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Quantization of a String of Spinning Material—Hamiltonian and Lagrangian Formulations*

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The dynamics of a relativistic string made of spinning material is discussed in two different formulations. The first is a manifestly covariant formulation under a gauge transformation. The second is a Hamiltonian formalism which enables us to make the transition from the classical to the quantum description in a coherent way. The Lagrangian is also constructed. The results of our investigations are as follows: (1) The mass spectra here coincide with those of the Neveu-Schwarz model. (2) The model is ghost-free. (3) The Poincaré generators $\mathcal{M}^{\mu\nu}$ and \mathcal{P}^μ are constructed. The quantization is shown to be consistent with Lorentz covariance if the dimension of space-time is 10 and the Regge intercept is $\frac{1}{2}$.

I. INTRODUCTION

The string picture^{1,2} of dual models has two important aspects. The first is the substantiality of the picture, which helps us to figure out intuitive images and gain an insight into a dynamical mechanism. The second is the mathematical refinement of the formulation. In particular, in the treatment of gauge invariance, which is the fundamental clue in understanding dual models, one can take advantage of techniques developed in gravitation and Yang-Mills theories.^{3,4} In a recent work Goddard, Goldstone, Rebbi, and Thorn⁵ have greatly improved our understanding of the Veneziano model. One improvement is the simplification of the ghost-eliminating mechanism by Brower, and Goddard and Thorn,⁶ which is now understood as given by the existence of a certain gauge where no ghost appears.⁷ Another is the relation between Lorentz covariance and the dimensionality

of space-time, d . In their string model they have shown that the quantization is consistent with Lorentz covariance if $d=26$.

In view of these aspects, it is a challenging problem to extend the string picture to the Neveu-Schwarz model⁸ (NSM). In a previous paper⁹ we showed a manifestly gauge-invariant formalism of the string model, which reduces to the NSM in a special gauge. In the present article, we further develop the argument and give the Hamiltonian and the Lagrangian formulations, which are useful for various purposes, i.e., the quantization of the string motion, the incorporation of the interaction with external sources, etc.

The model we consider is the one based on a string on which Lorentz vector quantities (spins) are continuously distributed.¹⁰ The system is invariant under a gauge group. The generalized Hamiltonian formalism developed by Dirac,³ then, enables us to provide a quantization procedure.

The spectra of the string motion are shown to be those of Neveu and Schwarz. All the negative-normed particles (ghosts) and the tachyon on the leading trajectory are eliminated by a certain choice of the gauge. Lorentz covariance requires that the dimension of space-time is 10 and the Regge intercept is $\frac{1}{2}$, as expected from the dual-model analysis.¹¹

In Sec. II, we give a manifestly covariant formulation in which the gauge transformation properties are transparent. Although this formulation was already presented in a previous report⁹ except for some refinements, we include it here to make the discussion self-contained. The reader who read the previous work can go directly to Sec. III.

In Sec. III, we construct the Hamiltonian which contains all the arbitrariness due to gauge invariance. This Hamiltonian formalism provides a way to quantize the model and defines the path integration in the functional formalism. The gauge-invariant Lagrangian is also constructed from the Hamiltonian. The explicit form, which turns out to be very complicated, is shown in Appendix A.

The quantization and the Lorentz covariance are discussed in Sec. IV. The final section is devoted to comments and further outlooks.

II. MANIFESTLY COVARIANT FORMALISM

In this section we present, for the mechanics of a string, a formalism in which the gauge invariance is obvious.

In order to construct a model which reproduces the NSM in a special gauge, one has to consider the Lorentz-vector position field $X^\mu(\xi)$ ($\mu = 0, 1, \dots, d-1$) together with a two-component Lorentz-vector field

$$S^\mu(\xi) = \begin{pmatrix} S_1^\mu(\xi) \\ S_2^\mu(\xi) \end{pmatrix}, \quad (2.1)$$

where $\xi = (\xi^1, \xi^2)$ is a parameter specified below. We know, moreover, that S^μ has to be transformed with the dimensionality index $-\frac{1}{2}$ under the conformal transformation. In what follows such a quantity is referred to as the conformal spinor.

One of the difficulties of incorporating the conformal spinor into the dynamical system is the nonexistence of the spinor representation under a nonlinear coordinate transformation. However, we can overcome this trouble, just as in gravitation theory, by introducing an extra field $V_i^a(\xi)$ ($a, i = 1, 2$), the *zweibein* field, an analog to the *vierbein* field in gravitation theory.¹²

In the d -dimensional Lorentz space, we con-

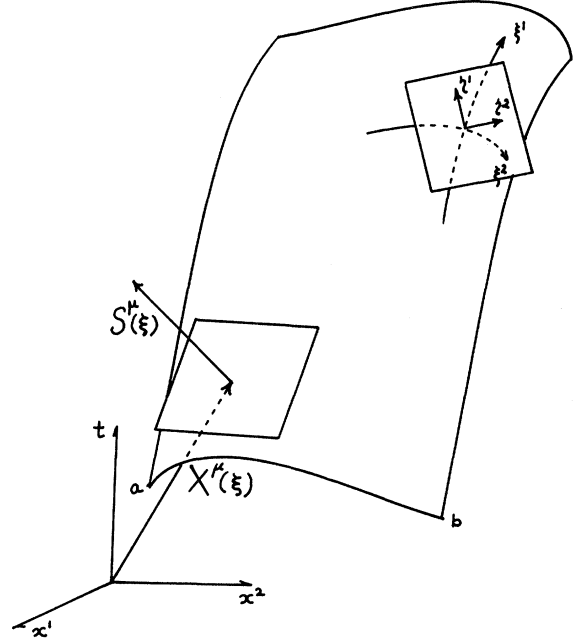


FIG. 1. The world sheet $X^\mu(\xi)$ swept out by the string $a-b$. At each point on the world sheet, a tangent surface (τ_ξ^1, τ_ξ^2) is associated. $S^\mu(\xi)$ is the conformal spinor on the tangent surface.

sider the motion of a string on which S^μ is distributed. The two-dimensional manifold swept out by the string is called the world sheet (Fig. 1). Furthermore, we assume that the parametrization (ξ^1, ξ^2) of the world sheet is given by the metric tensor

$$g^{ij} = \frac{1}{4} i [\bar{S} \hat{V}_i \partial_j S - \partial_j \bar{S} \hat{V}_i S + (i \leftrightarrow j)] + \frac{1}{2} \partial_i X \partial_j X \quad (2.2)$$

with

$$\bar{S}^\mu = S^\mu T_{\sigma_1}, \quad (2.3)$$

where S^μ is an anticommuting field even in the unquantized system [otherwise (2.2) is meaningless]. The *zweibein* field V_i^a is employed in (2.2) as the 2×2 matrix

$$\hat{V}_i = \sigma_a V_i^a \quad (2.4)$$

with

$$\sigma^1 = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = -\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (2.5)$$

The fields X^μ , S^μ , and V_i^a are all assumed to be real.

