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Proton decay and lattice QCD

Yasuo Hara, Satoshi Itoh, Yoichi Iwasaki, and Tomoteru Yoshié
Institute of Physics, University of Tsukuba, Ibaraki 305, Japan
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Proton decay rates in grand unified theories are calculated by using numerical results obtained by Monte Carlo simulation in lattice QCD. The decay rates are calculated in such a way that they are independent on the renormalization point. By comparing the theoretical and experimental results on $p \to \pi^0 e^+$ decay, we conclude that the simple SU(5) model is excluded as a realistic model.

I. INTRODUCTION

Since the instability of the proton was predicted by grand unified theories[1] (GUT's), several experimental groups have been trying hard to detect proton decay. Until now they have discovered several candidate events, but they have not been able to obtain definite evidence that the proton decays.[2] Instead they have obtained upper bounds for the rates of various decay modes of the proton.[2]

Though many articles on theoretical estimation of rates of various decay modes of the proton have been published[1,3] we cannot predict any of the decay rates definitely. The main reason for this situation is the fact that there are various grand unified theories based on various symmetry groups, various types of symmetry breaking, and various Higgs sectors.

Even if we choose a particular grand unified theory, for example, the simple SU(5) model,[4] there still exists an uncertainty by a factor of more than 10 in the calculation of hadronic matrix elements of the proton-decay processes.

The purpose of this article is to calculate the decay rates of the proton in grand unified theories as accurately as possible in order to exclude some grand unified theories by comparing the predicted decay rates in the grand unified theories with experimental upper bounds for the decay rates.

For this purpose we use the numerical results of Monte Carlo simulation in lattice QCD obtained by three of the present authors (S.I., Y.I., and T.Y.).[5,6]

For simplicity we study only the $p \to \pi^0 + e^+$ decay in the simple SU(5) model.[4] Application to other decay modes and to other grand unified theories is straightforward.

In the simple SU(5) model proton decay is mediated by superheavy leptoquark gauge bosons X and Y. The effective Lagrangian density which induces the $p \to \pi^0 + e^+$ decay is expressed as

$$L_{\rm eff} = (g^2/m_X{}^2)\epsilon_{ijk}[\,2(\bar{u}^{\,c}_{kR} d_{iL})(\bar{e}^{\,+}_{L} u_{jR}) + (\bar{u}^{\,c}_{kL} d_{iR})(\bar{e}^{\,+}_{R} u_{jL})]\;, \qquad (1.1)$$

if we neglect the Higgs-boson contribution, generation mixing, and radiative corrections due to strong and electroweak interactions.

Radiative corrections modify the effective Lagrangian density from the form in Eq. (1.1). Since the effective Lagrangian density (1.1) is multiplicatively renormalized, it becomes

$$(L_{\rm eff})_\mu = 4\pi(\alpha_{\rm GUT}/m_X{}^2)A_3\epsilon_{ijk} \times [\,2A^L_{12}(\bar{u}^{\,c}_{kR} d_{iL})(\bar{e}^{\,+}_{L} u_{jR}) + A^R_{12}(\bar{u}^{\,c}_{kL} d_{iR})(\bar{e}^{\,+}_{R} u_{jL})]\;, \qquad (1.2)$$

where μ ($\ll m_W$) is the renormalization point and

$$\alpha_{\rm GUT} = g^2(m_X)/4\pi = \alpha_3(m_X) \approx \tfrac{1}{41}\;. \qquad (1.3)$$

In one-loop order the renormalization factors A^L_{12}, A^R_{12}, and A_3 are expressed as[7,8]

$$A^L_{12} = \left[\frac{\alpha_2(m_W)}{\alpha_2(m_X)}\right]^{27/38}\left[\frac{\alpha_1(m_W)}{\alpha_1(m_X)}\right]^{-23/82} \approx 2.49\;, \qquad (1.4)$$

$$A^R_{12} = \left[\frac{\alpha_2(m_W)}{\alpha_2(m_X)}\right]^{27/38}\left[\frac{\alpha_1(m_W)}{\alpha_1(m_X)}\right]^{-11/82} \approx 2.20\;, \qquad (1.5)$$

and

$$A_3 = \left[\frac{\alpha_3(\mu)}{\alpha_{\rm GUT}}\right]^{2/(11-2N/3)}\;, \qquad (1.6)$$

where N is the number of quark flavors, $\alpha_i = g_i{}^2/4\pi$ and g_1, g_2, and g_3 are coupling constants of $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ interactions, respectively.

