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PROPs for involutive monoids and involutive bimonoids. (English) Zbl 07266037

Theory Appl. Categ. 35, 1564-1575 (2020).

The categorification of algebras over a unital commutative ring k was first introduced by *M. Markl* [J. Pure Appl. Algebra 113, No. 2, 195–218 (1996; Zbl 0865.18011)] as algebras over a PROP in order to study the deformation theory of algebras, where PROPs were defined in terms of generators and relations, whose categories of algebras are equivalent to that of associative algebras, that of commutative algebras and that of bialgebras over k.

T. Pirashvili [Cah. Topol. Géom. Différ. Catég. 43, No. 3, 221–239 (2002; Zbl 1057.18005)] presented an explicit description of a PROP categorifying associative algebras, commutative algebras and bialgebras within the category of vector spaces over a field, which was constructed from the category of non-commutative sets introduced by *B. L. Feigin* and *B. L. Tsygan* [Lect. Notes Math. 1289, 67–209 (1987; Zbl 0635.18008)], putting the generalized Q-construction of *Z. Fiedorowicz* and *J.-L. Loday* [Trans. Am. Math. Soc. 326, No. 1, 57–87 (1991; Zbl 0755.18005)] to use. On the other hand, an alternative approach, making use of distributive laws for PROPs, was given by *S. Lack* [Theory Appl. Categ. 13, 147–163 (2004; Zbl 1062.18007)], where Pirashvili's PROP was described as a composite of the PRO of finite ordinals and its opposite category, the result being shown to be so general as to hold for bimonoids in a symmetric monoidal category.

This paper combines both of these methods, introducing the PROP of *involutive non-commutative sets*, from which the algebras in a symmetric monoidal category constructed off a composite PROP are *involutive bimonoids*.

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MSC:

16T10 Bialgebras
18M05 Monoidal categories, symmetric monoidal categories
18M85 Polycategories/dioperads, properads, PROPs, cyclic operads, modular operads

Keywords:

involutive non-commutative sets; bimonoid; bialgebras; PROP; symmetric monoidal categories

Full Text: Link

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