The transformation properties of g_{ij} are determined on two distinct gauge groups defined below.

To a given parametrization of the world sheet (ξ^1, ξ^2) , we associate a conformal space at each point ξ , which we call the tangent surface at ξ . If the coordinate on the tangent surface at ξ is represented by $\eta_\xi = (\eta_\xi^1, \eta_\xi^2)$, the conformal transformation C_ξ on the tangent surface¹² is defined by

$$C_\xi: d\eta_\xi^a = \Lambda^a_b(\eta_\xi) d\eta_\xi^b, \quad (2.6)$$

with

$$\Lambda^a_b(\eta_\xi) = \exp[\mu_\xi(\eta_\xi)] \times \begin{pmatrix} \cosh\theta_\xi(\eta_\xi) & \sinh\theta_\xi(\eta_\xi) \\ \sinh\theta_\xi(\eta_\xi) & \cosh\theta_\xi(\eta_\xi) \end{pmatrix}_{ab}, \quad (2.7)$$

where μ_ξ and θ_ξ are real and arbitrary functions. The functional forms of μ_ξ and θ_ξ depend on ξ also. In the following we use simplified notations $\Lambda^a_b(\xi)$, $\mu(\xi)$, and $\theta(\xi)$ for $\Lambda^a_b(\eta_\xi)$, $\mu_\xi(\eta_\xi)$, and $\theta_\xi(\eta_\xi)$, respectively. The conformal spinor $S(\xi)$ is defined on this tangent surface. Under (2.6), related quantities are transformed as

$$\begin{aligned} \xi^i &\rightarrow \xi'^i, \\ X^\mu(\xi) &\rightarrow X^\mu(\xi'), \\ V_i^a(\xi) &\rightarrow \Lambda^a_b(\xi) V_i^b(\xi'), \\ S^\mu(\xi) &\rightarrow \Omega(\xi) S^\mu(\xi'), \end{aligned} \quad (2.8)$$

where

$$\Omega(\xi) = \exp[-\frac{1}{2}\mu(\xi)] \exp[-\frac{1}{2}\theta(\xi)\sigma_3]. \quad (2.9)$$

The metric tensor g_{ij} is invariant under $\Pi_\xi C_\xi$.

Under the reparametrization T of the world sheet

$$T: \xi^i \rightarrow \xi'^i(\xi), \quad (2.10)$$

where ξ'^i is an arbitrary function, and both X^μ and S^μ are assumed to be scalar, whereas V_i^a is to be a vector with respect to the suffix i only, i.e.,

$$\begin{aligned} T: X^\mu(\xi) &\rightarrow X^\mu(\xi'), \\ S^\mu(\xi) &\rightarrow S^\mu(\xi'), \\ V_i^a(\xi) &\rightarrow \frac{\partial \xi'^j}{\partial \xi^i} V_j^a(\xi'). \end{aligned} \quad (2.11)$$

Therefore, g_{ij} is a second-rank tensor under T .

It may be worthwhile to point out that, in contrast with the case in gravitation theory, the covariant derivative is unnecessary in (2.2) due to the special situation in two-dimensional space.

A. The Model

The equations of motion for our string are given by

$$\begin{aligned} \frac{\partial}{\partial \xi^i} \left[\frac{1}{(-\det g)^{1/2}} \left(2g_{12} \frac{\partial g_{12}}{\partial (\partial_i S)} \right. \right. \\ \left. \left. - g_{11} \frac{\partial g_{22}}{\partial (\partial_i S)} - g_{22} \frac{\partial g_{11}}{\partial (\partial_i S)} \right) \right] \\ - \frac{1}{(-\det g)^{1/2}} \left(2g_{12} \frac{\partial g_{12}}{\partial S} - g_{11} \frac{\partial g_{22}}{\partial S} - g_{22} \frac{\partial g_{11}}{\partial S} \right) = 0, \end{aligned} \quad (2.12)$$

$$\begin{aligned} \frac{\partial}{\partial \xi^i} \left[\frac{1}{(-\det g)^{1/2}} \left(2g_{12} \frac{\partial g_{12}}{\partial (\partial_i X)} \right. \right. \\ \left. \left. - g_{11} \frac{\partial g_{12}}{\partial (\partial_i X)} - g_{22} \frac{\partial g_{11}}{\partial (\partial_i X)} \right) \right] = 0. \end{aligned} \quad (2.13)$$

One can easily confirm the invariance of (2.12) and (2.13) under

$$\Pi_\xi C_\xi \times T \quad (2.14)$$

because (2.12) and (2.13) are formally obtained from the Euler equation if one assumes the action

$$I = 2 \int (-\det g)^{1/2} d^2 \xi.$$

Now, the model will be determined if we specify what the *zweibein* field V_i^a is. We require that V_i^a is determined by the following subsidiary conditions:

$$g_{ij} = h(\xi) \eta_{ab} V_i^a V_j^b, \quad (2.15)$$

$$\partial_i X \hat{V}^i \sigma_1 S = 0, \quad (2.16)$$

where g_{ij} is given by (2.2) and

$$\hat{V}^i = \sigma^a V_a^i, \quad V_a^i V_i^b = \delta_a^b. \quad (2.17)$$

The inverse *zweibein* field V_a^i to V_i^a is necessary to make (2.16) covariant under (2.14). The new quantity $h(\xi) \eta_{ab}$ ($\eta_{11} = -\eta_{22} = 1$, $\eta_{12} = \eta_{21} = 0$) is the metric tensor on the tangent surface at ξ . Since we have five unknowns (V_i^a and h) and five independent relations [(2.15) and (2.16)], the *zweibein* field is solvable as a function of S^μ , X^μ , and their first derivatives.