Now the strong-interaction problem is to calculate hadronic matrix elements $\langle \bar{l},{\rm meson}\,|\,(L_{\rm eff})_\mu\,|\,N\rangle$ with an appropriate value of μ.

In the past various models such as the bag models and the nonrelativistic SU(6) model together with $A_3(\mu)$ with $\mu \approx 1$ GeV chosen as a typical hadronic scale were used to calculate the matrix elements. However, in these models it is difficult to calculate the matrix elements reliably. There are many arbitrary parameters in these models, and it is not possible to determine the renormalization point μ corresponding to these models.

In this article we assume the validity of current algebra and the PCAC (partial conservation of axial-vector current) relation.[9] In this model all nucleon$\to$antilepton + pseudoscalar meson decay amplitudes are related to the three-quark annihilation matrix element:

$$\langle 0\,|\,\epsilon_{ijk}[\bar{u}^{\,T}_{k}(0)C^{-1}\gamma_5 d_i(0)]u_j(0)\,|\,p;\mathbf{q}_p = 0\rangle = 2\beta\psi_p(0)\;, \qquad (1.7)$$

where $u_j(0)$, $u_k(0)$, and $d_i(0)$ are annihilation operators of the u and d quarks at the origin, i,j,k are color indices, C is the charge-conjugation matrix, and $\psi_p(0)$ is the Dirac spinor of a proton with zero momentum.

In this article we calculate the hadronic matrix element β and the renormalization factor A_3 by making use of the numerical results of Monte Carlo simulation in lattice QCD. In Sec. II we give decay amplitudes and decay widths of the nucleon in the current-algebra model. In Sec. III we calculate β and in Sec. IV we calculate A_3. The ambiguity in the choice of $\alpha_3(\mu)$ in $A_3(\mu)$ will be found to be removed if we use the lattice QCD in the calculation of β. In Sec. V we evaluate the mass of the leptoquark gauge boson X in the simple SU(5) model. In Sec. VI we calculate the $p \to \pi^0 + e^+$ decay rate in the simple SU(5) model. A discussion and conclusions are given in Sec. VII.

II. THE CURRENT ALGEBRA AND PCAC RELATION

In this article we assume the validity of current algebra and the PCAC relation. In this model all nucleon→antilepton + pseudoscalar meson decay amplitudes are expressed as sums of current-commutator terms and nucleon-pole terms (Fig. 1) (Refs. 9 and 10).

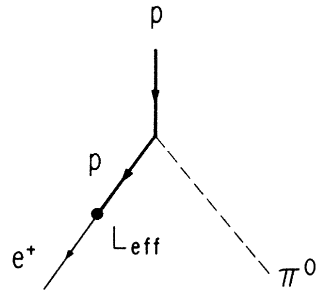

FIG. 1. The proton-pole term in $p \to \pi^0 e^+$ decay.

Both the current-commutator terms and the nucleon-pole terms are related to the three-quark annihilation matrix element (Fig. 2):

$$\langle 0 | \epsilon_{ijk}[u_k^T(0)C^{-1}(\gamma_5+\text{const})d_i(0)]u_j(0) | p;\mathbf{q}_p=\mathbf{0}\rangle = \langle 0 | \epsilon_{ijk}[u_k^T(0)C^{-1}\gamma_5 d_i(0)]u_j(0) | p;\mathbf{q}_p=\mathbf{0}\rangle \equiv 2\beta\psi_p(0) , \quad (2.1)$$

which corresponds to the matrix element

$$\langle 0 | \epsilon_{ijk}[u_{k\uparrow}(0)d_{i\downarrow}(0) - u_{k\downarrow}(0)d_{i\uparrow}(0)]u_j(0) | p;\mathbf{q}_p=\mathbf{0}\rangle \quad (2.2)$$

in the nonrelativistic limit. In (2.1) space reflection invariance is used.

