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Diffeological vector spaces. (English summary)
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This paper considers several classes of diffeological vector spaces, the containment between some two of which is the main topic of the paper. Any vector space has a smallest diffeology making it a diffeological vector space (called the fine diffeology). The collection of fine diffeological vector spaces is denoted by $\mathcal{FV}$. The collection of diffeological vector spaces whose finite-dimensional subspaces are all fine is written $\mathcal{FFV}$. A diffeological vector space V is said to be in $\mathcal{SD}$ (resp. $\mathcal{SV}$) if the smooth (resp. smooth linear) functionals $V \to \mathbb{R}$ separate points of V. A diffeological vector space V is said to be in $\mathcal{PV}$ if for every linear subduction $f\colon W_1 \to W_2$ and every smooth linear map $g\colon V \to W_2$, there exists a smooth linear map $h\colon V \to W_1$ such that $g = f \circ h$. The authors write $\mathcal{DV}$ for the collection of diffeological vector spaces V such that a function $p\colon \mathbb{R}^n \to V$ is smooth if and only if $l \circ p\colon \mathbb{R}^n \to \mathbb{R}$ is smooth for each smooth linear functional $l\colon V \to \mathbb{R}$. The authors write $\mathcal{HT}$ for the collection of diffeological vector spaces whose natural topologies (called D-topologies) are Hausdorff.

The main results of the paper are:

1. It holds that $\mathcal{FV} \subset \mathcal{PV} \subset \mathcal{SV} \subseteq \mathcal{SD} \subset \mathcal{FFV}$, and $\mathcal{SD} \subset \mathcal{HT}$.
2. When restricted to finite-dimensional vector spaces, the collections $\mathcal{FV}$, $\mathcal{PV}$, $\mathcal{SV}$, $\mathcal{SD}$ and $\mathcal{FFV}$ agree.
3. When restricted to V in $\mathcal{DV}$, the collections $\mathcal{SV}$, $\mathcal{SD}$, $\mathcal{FFV}$ and $\mathcal{HT}$ agree.

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