Then, $V_i^a(S, X)$ so obtained must be substituted into (2.12) and (2.13) after the partial derivatives $\partial/\partial S^\mu$, $\partial/\partial X^\mu$, etc., are performed with the V 's being regarded as independent of S^μ and X^μ . This determines the model.

The equations of motion defined above are no longer derivable from a simple Lagrangian such as $L = 2(-\det g)^{1/2}$ because of the conditions (2.15) and (2.16). The invariance of (2.12) and (2.13) under (2.14), however, is guaranteed by the covariance of the subsidiary conditions.

The dynamical content of (2.15) may be inferred from an analogous relation $g_{\mu\nu} = V_\mu^a V_\nu^b \eta_{ab}$ [$a, \dots, \nu = 0, 1, 2, 3$] in gravitation theory. The

relation (2.16) implies the orthogonality of the polarization of the string constituent to the velocity at a given point of the string.

B. Choice of the Gauge

Contrary to the complicated appearance of the equations, the model is exactly solvable if a suitable gauge is chosen.

To do this one recalls that, in any two-dimensional geometry, one is able to choose the T -gauge where the metric tensor is diagonal, i.e.,

$$g_{ij} = \hbar \eta_{ab} V_i^a V_j^b = g(\xi) \eta_{ij}, \quad (2.18)$$

where $\eta_{11} = -\eta_{22} = 1$ and $\eta_{12} = \eta_{21} = 0$. Moreover, one has an extra gauge C_ξ at each ξ . It is an easy exercise to show that, if the ξ parametrization is orthogonal as in (2.18), the η_ξ coordinate on the tangent surface at ξ can be taken to be

$$V_i^a(\xi) = \delta_i^a \quad (2.19)$$

for all ξ .

In this gauge Eqs. (2.12), (2.13), (2.15), and (2.16) turn out to be

$$\partial_- S_1^\mu = 0, \quad (2.20)$$

$$\partial_+ S_2^\mu = 0, \quad (2.21)$$

$$\partial_+ \partial_- X^\mu = 0, \quad (2.22)$$

$$i S_1 \partial_+ S_1 + \frac{1}{2} (\partial_+ X)^2 = 0, \quad (2.22)$$

$$i S_2 \partial_- S_2 + \frac{1}{2} (\partial_- X)^2 = 0, \quad (2.22)$$

$$\partial_+ X S_1 + S_1 \partial_+ X = 0, \quad (2.23)$$

$$\partial_- X S_2 + S_2 \partial_- X = 0, \quad (2.23)$$

where we have introduced new parameters

$$\sigma^\pm = \frac{1}{2}(\xi^1 \pm \xi^2) \quad (2.24)$$

and

$$\partial_\pm = \frac{\partial}{\partial \sigma^\pm}.$$

It should be noted at this point that our model still has a gauge invariance under an over-all conformal transformation T_c , in which C_ξ and T are performed simultaneously keeping (2.19) unchanged. The transformation property of various fields are obtained from (2.8) and (2.11). Namely, corresponding to T_c ,

$$d\xi'^i = \Lambda^i_j(\xi) d\xi^j, \quad (2.25)$$

one obtains

$$X^\mu(\xi) \rightarrow X^\mu(\xi'), \quad (2.26)$$

$$S^\mu(\xi) \rightarrow \Omega(\xi') S^\mu(\xi'),$$

where

$$\Omega(\xi) = \exp\left[-\frac{1}{2}\lambda(\xi)\right] \exp\left[-\frac{1}{2}\varphi(\xi)\sigma_3\right], \quad (2.27)$$

$$\Lambda^i_j(\xi) = \exp[\lambda(\xi)] \begin{pmatrix} \cosh \varphi(\xi) & \sinh \varphi(\xi) \\ \sinh \varphi(\xi) & \cosh \varphi(\xi) \end{pmatrix}_{ij}.$$

It is (2.26) that is adopted in the usual formulation of NSM.

Finally we remark that (2.22) and (2.23) are conditions corresponding to L and G in dual models, respectively.

C. The Super Gauge

There remains a crucial point to be settled. A remarkable aspect of our model is the existence of an extra invariance.¹³ The Eqs. (2.20)–(2.23) are invariant under a new transformation

$$S_1 \rightarrow S_1 + f(\sigma_+) \partial_+ X, \quad (2.28)$$

$$S_2 \rightarrow S_2 + g(\sigma_-) \partial_- X,$$

$$X \rightarrow X - 4i \left(\int^{\sigma_+} f S'_1 d\sigma + \int^{\sigma_-} g S'_2 d\sigma \right), \quad (2.29)$$

where f and g are small but arbitrary functions which anticommute with S_i .¹⁴

The invariance of (2.20) and (2.21) is obvious.

The variations of (2.22) and (2.23) can be calculated straightforwardly to be

$$\delta[\text{Eq. (2.22)}] = -2i \partial_+ [f(\partial_+ X \cdot S)] = 0, \quad (2.30)$$

$$\delta[\text{Eq. (2.23)}] = 4i [i S_1 \partial_+ S_1 + \frac{1}{2} (\partial_+ X)^2] = 0, \quad (2.31)$$

and similar relations hold for S_2 and $\partial_- X$.

Note that the right-hand sides of (2.30) and (2.31) are proportional to the subsidiary conditions (2.23) and (2.22), respectively. The invariance implies that the Hamiltonian (although it is not explicitly given here) and the right-hand sides of (2.22) and (2.23) form a closed algebra through the Poisson bracket (see Sec. III).

As will be discussed in the next section, it is this algebraic property which allows us to choose a component of S_i^μ , say $S_i^+ = \frac{1}{2}(S_i^0 + S_i^{d-1})$, to be

$$S_i^+ = 0. \quad (2.32)$$

Similarly, the existence of T_c allows us to choose, say, $X^+ = \frac{1}{2}(X^0 + X^{d-1})$ to be

$$X^+ = \sqrt{2} p^+ \xi^1, \quad (2.33)$$

where p^+ can be identified with a light-cone component of the total string momentum.