In the nonrelativistic limit the matrix element β is related to the wave function of the nucleon in the quark model $\psi_N(\mathbf{r}_a+\mathbf{r}_b-2\mathbf{r}_c, \mathbf{r}_a-\mathbf{r}_b)$ through

$$\psi_N(\mathbf{0},\mathbf{0}) = (\sqrt{2}/3)\beta . \quad (2.3)$$

Then, we can show that $N \to \bar{l}\pi$ decay amplitudes and widths are expressed as

$$(\alpha_{\text{GUT}}/m_X{}^2)A\beta[\bar{u}_l(b+d\gamma_5)u_N] \quad (2.4)$$

and

$$\Gamma \approx (\alpha_{\text{GUT}}/m_X{}^2)^2 A^2\beta^2 m_N(b^2+d^2)/16\pi , \quad (2.5)$$

where

$$A \equiv A_{12}^L A_3 \approx A_{12}^R A_3 \quad (2.6)$$

and

$$b = -d/3 = \sqrt{2}\pi(1+g_A)/f_\pi \quad \text{for } p \to \pi^0 e^+ . \quad (2.7)$$

We consider that the model is reliable semiquantitatively[11] with a correction factor of $\times(\sim 2)^{\pm 1}$.

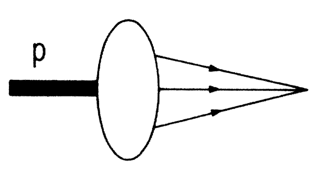

FIG. 2. The three-quark annihilation matrix element.

III. CALCULATION OF THE MATRIX ELEMENT β

In Ref. 6 the propagators of hadrons

$$A(\tau) = \sum_{\mathbf{n}} \langle P(n)P^\dagger(0)\rangle \quad [n=(\mathbf{n},0), 0=(\mathbf{0},0)] \quad (3.1)$$

have been calculated in the quenched approximation to lattice QCD with a renormalization-group-improved lattice SU(3) gauge action and Wilson's quark action on a $16^3\times 48$ lattice at $\beta=2.4$ ($\beta=6/g^2$), where $P(n)$ is an annihilation operator of a hadron at site n. For the proton the annihilation operator is

$$P(n) = \epsilon_{ijk}[u_k^T(n)C^{-1}\gamma_5 d_i(n)]u_j(n) , \quad (3.2)$$

which is an annihilation operator of a three-quark state with $J^P = \frac{1}{2}^+$ at site n.

Though the value of the coupling constant, $\beta=2.4$, in which the calculations have been done is not in the asymptotic scaling region, we believe that it is in the scaling region. The reason is as follows. In the study of the string tension the values of the coupling constant $2.6 \le \beta \le 2.9$ have been found to be in the asymptotic scaling region.[5] Unfortunately we cannot choose these values in our calculation since the size of the proton becomes bigger than our lattice size. However, we expect that the scaling sets in at smaller β and $\beta=2.4$ is very close to $\beta=2.6$ where the asymptotic scaling sets in.

The hadron propagators calculated for five values of the hopping parameter $K=0.14$, 0.145, 0.15, 0.1525, and 0.154 are shown in Fig. 3. The hopping parameter K is related to the bare-quark mass m_0 and the lattice spacing a through the relation[6]

