spectrum and localization of the fluctuations from the
interferometric data by the numerical simulations.

The heterodyne characteristics of a new monolithic detector are evaluated for
various values of intermediate frequency and LO power. The response up to 10 GHz is
confirmed and the low LO power requirement is identified. These are quite attractive to
the measurement of ECE in large plasma devices. The first ECE measurement on
GAMMA10 using the monolithic detector is succeeded.

Using the imaging array system, further precise data about the relation between
plasma density and confining potential will make it possible to study more detailed
mechanism of potential formation.
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References


[18] M. Bornatici, R. Cano, O. De Barbieri, and F. Engelmann,


     Y. Harada and K. Honda, Proceedings of the Symposium on Polymeric
     Materials for Electronic Packaging & High Technology Applications, 1988

[39] P. F. Goldsmith, in Infrared and Millimeter Waves, Vol. 6,


[41] J. Fujita and the LHD Diagnostics Group, Diagnostics for Experimental
     Thermonuclear Fusion Reactors, edited by P. E. Stott et. al. (Plenum


Appendix

Abel Inversion Methods

The brief descriptions about Abel inversion technique are described in Sec. 2.3. In this Appendix, the specification of two inversion methods is presented.

In GAMMA10, line-density profiles at the central cell are well fitted to the following function as

\[ nl(x) = n_0 + n_1x^2 + n_2x^4 + n_3x^6 + n_4x^8, \]  

(A.1)

where \( x \) is the vertical axis shown in Fig. A-1. Using Eq. (2.23), we can obtain the Abel-inverted density profile \( n_e(r) \) written by

\[ n_e(r) = -\frac{2}{\pi} \int \frac{2n_1x + 4n_2x^3 + 6n_3x^5 + 8n_4x^7}{\sqrt{x^2 - r^2}} dx, \]  

(A.2)

\[ = n_{e0} + n_{e1}r^2 + n_{e2}r^4 + n_{e3}r^6 + n_{e4}r^8 \]

where \( a \) is the plasma radius. Since Eq. (A.2) can be solved analytically, the coefficients of the Abel-inverted density profile, \( n_{e0}-n_{e4} \), are directly obtained by using fitting parameters of \( n_1-n_a \). This is the analytical method.

On the other hand, the numerical method is based on the Matoba's method [20]. When we consider the situation to observe at the position of \( x_k = r_k \) with a width of \( \Delta r \) shown in Fig. A-1, the observed line-integrated signal \( I_k \) can be written by
\[
I_k = \frac{1}{\Delta r} \sum_{i=k}^{N} S_{ki} n_r(r_i), \quad (A.3)
\]

where \( S_{ki} \) is the observed area between concentric circles \( k \) and \( k-1 \) shown as a gray region in Fig. A-1.

\[\text{Figure A-1} \quad \text{Coordinate system for Abel inversion.}\]

Equation (A.3) can be written explicitly by using matrix \( A \) as

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_k \\
\vdots \\
I_N
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & \cdots & \cdots & A_{1N} \\
0 & A_{22} & \cdots & \cdots & A_{2N} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & A_{kk} & \cdots & A_{kN} \\
0 & \cdots & \cdots & 0 & A_{NN}
\end{bmatrix} \begin{bmatrix}
n_r(r_1) \\
n_r(r_2) \\
\vdots \\
n_r(r_k) \\
\vdots \\
n_r(r_N)
\end{bmatrix}, \quad (A.4)
\]

where \( A_{ki} = S_{ki} / \Delta r \). The matrix element of \( A \) is written by
The electron density obtained by calculating the inverse of the matrix $A$ is written by

$$
[n_e(r)] = [A]^{-1}[I_k]. 
$$

In the experiment, the line-density profile is complemented by a cubic spline interpolation with $\Delta r = 1$ mm before inversion process.

Figure A-2 shows a result of Abel inversion at the central cell. In a line-density profile (top), open circle and solid line are the measured data and its fitting line respectively. The numbers $r_0$ and ans1-5 correspond to the plasma shift and fitting coefficient of $n_0-n_4$ in Eq. (A.1) respectively. Crosses are the line-integrated density of the Abel-inverted profiles using the result of numerical method, and are used to confirm the accuracy of the result. In the Abel-inverted density profile (bottom), a solid line and asterisks are the results obtained by the numerical and analytical methods, respectively. The numbers of ans1-5 correspond to the fitting coefficient of $n_{e0}$-$n_{e4}$ in Eq. (A.2). The results using each method are almost identical in the standard operation. For the plug-
cell plasma, almost same algorithm is used.

![Graph showing line density and electron density profiles at t = 200 ms.](image)

**Figure A-2** Line-density (top) and Abel-inverted density (bottom) profiles at the central cell measured at \( t = 200 \) ms. In the bottom figure, the solid line and asterisks are the results obtained by the numerical and analytical methods.