We are now ready to solve the system if we impose the boundary conditions

$$\partial_2 X^\mu|_{\xi_2=0,\pi} = 0, \quad (2.34)$$

$$(S_1^\mu - S_2^\mu)|_{\xi_2=0,\pi} = 0. \quad (2.35)$$

Taking the transverse components \vec{X} and \vec{S}_i as independent, one then obtains from (2.20)–(2.23)

$$S_i^- = \frac{1}{\sqrt{2}p^+} \partial_+ \vec{X} \cdot \vec{S}_i, \\ \partial_+ X^- = \frac{1}{\sqrt{2}p^+} [i \vec{S}_1 \cdot \vec{\partial}_+ \vec{S}_1 + \frac{1}{2} (\partial_+ \vec{X})^2], \quad (2.36)$$

$$S_2^\mu(\xi_1, \xi_2) = S_1^\mu(\xi_1, -\xi_2),$$

where $X^- = \frac{1}{2}(X^0 - X^{d-1})$ and $S_1^- = \frac{1}{2}(S_1^0 - S_1^{d-1})$.

III. HAMILTONIAN AND LAGRANGIAN

The purpose of this section is to investigate the inverse problem to the one developed in the previous section. In a certain gauge, we have a set of simple equations of motion (2.20) and (2.21), and a set of subsidiary conditions (2.22) and (2.23). Starting from these equations as given, we try to obtain a Hamiltonian which contains all the necessary arbitrariness caused by the gauge transformations, i.e., both the conformal and the super gauges. Then the gauge-invariant action will be obtained. To do this let us first review the generalized Hamiltonian formalism developed by Dirac³ and Faddeev,⁴ on which our argument is based.

A. Generalized Hamiltonian Formalism

For simplicity, we restrict the discussion to a system having a finite number of degrees of freedom.

Let us consider a Lagrangian $L(q, \dot{q})$ having a number of gauge invariances. The canonical momentum is defined by

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \quad (i = 1, 2, \dots, N). \quad (3.1)$$

When L has gauge invariances, (3.1) is generally singular, so that one cannot solve for the \dot{q}_i 's as functions of q and p . Instead, one obtains a set of conditions among the canonical variables

$$\varphi_a(q, p) = 0 \quad (a = 1, 2, \dots, M), \quad (3.2)$$

which defines a surface A in the phase space. Dirac showed that one can construct such a Hamiltonian and subsidiary conditions that obey the following rules:

(a) φ_a ($a = 1, \dots, M$) are independent and irreducible in the sense that an arbitrary function f vanishing on the surface A can be expressed as

$$f = \sum_{a=1}^M u_a(q, p) \varphi_a(q, p). \quad (3.3)$$

(b) The φ 's and H form a closed algebra through Poisson brackets,

$$[\varphi_a, \varphi_b]_P = \sum_c C_{ab}^c \varphi_c, \\ [H, \varphi_a]_P = \sum_b C_{ab} \varphi_b, \quad (3.4)$$

where the C 's are arbitrary functions, and $[\ , \]_P$ stands for the Poisson bracket.

Dirac then showed that the Hamiltonian in the general gauge is given by

$$H' = H + \sum_{a=1}^M v_a \varphi_a, \quad (3.5)$$

with arbitrary functions v_a . To choose a special gauge means to choose a special function for v_a .

The equation of motion for any function of the canonical variables is given by

$$\dot{F} = [F, H']_P \\ = [F, H]_P + \sum_a v_a [F, \varphi_a]_P. \quad (3.6)$$

The subsidiary conditions $\varphi_a = 0$ must be imposed after the Poisson-bracket calculations are performed in (3.6). The observable f , which is defined to be gauge-independent on the surface (3.2), satisfies

$$[f, \varphi_a]_P = \sum_{b=1}^M d_{ab} \varphi_b. \quad (3.7)$$

Then, the equation of motion for the observable is independent of v_a on the surface (3.2) since the last term of (3.6) vanishes due to (3.7).

Properly adjusting the arbitrariness v_a ($a = 1, \dots, M$), one can choose the gauge-dependent quantities

$$\chi_a \quad (a = 1, 2, \dots, M) \quad (3.8)$$

to be certain fixed values, say, zero, if the condition

$$\det |[\chi_a, \varphi_b]_P| \neq 0 \quad (3.9)$$

is satisfied.

The condition (3.9) implies that the χ 's are gauge-dependent and mutually independent. Indeed, when a transformation is induced by $\sum u_m \varphi_m$, the variation of the χ 's is given by

$$\delta \chi_a = \sum_m u_m [\chi_a, \varphi_m]_P.$$

Inversely, for any given $\delta \chi_a$, u_m is uniquely determined if (3.9) is satisfied.

In our case, the problem is the inverse. As is seen below, we have H and φ_a satisfying Dirac's conditions (a) and (b). The Hamiltonian in the general gauge is given by (3.5). The gauge-invariant action can be determined by two steps. First one defines the Lagrangian in the phase space by

$$L_{ph} = \sum_{i=1}^N p_i \dot{q}_i - H' \\ = \sum_i p_i \dot{q}_i - H - \sum_a v_a \varphi_a. \quad (3.10)$$

Second, one eliminates p_i and v_a from L_{ph} using

the Euler equations for (3.10). An elegant method for this procedure will be shown in Appendix A.

B. The Hamiltonian of the Model

The Hamiltonian and the φ conditions in our case are

$$H = iS_1 \tilde{\partial}_2 S_1 - iS_2 \tilde{\partial}_2 S_2 + \frac{1}{2}[P^2 + (\partial_2 X)^2], \quad (3.11)$$

$$L_+ = 2iS_1 \tilde{\partial}_2 S_1 + \frac{1}{2}(P + \partial_2 X)^2, \quad (3.12)$$

$$L_- = -2iS_2 \tilde{\partial}_2 S_2 + \frac{1}{2}(P - \partial_2 X)^2, \quad (3.13)$$

$$G_+ = (P + \partial_2 X)S_1, \quad (3.14)$$

$$G_- = (P - \partial_2 X)S_2, \quad (3.15)$$

where L_{\pm} and G_{\pm} play the role of φ 's.