$$2m_0 a = \left[\frac{1}{K} - \frac{1}{K_c}\right] , \quad (3.3)$$

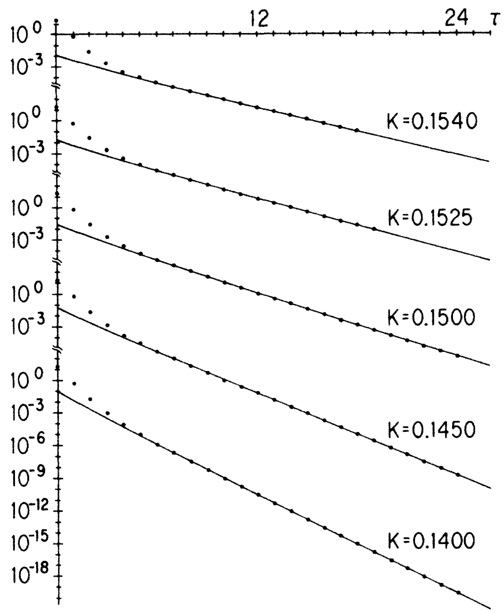

FIG. 3. The proton propagators with two mass fit (solid lines).

where K_c is the value of the hopping parameter for which the pion mass vanishes.

The baryon propagators for large τ have not been obtained for $K=0.1525$ and 0.154 because of large fluctuation. Because it is very time consuming to calculate the baryon propagators for the hopping parameter between 0.154 and K_c, they have not been calculated.

The hadron propagators for large Euclidian time τ have been fitted to $A_0 e^{-m\tau}$. The results for the hadron masses are shown in Fig. 4. By assuming that the masses (mass squared for the pion) are quadratic functions of $1/K$, K_c and a are determined as

$$K_c = 0.1569(2) \tag{3.4}$$

and

$$a^{-1} = 1810(60)\ \text{MeV},\quad a = 0.109(4)\ \text{fm} \tag{3.5}$$

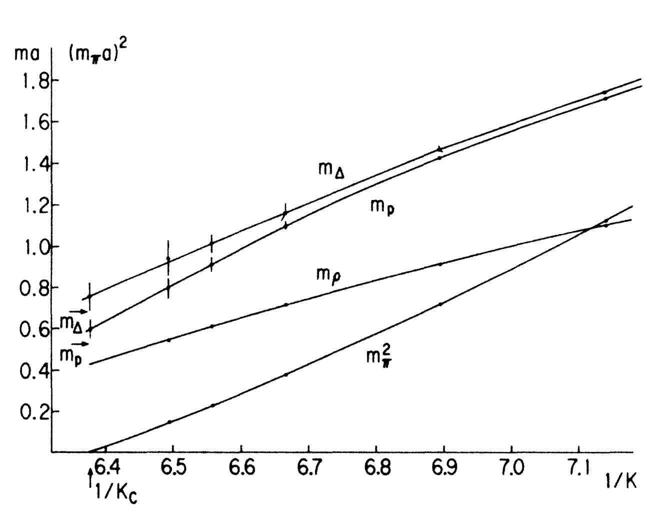

FIG. 4. The masses of ρ, pion, proton, and Δ vs $1/K$.

from the condition $m_\rho(K=K_c)a = 0.426(15)a = 770$ MeV. Theoretically estimated values $m_p = 1080(80)$ MeV and $m_\Delta = 1370(120)$ MeV should be compared with the experimental values 940 and 1232 MeV, respectively.

The matrix element β^2, which is related to the amplitudes of the proton propagators through the relation

$$\begin{aligned} 4\beta^2 &= [(2K)^3/a^9]\,|\langle 0|P(0)|\text{proton};\mathbf{q}_p=\mathbf{0}\rangle|^2 \\ &= [(2K)^3/a^6]A_0\ , \end{aligned} \tag{3.6}$$

are shown in Fig. 5. [In deriving this relation, the relation between the lattice fermion ψ_{lattice} and the fermion field in continuous space-time ψ, $\psi_{\text{lattice}}(n) = (a^3/2K)^{1/2}\psi(x)$, has been used.] The expectation value and the statistical error at each K have been estimated by neglecting the statistical error of the proton mass at each K.

