

PREFACE

This thesis is concerned with the well-posedness of the Cauchy problem for weakly hyperbolic equations of third order and weakly hyperbolic systems.

In 1983 F. Colombini, E. Jannelli and S. Spagnolo showed that the Cauchy problem for the weakly hyperbolic equations of second order

$$u_{tt} - \sum_{i,j=1}^n a_{ij}(t)u_{x_i x_j} = 0$$

is wellposed in the Gevrey classes of order $1 \leq s < 1 + \frac{\alpha}{2}$, if the coefficients $a_{ij}(t)$ ($1 \leq i, j \leq n$) belong to the Hölder class $C^\alpha([0, T])$ (see [CJS]). After that their result was generalized by many authors as follows.

i) In [CJS] the coefficients depend only on the time variable t . Therefore T. Nishitani and P. D'Ancona independently derived the same result for the weakly hyperbolic equations of second order with the coefficients depending also on the space variable x (see [N] and [D]).

ii) Y. Ohya and S. Tarama considered the weakly hyperbolic equations of higher order, and derived the similar kind of result(see [OT]). From the proof of their theorem we find that the regularity of the characteristic roots plays an important role in the treatment of the higher order equations(see [B] and [W]). Then it is possible to relax the regularity of the coefficients according to the principal parts of equations. In Chapter 1 we especially treat the third order equations and investigate the relation between the Gevrey wellposedness and the Hölder continuity of the coefficients(see [Ki5] and [CK]).

iii) By the Cauchy-Kovalevski Theorem we know that any type of systems can be solvable locally in the analytic class. As for weakly hyperbolic systems, K. Kajitani showed the global solvability in the analytic class(see [Ka2]). But in general it is difficult to prove the global solvability in the Gevrey classes. Therefore some authors impose the additional conditions on the systems(see [Ka3] and [DS2]). In Chapter 2 we study the weakly hyperbolic systems under the conditions concerned with the structures of matrices(see [Tag],[Yh] and [V]). Roughly speaking we investigate the relation among the Gevrey wellposedness

and the Hölder continuity of the coefficients and the sizes of the Jordan blocks (see [Ki2]).

While V. Ya. Ivrii showed that the Cauchy problem for the second order equations

$$u_{tt} - t^\alpha u_{xx} + t^\lambda u_x = 0$$

is wellposed in the Gevrey classes of order $1 \leq s < \frac{2(\alpha-\lambda)}{\alpha-2\lambda-2}$ where α and λ are integers satisfying $\lambda \geq 0$, $\alpha-2\lambda-2 > 0$ (see [I2] and [Ki3]). Moreover F. Colombini and N. Orrú generalized his result to the third order equations(see [CO1]). In Chapter 3 with a different method we also consider the third order equations and derive the further results(see [Ki6]).

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NOTATIONS

$$\langle \xi \rangle_\nu = (|\xi|^2 + \nu^2)^{\frac{1}{2}} \quad (\nu > 0). \quad \langle \xi \rangle = (|\xi|^2 + 1)^{\frac{1}{2}} (= \langle \xi \rangle_1).$$

$$\hat{f}(\xi) = \mathcal{F}[f](\xi) = \int e^{-ix \cdot \xi} f(x) dx.$$

$C^{k+\alpha}([0, T])$ ($k \in \mathbf{N}^1$, $0 \leq \alpha \leq 1$) is the space of functions $f(t)$ having k derivatives continuous, and the k -th derivative Hölder continuous with exponent α on $[0, T]$.

$L^2_{\rho, \kappa, \nu}(\mathbf{R}_x^n)$ ($\rho > 0$, $0 < \kappa \leq 1$) is the space of Gevrey functions $f(x)$ satisfying $e^{\rho \langle \xi \rangle^\kappa} \hat{f}(\xi) \in L^2(\mathbf{R}_\xi^n)$.

$G^s(\mathbf{R}_x^n)$ ($s \geq 1$) is the space of Gevrey functions $f(x)$ satisfying for any compact set $K \subset \mathbf{R}^n$, $\sup_{x \in K} |D^\alpha f(x)| \leq C_K \rho_K^{|\alpha|} |\alpha|!^s$ for $\forall \alpha \in \mathbf{N}^n$.

$G^s_0(\mathbf{R}_x^n)$ ($s > 1$) is the space of Gevrey functions $f(x)$ of the order s having compact support.