The relations (3.12)–(3.15) are equivalent to the left-hand sides of (2.22) and (2.23) except that $\partial_1 S_i^\mu$ is eliminated by the use of (2.20) and $\partial_1 X^\mu$ is replaced by P^μ . H is constructed in such a way that Hamilton's principle reproduces (2.20) and (2.21).

The Poisson brackets are required to be

$$\begin{aligned} [X^\mu(\xi^2), P^\nu(\xi^{2'})]_P &= 2\pi\delta(\xi^2 - \xi^{2'})\eta^{\mu\nu}, \\ [S_i^\mu(\xi^2), S_j^\nu(\xi^{2'})]_P &= -\frac{1}{2}i\pi\delta(\xi^2 - \xi^{2'})\eta^{\mu\nu}\delta_{ij} \end{aligned} \quad (3.16)$$

for $\xi^1 = \xi^{1'}$.

Some comments on the second relation may be necessary. The Poisson bracket for the anti-commuting field is not familiar in classical mechanics. We simply regard (3.16) as an algebraic rule for S_i , since we know that no trouble arises once the field is quantized. As a natural consequence, we assume

$$[S_i S_j, S_k]_P = S_i [S_j, S_k]_P - [S_i, S_k]_P S_j$$

for anticommuting fields. Another point we would like to mention is that the canonical momentum to S_i^μ is S_i^μ itself because it is a Majorana-type field. Finally, the factors 2π and $\frac{1}{2}\pi$ in (3.16) arise owing to our unorthodox normalization.

The algebraic condition (b) can now be confirmed by the use of (3.15) and (3.16);

$$[L_{\pm}(f), L_{\pm}(g)]_P = 4\pi L_{\pm}(fg' - f'g), \quad (3.17)$$

$$[L_+(f), L_-(g)]_P = 0, \quad (3.18)$$

$$[G_{\pm}(f), G_{\pm}(g)]_P = -i\pi L_{\pm}(fg), \quad (3.19)$$

$$[G_+(f), G_-(g)]_P = 0, \quad (3.20)$$

$$[L_{\pm}(f), G_{\pm}(g)]_P = -4\pi G_{\pm}(f'g), \quad (3.21)$$

$$[L_{\pm}(f), G_{\mp}(g)]_P = 0, \quad (3.22)$$

where

$$L_{\pm}(f) = \int L_{\pm}(\xi^1, \xi^2) f(\xi^2) d\xi^2, \quad (3.23)$$

$$G_{\pm}(g) = \int G_{\pm}(\xi^1, \xi^2) g(\xi^2) d\xi^2,$$

with arbitrary test functions $f(\xi^2)$ and $g(\xi^2)$.

Throughout the arguments in this and the next section, we fix ξ^1 at a common value, say, $\xi^1 = 0$, unless specified otherwise. Since $H = \frac{1}{2}(L_+ - L_-)$, (3.17)–(3.22) prove Dirac's second condition (b). As far as the first condition is concerned, we can satisfy it by restricting the functional space properly.

As a consequence, the Hamiltonian including the arbitrariness of L_{\pm} and G_{\pm} gauges turns out to be

$$H' = H + \lambda_+ L_+ + \lambda_- L_- + \rho_+ G_+ + \rho_- G_-. \quad (3.24)$$

As a special case where $\lambda_{\pm} = \rho_{\pm} = 0$, H' gives (2.20) and (2.21). As a matter of fact, H can be taken to be zero without losing generality, because the substitution of $H = \frac{1}{2}(L_+ - L_-)$ eliminates the first term in the right-hand side of (3.24), and subsequently changes λ_{\pm} to new values.

The equation of motion for any function F of the canonical variables is given by

$$\frac{d}{d\xi^1} F \equiv \dot{F} = [F, \bar{H}]_P, \quad (3.25)$$

with

$$\bar{H} = \frac{1}{2\pi(\xi^2 - \xi'^2)} \int_{\xi^2}^{\xi'^2} H'(\xi^1, \xi'^2) d\xi'^2. \quad (3.26)$$

The subsidiary conditions

$$L_{\pm} = 0, \quad (3.27)$$

$$G_{\pm} = 0 \quad (3.28)$$

must be imposed *after* the Poisson-bracket calculations are performed in (3.25).

Finally we emphasize that it is the Hamiltonian (3.24) that defines the path integration in the functional formalism.⁴ The NSM scattering amplitude based on this formalism will be discussed in a separate paper.

C. The Ghost-Free Gauge

So far the discussion has been free from any particular choice of gauge, i.e., λ_{\pm} and ρ_{\pm} are arbitrary. What one has to do next is to find a particular gauge where (i) the ghost-free conditions must be satisfied for arbitrary ξ^1 and ξ^2 [this corresponds to the choice of χ_a in (3.8)], and (ii) the equations of motion must be simple enough to be solvable. The second condition is, of course, purely technical.

In our case the gauge with $\lambda_{\pm} = \rho_{\pm} = 0$ turns out to be the correct choice. The equations of motion are determined from (3.25) as

$$\begin{aligned} \dot{X}^\mu &= P^\mu, \\ \dot{P}^\mu &= X^{\mu'}, \end{aligned} \quad (3.29)$$

$$\begin{aligned}\dot{S}_1^\mu - S_1^{\mu'} &= 0, \\ \dot{S}_2^\mu + S_2^{\mu'} &= 0,\end{aligned}\quad (3.30)$$

where the dot and prime mean $\partial/\partial\xi^1$ and $\partial/\partial\xi^2$, respectively.

Now we specify our χ 's which are defined in (3.8). In order to make the model ghost-free, we take the following Lorentz-noncovariant χ conditions:

$$P^+ = \frac{1}{\sqrt{2}}(P^0 + P^{d-1}) = \sqrt{2}p^+, \quad (3.31)$$

$$X^{+'} = \frac{1}{\sqrt{2}}(X^{0'} + X^{d-1'}) = 0, \quad (3.32)$$

$$S_1^+ = \frac{1}{\sqrt{2}}(S_1^0 + S_1^{d-1}) = 0, \quad (3.33)$$

$$S_2^+ = \frac{1}{\sqrt{2}}(S_2^0 + S_2^{d-1}) = 0,$$

for all ξ^1 and ξ^2 .

