Pion decay constant in quenched QCD with Kogut-Susskind quarks*

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We report results on the flavor breaking effect in the pion decay constant f_π calculated with the Kogut-Susskind quark action. Numerical simulations are carried out at $\beta = 6.0$ and 6.2. We find that the use of non-perturbative renormalization factor leads to results in the continuum limit well convergent among various KS flavors. As the best value for f_π we obtain $f_\pi = 89(6)$ MeV from the conserved axial vector current.

1. Introduction

The Kogut-Susskind (KS) quark action has the well-known feature that SU(4) flavor symmetry is broken down to U(1) subgroup at finite lattice spacing. The restoration of full flavor symmetry toward the continuum limit has been previously examined for pion mass[1,2]. Here we extend the examination to the pion decay constant by comparing results for various KS flavors to that in the U(1) channel for which the renormalization constant equals unity.

This
comparison is made both for perturbative[3] and non-perturbative renormalization factors, the latter evaluated[4] with the method of Ref. [5].

Numerical simulations are carried out in quenched QCD at $\beta = 6.0$ and 6.2 employing $32^3 \times 64$ and $48^3 \times 64$ lattices. Other lattice parameters are summarized in Table 1.

2. Formalism and simulation

In the hypercubic notation for the KS fermion fields, the axial vector current in the KS flavor channel F has the form

$$A_\mu^F = \overline{\phi}(\gamma_\mu\gamma_5 \otimes \xi_F)\phi. \quad (1)$$

The fields $\overline{\phi}$ and ϕ in (1) are generally separated over a 4-dimensional hypercube. We use both gauge-invariant and non-invariant operators, inserting the product of link variables between $\overline{\phi}$ and ϕ for the former, and employing the Landau gauge fixing to deal with the latter.

The one-loop perturbative results for the renormalization constants Z_A^F have been worked out in Ref. [3]. We use tadpole-improved values employing the tadpole-improved $\overline{\mathrm{MS}}$ coupling with $q^* = 1/a$; they will be denoted as $Z_A^{(\mathrm{P})F}$. For non-perturbative renormalization constants $Z_A^{(\mathrm{N})F}$, we take results of Ref. [4] at $(ap)^2 = 1.0024$.

Quark propagators are calculated for 16 wall sources, each corresponding to a corner of a hypercube. These propagators are combined to form the correlator $\langle A_\mu^F(t)\pi_W^F(0)\rangle$ for each flavor

Table 1
Lattice parameters of simulation.

β	$L^3\cdot T$	$m_q a$	#conf.	a^{-1}(GeV)	m_π/m_ρ
6.0	$32^3\cdot 64$	.010–.030	100	1.93(2)	.53–.72
6.2	$48^3\cdot 64$	.008–.023	60	2.70(5)	.55–.75

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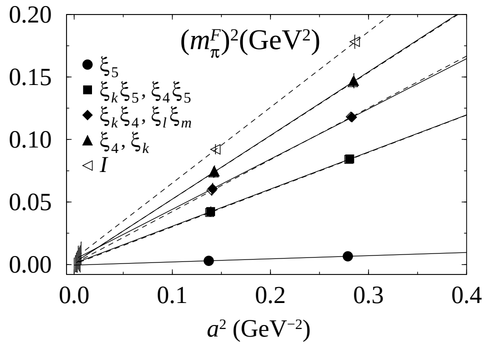

Figure 1. Continuum limit of pion masses.

F, where π_W^F is the pion field for the wall source, thereby enhancing signals[6].

We find the quark mass dependence of $(m_\pi^F)^2$, f_π^F and vector meson masses to be well described by a linear function. Hence a linear fit is employed for the chiral extrapolation in the quark mass m_q. The scale is set by the ρ meson mass in the spin-flavor channel $\gamma_k \otimes \xi_k$. Errors are estimated by the single elimination jackknife procedure.

3. Pion masses

We show in Fig. 1 values of $(m_\pi^F)^2$ at $m_q = 0$ as a function of a^2. The 16 KS flavors are grouped into 8 irreducible representations (irreps) given by[7] $\xi_5, \xi_4\xi_5, \xi_4, I$ (1-d irreps) and $\xi_k\xi_5, \xi_k\xi_4, \xi_\ell\xi_m, \xi_k$ (3-d irreps). We observe that the 8 irreps form a degeneracy pattern $\xi_5, (\xi_k\xi_5, \xi_4\xi_5), (\xi_k\xi_4, \xi_\ell\xi_m), (\xi_4, \xi_k), I$ at finite lattice spacing. This feature was initially observed numerically in Ref. [6], and recently theoretically explained in Ref. [8]. We also see that non-zero values of $(m_\pi^F)^2$ for the channels other than ξ_5 vanish quadratically in a toward the continuum limit as expected.

4. Pion decay constants

In Fig. 2 we illustrate how the bare values of f_π^F change under renormalization, taking results for gauge invariant current at $\beta = 6.2$. The bare values for the 8 irreps again form a degeneracy pattern. The pattern reflects the distance of $\overline{\phi}$ and ϕ in (1), and is different from that of m_π^F.