In order to obtain β at $K=K_c$, we have to extrapolate the results for A_0 to $K=K_c$. Since we do not know the dependence of A_0 on K, we assume that the amplitude A_0 is a quadratic function of $1/K$ for simplicity. Thus, we obtain

$$\beta^2 = [0.029\ (\text{GeV})^3]^2 \times (1 \pm 0.44)\ . \tag{3.7}$$

As is shown in Fig. 5, our approximation on the K dependence of A_0 is satisfactory. Unfortunately we cannot estimate the uncertainty introduced due to this approximation. However, we would like to stress that the uncertainty is not inherent in our formalism, but is rather technical. We hope that we are able to calculate A_0 at $K=K_c$ in the near future. Then, this uncertainty will disappear. On the other hand, in other models the uncertainty related to the choice of the renormalization point cannot be removed.

As is shown in Fig. 5, the amplitude decreases as K increases. This behavior indicates that attraction among three quarks at short distance decreases as the bare-quark mass decreases.

As a reference we show a compilation[11] of previous calculations of β based on the bag model and other models in Table I. We find that our result (3.7) is much bigger than

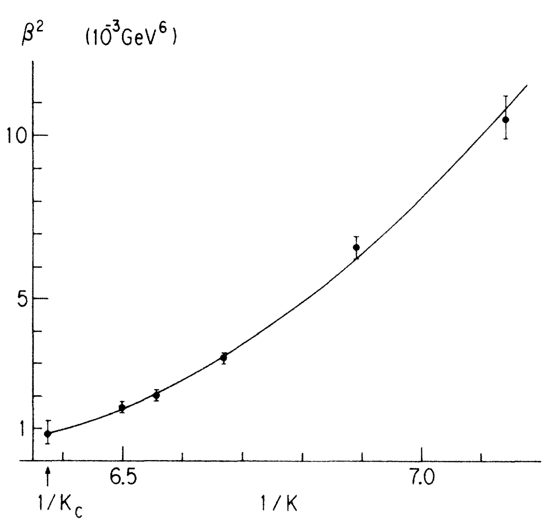

FIG. 5. The matrix element β^2 vs $1/K$.

TABLE I. Compilation (Ref. 11) of calculations of β. The first three values are obtained by making use of the bag model. The results depend sensitively on the size of the bag assumed. The next three values are obtained by using QCD and finite-energy sum rules. Tomozawa's result is based on the relativistic quark model with a harmonic-oscillator potential ($\hbar\omega=250$ MeV and $m_u=330$ MeV). The value of Brodsky *et al.* is based on knowledge about the short-distance structure of baryon wave functions gleaned from QCD form-factor calculations and the $J/\psi\to p\bar{p}$ decay rate.

Authors	β (GeV^3)
Donoghue and Golowich	0.003
Thomas and McKellar	0.02
Milosevic *et al.*	0.005
Ioffe	0.009
Krasniikov *et al.*	0.006
Ioffe and Smilga	0.006
Tomozawa	0.009
Brodsky *et al.*	0.03
This paper	0.03

the results in the bag model and the quark model. These models are not suitable for the estimation of three-quark correlation in the nucleon, while they may be useful for studying single-quark and two-quark properties of hadrons such as magnetic moments and mass differences.

IV. CALCULATION OF THE RENORMALIZATION FACTOR $A_3(\mu)$

In the past there was an ambiguity in the choice of $\alpha_3(\mu)$ in the renormalization factor $A_3(\mu)$. We can remove this ambiguity in lattice QCD.

Since the decay amplitude is independent of the renormalization point μ, μ dependence of $A_3(\mu)$ must be canceled by μ dependence of the matrix element β.

The matrix element β evaluated in lattice QCD, β_{lattice}, is related to the matrix element β evaluated in QCD with a renormalization point μ, β_μ, through a renormalization factor $[\alpha_3(\text{lattice})/\alpha_3(\mu)]^\gamma$:

$$\beta_\mu=\left[\frac{\alpha_3(\text{lattice})}{\alpha_3(\mu)}\right]^\gamma \beta_{\text{lattice}}\ , \tag{4.1}$$

where $\gamma=2/(11-2N/3)$. Thus, because of the relation

$$\begin{aligned}A_3(\mu)\beta_\mu&=\left[\frac{\alpha_3(\mu)}{\alpha_3(m_x)}\right]^\gamma \beta_\mu\\&=\left[\frac{\alpha_3(\text{lattice})}{\alpha_3(m_x)}\right]^\gamma \beta_{\text{lattice}}\ ,\end{aligned} \tag{4.2}$$

we find that we have to use α_3(lattice) as $\alpha_3(\mu)$ in Eq. (1.6) for A_3 when we use the lattice QCD to evaluate the matrix element β.