It may be instructive to examine the relation between these conditions and the equations of motion. First, the relations (3.31)–(3.33) are possible at the initial point $\xi^1 = \xi_0^1$: The equations of motion (3.29) and (3.30) and the subsidiary conditions (3.27) and (3.28) are invariant under the transformation

$$F(\xi^1, \xi^2) \rightarrow F(\xi^1, \xi^2) + \int [F(\xi^1, \xi^2), u_+(\sigma)L_+(\xi^1, \sigma) + u_-(\sigma)L_-(\xi^1, \sigma) + w_+(\sigma)G_+(\xi^1, \sigma) + w_-(\sigma)G_-(\xi^1, \sigma)]_P d\sigma, \quad (3.34)$$

where F is any canonical variable, and u_\pm and w_\pm are arbitrary ξ^1 -independent functions. One can see that (3.31)–(3.33) can be satisfied by appropriate choices of u_\pm and w_\pm , without changing the form of the equations of motion. Second, Eqs. (3.29) and (3.20) tell us that (3.31)–(3.33) hold for arbitrary ξ^1 .

Substituting (3.31)–(3.33) into (3.27) and (3.28), we arrive at the same results (2.36) as obtained in the previous section.

D. The Invariant Lagrangian

As is shown above, practical problems do not necessarily require the action principle. It may be, however, interesting to construct the invariant Lagrangian. A possible method is to start from the Lagrangian in the phase space defined by

$$L_{ph} = P\dot{X} + i[S_1, \dot{S}_1] + i[S_2, \dot{S}_2] - H'. \quad (3.35)$$

The Euler equations are obtained by regarding P , X , S_i , λ_\pm , and ρ_\pm as independent. As demonstrated in Ref. 5, one can construct the invariant Lagrangian if one eliminates P , λ_\pm , and ρ_\pm from L_{ph} by the use of the Euler equations. Although we have obtained the Lagrangian, we are skeptical of its usefulness because of its complicated structure. The interested reader can refer to Appendix A.

IV. QUANTIZATION AND LORENTZ COVARIANCE

We are ready to quantize our model. According to the usual rule, we replace the Poisson brackets (3.15) and (3.16) by the commutator for X^μ and anticommutator for S^μ ;

$$[X^i(\xi^2), P^j(\xi^{2'})] = 2\pi i g^{ij} \delta(\xi^2 - \xi^{2'}), \quad (4.1)$$

$$\{S_a^i(\xi^2), S_b^j(\xi^{2'})\} = \frac{1}{2}\pi \delta_{ab} g^{ij} \delta(\xi^2 - \xi^{2'}). \quad (4.2)$$

The quantization condition is imposed only for the transverse components of X and S , since the others are expressed in terms of the former as shown in (2.36).

Taking the world sheet as a strip with width $0 \leq \xi^2 \leq \pi$, and imposing the boundary conditions (2.34) and (2.35), we make a normal-mode expansion of the canonical fields as

$$X^i(\xi^1, \xi^2) = \sum_{n \neq 0} \frac{i}{n} a_n^i e^{-in\xi^1} \cos(n\xi^2) + \frac{x^i}{\sqrt{2}} + \sqrt{2}p^i \xi^1, \quad (4.3)$$

$$S_1^i(\xi^1, \xi^2) = \frac{1}{2} \sum_k b_k^i e^{-ik(\xi^1 + \xi^2)}, \quad (4.4)$$

$$S_2^i(\xi^1, \xi^2) = \frac{1}{2} \sum_k b_k^i e^{-ik(\xi^1 - \xi^2)}, \quad (4.5)$$

where n runs through all integers except zero, while k runs through all half-integers. Hermiticity requires $a_{-n} = a_n^\dagger$ and $b_{-k} = b_k^\dagger$. In terms of these operators, the commutation relations are

$$[a_m^i, a_n^j] = m \delta_{m, -n} \delta^{ij}, \quad (4.6)$$

$$\{b_k^i, b_l^j\} = \delta_{k, -l} \delta^{ij}, \quad (4.7)$$

$$[p^i, x^j] = -i \delta^{ij}, \quad (4.8)$$

$$[p^i, p^j] = [x^i, x^j] = 0.$$

From (2.36) and (3.31)–(3.33), the light-cone components are

$$X^- = \frac{i}{\sqrt{2}p^+} \sum_{n \neq 0} \frac{L_n}{n} e^{-in\xi^1} \cos(n\xi^2) + \frac{(L_0 - \alpha_0)\xi^1}{\sqrt{2}p^+} + \frac{x^-}{\sqrt{2}}, \quad (4.9)$$

$$S_1^- = \frac{1}{2\sqrt{2}p^+} \sum_i G_i e^{-iI(\xi^1 + \xi^2)}, \quad (4.10)$$

$$S_2^- = \frac{1}{2\sqrt{2}p^+} \sum_i G_i e^{-iI(\xi^1 - \xi^2)},$$

where

$$L_n = \frac{1}{2} \left[\sum_{m,i} : a_{n-m}^i a_m^i : + \sum_{i,i} (l - \frac{1}{2}n) : b_{n-i}^i b_i^i : \right], \quad (4.11)$$

$$G_l = \frac{1}{2} \sum_k : a_{l-k}^i b_k^i :. \quad (4.12)$$

Here we write $\sqrt{2} p^i = a_0^i$.

In obtaining the above expressions, the only point one has to be careful of is α_0 in (4.9). As will be shown in (4.25) below, Lorentz covariance requires $\alpha_0 = \frac{1}{2}$.