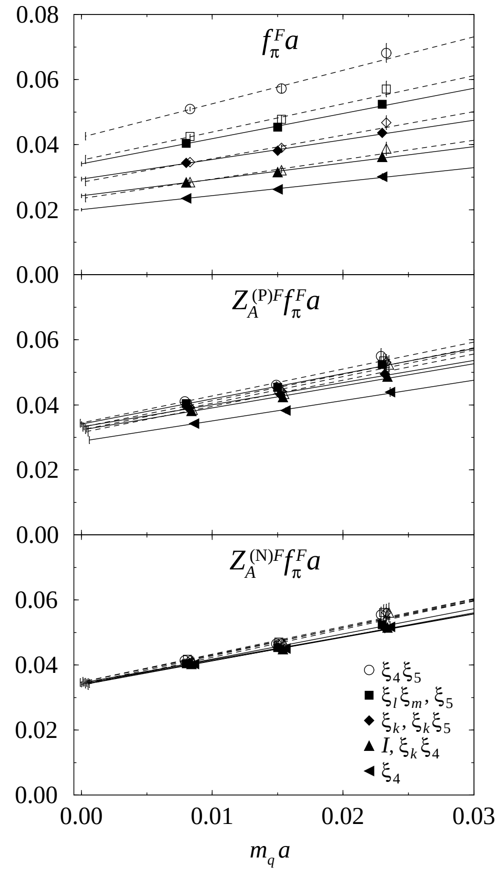

Figure 2. f_π^F obtained with gauge-invariant axial vector current as functions of m_q at $\beta = 6.2$ before renormalization (top), with perturbative (middle) and non-perturbative (bottom) renormalization factors.

The perturbative renormalization constants (middle frame in Fig. 2) help to reduce the discrepancy among the bare values of f_π^F. It is clear, however, that the discrepancy is much more reduced with the non-perturbative renormalization constants (bottom frame).

In Fig. 3 we plot the continuum extrapolation of the pion decay constants obtained with gauge-invariant currents. The top figure shows results with perturbative renormalization factors, and the bottom figure with non-perturbative factors. Lines are quadratic fits in a^2 following the $O(a^2)$ scaling violation expected for the KS quark action. Similar figures for the decay constants ob-

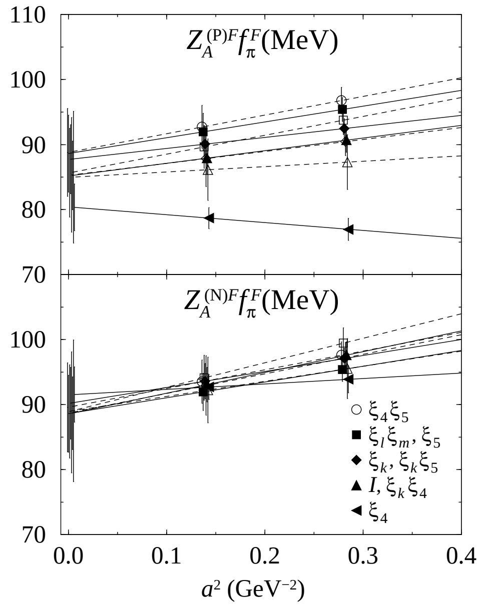

Figure 3. Continuum limit of f_π^F with perturbative (top) and non-perturbative (bottom) renormalization factor obtained with gauge-invariant axial vector current.

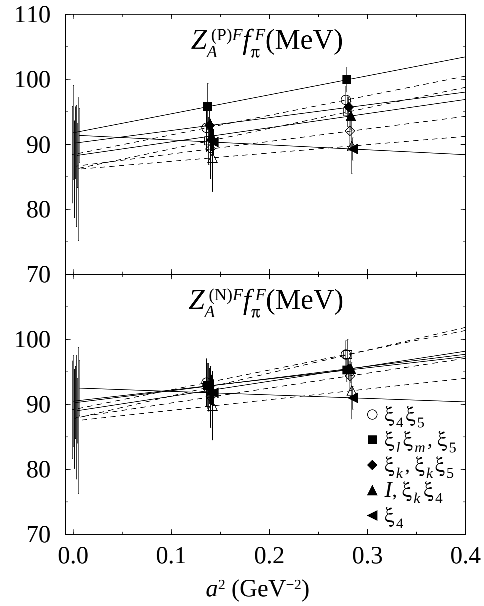

Figure 4. Same as Fig. 3 obtained with gauge non-invariant axial vector current.

tained with the gauge non-invariant currents are shown in Fig. 4.

The gauge-invariant current for the flavor ξ_5 does not require renormalization, from which we obtain $f_\pi = 89(6)$ MeV in the continuum limit. This result is consistent with the experimental value of 92.4(3) MeV[9]; possible deviations due to quenching is not visible within our 7% error.

We observe in Figs. 3 and 4 that the perturbative renormalization factors at one-loop order are not sufficient to ensure restoration of flavor symmetry in the continuum limit. In contrast, the decay constants evaluated with the non-perturbative renormalization factors agree much better already at finite lattice spacings, and their continuum limits are well convergent, the central values coinciding with each other well within the errors.

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