Since $\alpha_3(\text{lattice})=0.20$ is used[6] in the calculation of β in Sec. III, the renormalization factor

$$A_3\approx\left[\frac{\alpha_3(\text{lattice})}{\alpha_3(m_x)}\right]^\gamma\approx[0.20/(1/41)]^{2/7}\approx1.8 \tag{4.3}$$

in our case.

V. EVALUATION OF m_X IN THE SIMPLE SU(5) MODEL

The matrix element β is independent on the grand unified theories, but the mass of the X boson m_X is dependent on the grand unified theories. In the simple SU(5) model m_X is predicted to be[12]

$$m_X=1.3\times10^{14}\times(1.5)^{\pm1}(\Lambda_{\overline{\text{MS}}}/100\text{ MeV})\text{ GeV} \tag{5.1}$$

for three generations, a single Higgs doublet and $m_t=50$ GeV.

The strong scale parameter in the minimum-subtraction scheme $\Lambda_{\overline{\text{MS}}}$ has been estimated to be[5]

$$\Lambda_{\overline{\text{MS}}}=99\pm1\text{ MeV} \tag{5.2}$$

by studying the string tension by Monte Carlo simulation in lattice QCD in the quenched approximation for $2.6\le\beta\le2.9$, which has been found to be in the asymptotic scaling region (with $\sqrt{\sigma}=420$ MeV).

The result (5.2) does not include systematic errors due to the quenched approximation, finite lattice size, etc. By assigning an ample systematic error to (5.2), we conclude

$$(100\text{ MeV})\times\tfrac{2}{3}\le\Lambda_{\overline{\text{MS}}}\le(100\text{ MeV})\times\tfrac{3}{2}\ . \tag{5.3}$$

This result is consistent with recent experimental values[13,14] of $\Lambda_{\overline{\text{MS}}}$.

VI. DECAY RATE OF $p\to\pi^0e^+$ IN THE SIMPLE SU(5) MODEL

By making use of the estimated values of the parameters β and A_3, (3.7) and (4.3), we find

$$\begin{aligned}\Gamma(p\to e^+\pi^0)&=(5\pi/4)(1+g_A)^2A^2\beta^2(m_N/f_\pi^{\,2})\\&\quad\times(\alpha_{\text{GUT}}/m_X^{\,2})^2\text{ GeV}^5\\&=[4.1\times10^{-3}\times(4)^{\pm1}/m_X^{\,4}]\text{ GeV}^5\ ,\end{aligned} \tag{6.1}$$

where we have assumed that the error associated with the decay amplitude (2.7) in the current-algebra model is $\sim2^{\pm1}$ and that the sum of systematic and statistical errors of $\beta^2=(0.029\text{ GeV}^3)^2$ is also $\sim2^{\pm1}$.

From (5.1) and (6.1) we obtain

$$\begin{aligned}\tau(p\to e^+\pi^0)&=1.45\times10^{27}\\&\quad\times(4)^{\pm1}(m_X/1.3\times10^{14}\text{ GeV})^4\text{ yr}\\&=1.45\times10^{27}\times(20)^{\pm1}(\Lambda_{\overline{\text{MS}}}/100\text{ MeV})^4\text{ yr}\ .\end{aligned} \tag{6.2}$$

Thus, we obtain the upper bound for $p\to\pi^0e^+$ decay in the simple SU(5) model from Eqs. (5.3) and (6.2):

$$\tau(p\to e^+\pi^0)<1.5\times10^{29}\text{ yr}\ . \tag{6.3}$$

By comparing this result with recent experimental result on the proton-decay rate,[2]

$$\tau(p\to\pi^0e^+)>3.3\times10^{32}\text{ yr (90\% C.L.)}\ , \tag{6.4}$$

we find that the simple SU(5) model with three generations, a Higgs doublet, and $m_t=50$ MeV is excluded.