Since the gauge we adopted is not manifestly Lorentz-covariant, we must ensure the covariance by explicit construction of the Poincaré generators $\mathfrak{M}^{\mu\nu}$ and \mathcal{P}^μ , which obey the algebraic relations

$$[\mathcal{P}^\mu, \mathcal{P}^\nu] = 0, \quad (4.13)$$

$$[\mathcal{P}^\mu, \mathfrak{M}^{\alpha\beta}] = i(-g^{\mu\alpha}\mathcal{P}^\beta + g^{\mu\beta}\mathcal{P}^\alpha), \quad (4.14)$$

$$[\mathfrak{M}^{\mu\nu}, \mathfrak{M}^{\alpha\beta}] = i(g^{\mu\alpha}\mathfrak{M}^{\nu\beta} - g^{\nu\alpha}\mathfrak{M}^{\mu\beta} - g^{\mu\beta}\mathfrak{M}^{\nu\alpha} + g^{\nu\beta}\mathfrak{M}^{\mu\alpha}). \quad (4.15)$$

Since we have identified X^μ with the position of the string, the displacement generator should be given by

$$\mathcal{P}^\mu = \frac{1}{\sqrt{2}} \frac{1}{\pi} \int_0^\pi P^\mu(\xi^2) d\xi^2, \quad (4.16)$$

which gives, when the solutions are substituted,

$$\mathcal{P}^\mu = p^\mu, \text{ for } \mu = i \text{ and } +, \quad (4.17)$$

and

$$\mathcal{P}^- = \frac{1}{p^+} (L_0 - \alpha_0). \quad (4.18)$$

A natural choice for the Lorentz generators may be

$$\mathfrak{M}^{\mu\nu} = \frac{1}{\pi} \int_0^\pi d\xi^2 \left[\frac{1}{2} (X^\mu P^\nu - P^\mu X^\nu) - \frac{1}{2} (X^\nu P^\mu - P^\nu X^\mu) + 2(S_1^\mu S_1^\nu + S_2^\mu S_2^\nu) \right]. \quad (4.19)$$

In terms of normal-mode operators, $\mathfrak{M}^{\mu\nu}$ turns out to be

$$\mathfrak{M}^{ij} = \sum_{n=1}^\infty \frac{-i}{n} (a_{-n}^i a_n^j - a_{-n}^j a_n^i) + x^i p^j - x^j p^i - i \sum_k : b_{-k}^i b_k^j :, \quad (4.20)$$

$$\mathfrak{M}^{i-} = \frac{-i}{\sqrt{2} p^+} \sum_{n=1}^\infty \frac{a_{-n}^i L_n - L_{-n} a_n^i}{n} + \frac{1}{4p^+} [x^i (L_0 - \alpha_0) + (L_0 - \alpha_0) x^i] - \frac{1}{\sqrt{2}} x^- a_0^i + \frac{i}{\sqrt{2} p^+} \sum_{k=\frac{1}{2}}^\infty (G_{-k} b_k^i - b_{-k}^i G_k), \quad (4.21)$$

$$\mathfrak{M}^{i+} = x^i p^+, \quad (4.22)$$

$$\mathfrak{M}^{+-} = -\frac{1}{2} (x^- p^+ + p^+ x^-). \quad (4.23)$$

In the proof of (4.13)–(4.15), one finds a situation similar to the one encountered in Ref. 5. All algebras other than $[\mathfrak{M}^{i-}, \mathfrak{M}^{j-}]$ are easily confirmed as is shown in Appendix B. The latter turns out to be

$$[\mathfrak{M}^{i-}, \mathfrak{M}^{j-}] = -\frac{1}{2(p^+)^2} \left\{ \sum_{n=1}^\infty \left[n \left(1 - \frac{d-2}{8} \right) - \frac{1}{n} \left(2d_0 - \frac{d-2}{8} \right) \right] (a_{-n}^i a_n^j - a_{-n}^j a_n^i) + \sum_{k=\frac{1}{2}}^\infty \left[4k^2 \left(1 - \frac{d-2}{8} \right) - \left(2\alpha_0 - \frac{d-2}{8} \right) \right] (b_{-k}^i b_k^j - b_{-k}^j b_k^i) \right\}, \quad (4.24)$$

where d is the dimension of space-time. Lorentz covariance, therefore, is verified if

$$\alpha_0 = \frac{1}{2} \quad (4.25)$$

and

$$d = 10. \quad (4.26)$$

These results agree with those obtained in the dual resonance model.¹¹ Goddard, Rebbi, and Thorn¹⁵ also obtained (4.26) by constructing the generators of $O(9)$ in a different method. We obtain the generators of $O(9,1)$ as well as the translation operators \mathcal{P}_μ in the framework of the string picture.

Spectra of the string are determined by the

mass operator

$$M^2 = (\mathcal{P}^\mu)^2 = R - \frac{1}{2}, \quad (4.27)$$

where

$$R = \sum_{n=1}^\infty a_{-n}^i a_n^i + \sum_{i=\frac{1}{2}}^\infty l b_{-i}^i b_i^i. \quad (4.28)$$

The spectra coincide with those of NSM. The ghosts are not contained because all negative-normed components are eliminated by (4.9) and (4.10). It is interesting that the tachyon at $(\text{mass})^2 = -1$ also does not appear in the spectrum relation (4.27), and therefore the decoupling has nothing to do with the type of vertex operator adopted in NSM.

V. CONCLUDING REMARKS

Our model is that of a massless string made of spinning material. As is shown in (4.4) and (4.5), the collective motion S^μ of the constituents has two components, the right-going wave S_1^μ and the left-going wave S_2^μ , propagating with the speed of light. At both ends of the string S^μ is totally reflected. One may imagine a massless flexible tube in which photons are confined.

