Carqueville, Nils; Montiel Montoya, Flavio
Extending Landau-Ginzburg models to the point. (English) Zbl 07263740
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Fully extended Topological Quantum Field Theory (TQFT) is an attempt to capture the quantum field theoretic notion of locality in a simplified rigorous setting as well as a source of functorial topological invariants, being formulated, in dimension n, as a symmetric monoidal (∞, n)-functor from a certain category of bordisms with extra geometric structure to some symmetric monoidal (∞, n)-category $\mathcal{C}$. The requirement that such functors must respect structure and relations among bordisms of all dimensions from 0 to n is pretty restrictive. In particular, the *cobordism hypothesis* [*J. C. Baez* and *J. Dolan*, J. Math. Phys. 36, No. 11, 6073–6105 (1995; Zbl 0863.18004); *J. Lurie*, in: Current developments in mathematics, 2008. Somerville, MA: International Press. 129–280 (2009; Zbl 1180.81122); *D. Ayala* and *J. Francis*, "The cobordism hypothesis", Preprint, arXiv:1705.02240] claims that, in the case of bordisms with framings, a TQFT is aleady determined by what it assigns to the point, and that fully extended TQFTs with values in $\mathcal{C}$ are equivalent to fully dualizable objects in $\mathcal{C}$. On the other hand, fully extended TQFTs on oriented bordisms are argued to be described by homotopy fixed points of an induced $SO(n)$-action on fully dualizable objects.

This paper is concerned with fully extended TQFTs in dimension $n = 2$. Following [*C. J. Schommes-Pries*, "The classification of two-dimensional extended topological field theories", Preprint, arXiv:1112.1000; http://www.chimaira.org/archive/DualsTricategories_TheThesis.pdf], the authors take an *extended framed* (or *oriented*) 2 *-dimensional TQFT with values in a symmetric monoidal bicategory* $\mathcal{B}$ (called the *target*) to be a symmetric monoidal 2-functor

$$\mathcal{Z} : \mathrm{Bord}^{\sigma}_{2,1,0} \to \mathcal{B}$$

where $\sigma = \mathrm{fr}$ or $\sigma = \mathrm{or}$, and $\mathrm{Bord}^{\sigma}_{2,1,0}$ is the bicategory of points, 1-manifolds with boundary and 2-manifolds with corners.

On the one hand, the dominant example of the target $\mathcal{B}$ is the bicategory Alg_k of finite-dimensional k-algebras, finite-dimensional bimodules and bimodule maps, where k is some field. With due regard to the cobordism hypothesis, one can see that extended framed TQFTs with values in Alg_k are classified by finite-dimensional separable k-algebras [*C. J. Schommes-Pries*, "The classification of two-dimensional extended topological field theories", Preprint, arXiv:1112.1000; *J. Lurie*, in: Current developments in mathematics, 2008. Somerville, MA: International Press. 129–280 (2009; Zbl 1180.81122)], while in the oriented case the classification is in terms of separable symmetric Frobenius k-algebras [*J. Hesse* et al., Theory Appl. Categ. 32, 652–681 (2017; Zbl 1377.18003)].

On the other hand, non-separable algebras arise prominently in non-extended TQFTs

$$\mathcal{Z}_{\mathrm{ne}} : \mathrm{Bord}^{\sigma}_{2,1} \to \mathcal{V}$$

which are equivalent to commutative Frobenius algebras in a symmetric monoidal 1-category $\mathcal{V}$. Important examples are the categories of vector spaces, possibly with $\mathbb{Z}_2$- or $\mathbb{Z}$-grading. In $\mathcal{V} = \mathrm{Vect}_k^{\mathbb{Z}_2}$ or $\mathcal{V} = \mathrm{Vect}_k^{\mathbb{Z}}$, Dolbeault cohomologies of Calabi-Yau manifolds serve as exmaples of non-separable commutative Frobenius algebras describing B-twisted sigma models. The Jacobi algebras

$$k[x_1, \dots, x_n] / (\partial_1 W, \dots, \partial_n W)$$

of isolated singularities described by polynomials W are another class of examples of generically non-separable Frobenius algebras whose associated TQFTs are Landau-Ginzburg models with potential W.

The authors are interested in the question how sigma models and Landau-Ginzburg models relate to fully extended TQFTs. A non-extended 2-dimensional TQFT

$$\mathcal{Z}_{\mathrm{ne}} : \mathrm{Bord}^{\sigma}_{2,1} \to \mathcal{B}$$

is to be extended to the point provided that there is a symmetric monoidal bicategory $\mathcal{B}$ and an extended TQFT

$$\mathcal{Z}:\mathrm{Bord}^{\sigma}_{2,1,0}\to\mathcal{B}$$

with $\mathbb{I}_{\mathcal{B}}\in\mathcal{B}$ the unit object and $\phi=\mathbb{I}_{\mathrm{Bord}^{\sigma}_{2,1,0}}$ holding

$$\mathcal{V}\cong\mathrm{End}_{\mathcal{B}}\left(\mathbb{I}_{\mathcal{B}}\right)\ \text{ and }\ \mathcal{Z}_{\mathrm{ne}}\cong\mathcal{Z}\left|\mathrm{End}_{\mathrm{Bord}^{\sigma}_{2,1,0}}\left(\phi\right)\right.$$

The authors hold the creed that the extendability of the known classes of non-separable TQFTs is captured by the motto that if a non-extended 2-dimensional TQFT$\mathcal{Z}_{\mathrm{ne}}$ is a restriction of an appropriate defect TQFT $\mathcal{Z}^{\mathrm{def}}_{\mathrm{ne}}$, then $\mathcal{Z}_{\mathrm{ne}}$ can be extended to the point, at least as a framed theory, with the bicategory $\mathcal{B}_{\mathcal{Z}^{\mathrm{def}}_{\mathrm{ne}}}$ associated to $\mathcal{Z}^{\mathrm{def}}_{\mathrm{ne}}$ as target. This paper, consisting of three sections, aims to make this precise for Landau-Ginzburg models. §2 collects the data that the bicategory of Landau-Ginzburg models $\mathcal{LG}$ with a symmetric monoidal structure in which every object has a dual and every 1-morphism has left and right adjoints.

§3 addresses TQFTs with values in $\mathcal{LG}$ and $\mathcal{LG}^{\mathrm{gr}}$. The authors firstly review framed and oriented 2-1-0-extended TQFTs and their classification in terms of fully dualizable objects and trivializable Serre automorphisms, respectively. It is then observed that every object

$$W\equiv\left(k\left[x_1,\dots,x_n\right],W\right)$$

in $\mathcal{LG}$ or $\mathcal{LG}^{\mathrm{gr}}$ gives rise to an extended framed TQFT, and it is precisely shown when W determines an oriented theory. It is also shown how the extended framed or oriented TQFTs recover the Jacobi algebras Jac_W as commutative Frobenius k-algebras, and it is explained how a construction of *M. Khovanov* and *L. Rozansky* [Fundam. Math. 199, No. 1, 1–91 (2008; Zbl 1145.57009); Fundam. Math. 199, No. 1, 1–91 (2008; Zbl 1145.57009)] is to be recovered as a special case of the cobordism hypothesis.

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MSC:

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57R56 Topological quantum field theories (aspects of differential topology)
81T45 Topological field theories in quantum mechanics
16G20 Representations of quivers and partially ordered sets

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