Even for $\Lambda_{\overline{\text{MS}}}<400$ MeV we find

$$\tau(p\to\pi^0e^+)<7.4\times10^{30}\text{ yr}\ , \tag{6.5}$$

and (6.5) is still incompatible with the experimental result (6.4).

Even if we allow the existence of various Higgs particles with mass $<m_X$, they do not prolong the life of the proton by a factor of more than[12] 150 and the SU(5) model with $\Lambda_{\overline{\rm MS}} \lesssim 150$ MeV is still incompatible with the experimental result (6.4).

VII. DISCUSSION AND CONCLUSIONS

In this article we have calculated the three-quark annihilation matrix element β by using Monte Carlo simulation in lattice QCD in the quenched approximation. At present in order to evaluate β we have to extrapolate β calculated for smaller values of the hopping parameter K to K_c by assuming that β is a quadratic function of $1/K$. We hope that we are able to calculate β at K closer to K_c in the near future. Then our calculation would become more reliable.

While in principle it is possible to calculate matrix elements $\langle \pi l \,|\, L_{\rm eff} \,|\, p\rangle$ and $\langle \rho l \,|\, L_{\rm eff} \,|\, p\rangle$ by using Monte Carlo simulation, it is practically impossible to carry out the calculation at present because of the capacity of available computers.

We have estimated the magnitude of the matrix element $\langle \pi l \,|\, L_{\rm eff} \,|\, p\rangle$ by relating it to the three-quark annihilation matrix element β in the current-algebra model. In this model the $p\to\pi l$ decay amplitude is a sum of the current-commutator term and the proton-pole term.

We can include the contribution of $N^*(1440)$ to the $p\to\pi l$ decay amplitude if we replace the factor $1+g_A$ in (2.7) by

$$1+[1+0.18\times(1.4)^{\pm 1}\beta^*/\beta]g_A \;, \tag{7.1}$$

where β^* is defined by

$$\langle 0 \,|\, \epsilon_{ijk}[u_k^T(0)C^{-1}\gamma_5 d_i(0)]u_j(0) \,|\, p^*(1440);\mathbf{q}_{p^*}=0\rangle \equiv 2\beta^*\psi_{p^*}(0) \;. \tag{7.2}$$

The factor $(1.4)^{\pm 1}$ in (7.1) is due to the uncertainty of the $N^*\to N+\pi$ decay width. From the two-pole fit of the proton propagators shown in Fig. 3 we find

$$(\beta^*/\beta)^2 \approx 2-4 \;, \tag{7.3}$$

but it is impossible to extrapolate β^* to $K=K_c$ reliably. It is interesting to notice

$$|\,\beta^*/\beta\,| = \sqrt{3} \tag{7.4}$$

in the simple quark model with a harmonic-oscillator potential. If we include the contribution from $N^*(1440)$ pole, the decay rate of $p\to\pi^0 e^+$ increases by a factor of 1.2–1.6 for $\beta^*/\beta=(1.4-2.0)$.

In this article we have found that the simple SU(5) model is excluded as a realistic model by comparing the theoretical and experimental results on $p\to\pi^0 e^+$ decay.

Simple SO(10) models are also excluded if the SO(10) symmetry breaks down through the following paths:

$$\mathrm{SO}(10)\to\mathrm{SU}(5)\to\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}_Y(1)$$

and (7.5)

$$\begin{aligned}\mathrm{SO}(10)&\to\mathrm{SU}(5)'\times\mathrm{U}(1)_{B-L+2Y}\\ &\to\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}_Y(1)\;.\end{aligned}$$

In these cases the masses of the leptoquark gauge bosons which induce nucleon decays are predicted to be no bigger than the masses in the simple SU(5) model.[15,16]