We have demonstrated two different (but equivalent) formalisms concerning the string motion. Each has its own advantages. The manifestly covariant formalism is better, not only in the transparency of the gauge invariance, but also in its visual nature of description. In contrast, in the Hamiltonian formalism it is hardly possible to imagine that a Hamiltonian such as (3.24) has anything to do with the string motion. For practical purposes, however, the latter is superior to the former. Moreover, the formalism is so general that it is applicable to other dual models, i.e., the Bardakci-Halpern model,¹⁶ the Thirring-type model,¹⁷ etc. The invariant Lagrangian,

$$\int \mathcal{D}X^\mu \mathcal{D}P^\mu \mathcal{D}S_1^\mu \mathcal{D}S_2^\mu \mathcal{D}\lambda \mathcal{D}\omega \mathcal{D}\rho_+ \mathcal{D}\rho_- \times \exp\left(\frac{i}{2\pi} \int d^2\xi \{P\dot{X} + i[S_1, \dot{S}_1] + i[S_2, \dot{S}_2] + \frac{1}{2}\lambda(L_+ + L_-) + \frac{1}{2}\omega(L_+ - L_-) + \rho_+ G_+ + \rho_- G_-\}\right), \quad (\text{A1})$$

where $\lambda_\pm = \frac{1}{2}(\lambda \pm \omega)$. In the phase space, both X^μ and P^μ are considered to be independent. If one performs the functional integration with respect to P^μ , λ , ω , and ρ_\pm , the integral (A1) gives the exponential of the action with a certain integration measure. A convenient order of performing the integrations is to integrate first with respect to P^μ , then ω and ρ_\pm , and finally λ .

In performing the integration about ρ_\pm , we separate an anticommuting component $\bar{\kappa}$ out of ρ_\pm in such a way that $\rho_\pm = \rho'_\pm \bar{\kappa}$, where ρ'_\pm are the usual commuting functions and $\bar{\kappa}$ is assumed to be a constant satisfying $\bar{\kappa}^2 = 1$.

The action turns out to be

$$I = \frac{1}{2\pi} \int L d^2\xi \quad (\text{A2})$$

and

$$L = 2(AB)^{1/2} + C, \quad (\text{A3})$$

where

$$A = -\frac{1}{2}(a|W|a) + \frac{1}{2}\dot{X}^2, \quad (\text{A4})$$

$$B = -\frac{1}{2}(b|W|b) - i[S_1, S'_1] + i[S_2, S'_2] - \frac{1}{2}X'^2, \quad (\text{A5})$$

which mediates the two formalisms, usually turns out to be extremely complicated as in our case.

The construction of the scattering amplitude will be discussed in a separate paper.

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APPENDIX A

In this appendix we derive an invariant action. Since the Lagrangian in the phase space (3.35) contains all the arbitrariness λ_\pm and ρ_\pm induced by the gauge group (3.17)–(3.22), the gauge-invariant action may be obtained if λ_\pm and ρ_\pm are all eliminated. To do this we use the functional integration method.¹⁸

The path integral in the phase space will be given by

$$C = i[S_1, \dot{S}_1] + i[S_2, \dot{S}_2]. \quad (\text{A6})$$

Furthermore, a , b , and W are given by

$$W = M^{-1}, \quad (\text{A7})$$

$$M = \begin{bmatrix} (X')^2 & (\bar{\kappa}S_1 \cdot X') & (\bar{\kappa}S_2 \cdot X') \\ (\bar{\kappa}S_1 \cdot X') & -S_1^2 & -S_1 \cdot S_2 \\ (\bar{\kappa}S_2 \cdot X') & -S_1 \cdot S_2 & -S_2^2 \end{bmatrix}, \quad (\text{A8})$$

$$\tilde{a} = (\dot{X}X', \bar{\kappa}S_1 \cdot \dot{X}, \bar{\kappa}S_2 \cdot \dot{X}), \quad (\text{A9})$$

$$\tilde{b} = (i[S_1, S'_1] + i[S_2, S'_2], \bar{\kappa}S_1 \cdot X', -\bar{\kappa}S_2 \cdot X'). \quad (\text{A10})$$

At the final stage, $\bar{\kappa}$ can be eliminated due to $\bar{\kappa}^2 = 1$. One can readily confirm that (A3) reduces to $[(\dot{X}X')^2 - \dot{X}^2 X'^2]^{1/2}$ in the limit of $S_i \rightarrow 0$.

The integration measure in the path integral formula is now given by

$$\frac{1}{(\det M)^{1/2}} \left(\frac{1}{B}\right)^{1/2} \exp[-2(AB)^{1/2}] \times \left(\frac{\partial}{\partial A}\right)^{(d-2)/2} \exp[2(AB)^{1/2}]. \quad (\text{A11})$$

The Lagrangian so obtained may not be unique. From the covariant equations of motion (2.12) and (2.13), together with the subsidiary conditions (2.15) and (2.16), one can construct, in principle, a Lagrangian by eliminating the *zweibein* fields. Although both Lagrangians show all the gauge invariances required, the forms may be different.

APPENDIX B

We prove Eq. (4.14). The proof is straightforward, but tedious. Since the calculation is similar to that of Refs. 5 and 15, we do not give the details, but only comment that the crucial point in the calculation is the ordering of operators. To make the generators $\mathfrak{M}^{\mu\nu}$ well defined, we take normal-ordered products of operators. This is the reason why the commutator $[\mathfrak{M}^{i-}, \mathfrak{M}^{j-}]$ does not vanish in general.

We list some of the useful formulas to compute the commutator:

$$\begin{aligned} [a_m^i, a_n^j] &= m \delta_{m,-n} \delta^{ij}, \\ \{b_k^i, b_l^j\} &= \delta_{k,-l} \delta^{ij}, \\ a_0^i &\equiv \sqrt{2} p^i, \\ [p^i, x^j] &= -i \delta^{ij}, \\ [p^+, x^-] &= i, \\ [a_m^i, L_n] &= m a_{m+n}^i, \\ [b_k^i, L_n] &= (k + \frac{1}{2}) b_{k+n}^i, \\ [a_m^i, G_l] &= m b_{m+l}^i, \\ \{b_k^i, G_l\} &= a_{k+l}^i, \\ [x^i, L_n] &= i a_n^i, \\ [x^i, G_l] &= i b_l^i, \\ [L_n, L_m] &= (n-m) L_{n+m} + \frac{1}{8} (d-2) n(n^2-1) \delta_{n,-m}, \\ [L_n, G_l] &= (\frac{1}{2}n - l) G_{n+l}, \\ \{G_l, G_k\} &= 2 L_{l+k} + \frac{1}{2} (d-2) (l - \frac{1}{2}) (l + \frac{1}{2}) \delta_{l,-k}. \end{aligned}$$

Using these formulas, we can find Eq. (4.14) after some lengthy calculations.

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