If the SO(10) symmetry breaks down through the path

$$\begin{aligned}\mathrm{SO}(10)&\to\mathrm{SU}(4)_c\times\mathrm{SU}(2)_R\times\mathrm{SU}(2)_L\\ &\to\mathrm{SU}(3)_c\times\mathrm{U}(1)_{B-L}\times\mathrm{SU}(2)_R\times\mathrm{SU}(2)_L\\ &\to\mathrm{SU}(3)_c\times\mathrm{U}(1)_{B-L}\times\mathrm{U}(1)_R\times\mathrm{SU}(2)_L\\ &\to\mathrm{SU}(3)_c\times\mathrm{U}(1)_Y\times\mathrm{SU}(2)_L\;,\end{aligned} \tag{7.6}$$

the $\tau(p\to\pi^0 e^+)$ can be much longer than that in the simple SU(5) model[16,17] and, therefore, the SO(10) model can be compatible with the experimental result (6.4).

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[1]As a review, see P. Langacker, Phys. Rep. **72**, 185 (1981).

[2]As a review, see Y. Totsuka, in *Proceedings of the Twelfth International Symposium on Lepton and Photon Interactions at High Energies,* Kyoto, Japan, 1985, edited by M. Konuma and K. Takahashi (Research Institute for Fundamental Physics, Kyoto University, Kyoto, 1986), p. 120.

[3]A recent calculation is Y. Hara, Nucl. Phys. **B214**, 167 (1983); P. Falkensteiner, W. Lucha, and F. Schöberl, Z. Phys. C **27**, 477 (1985).

[4]H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[5]S. Itoh, Y. Iwasaki, and T. Yoshié, University of Tsukuba Report No. UTHEP-154, 1986 (unpublished). See also, S. Itoh, Y. Iwasaki, and T. Yoshié, Phys. Rev. D **33**, 1806 (1986).

[6]S. Itoh, Y. Iwasaki and T. Yoshié, Phys. Lett. B (to be published). See also, S. Itoh, Y. Iwasaki, and T. Yoshié, *ibid.* **167B**, 443 (1986) and S. Itoh, Y. Iwasaki, Y. Oyanagi, and T. Yoshié, Nucl. Phys. **B274**, 33 (1986).

[7]A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B135**, 66 (1978).

[8]J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Phys. Lett. **88B**, 320 (1979); F. Wilczek and A. Zee, Phys. Rev. Lett. **43**, 1571 (1979).

[9]Y. Tomozawa, Phys. Rev. Lett. **46**, 463 (1981); V. S. Berezinsky, B. L. Ioffe, and Ya. I. Kogan, Phys. Lett. **105B**, 33

(1981); M. Wise, R. Blankenbecler, and L. F. Abbott, Phys. Rev. D **23**, 1591 (1981).

[10]See, for example, R. E. Marshak, Riazzudin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969).

[11]S. J. Brodsky, J. Ellis, J. S. Hagelin, and C. T. Sachrajda, Nucl. Phys. **B238**, 561 (1984).

[12]W. J. Marciano, in *Proceedings of the Fourth Workshop on Grand Unification*, edited by H. A. Weldon *et al.* (Birkhauser, Boston, 1983), p. 13.

[13]G. P. Lepage, in *Proceedings of the 1983 International Symposium on Lepton and Photon Interactions at High Energies, Ithaca, New York, 1983*, edited by D. G. Kassel and D. L. Kreinick (Newman Laboratory of Nuclear Studies, Cornell University, New York, 1983), p. 565.

[14]F. Sciulli, in *Proceedings of the Twelfth International Symposium on Lepton and Photon Interactions at High Energies* (Ref. 2), p. 8.

[15]Y. Hara, T. Matsuki, and N. Yamamoto, Prog. Theor. Phys. **68**, 652 (1982).

[16]Y. Tosa, G. Branco, and R. E. Marshak, Phys. Rev. D **28**, 1731 (1983).

[17]D. Chang, R. N. Mohapatra, J. M. Gipson, R. E. Marshak, and M. K. Parida, Phys. Rev. D **31**, 1718 (1985).